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Payment Instruments and Collateral in the Interbank Payment System

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Abstract

This paper presents a three-period model to analyze the need for bank reserves in the presence of other liquid assets like Treasury securities. If a pair of banks settle bank transfers without bank reserves, they must prepare extra liquidity for interbank payments, because depositors' demand for timely payments causes a hold-up problem in the bilateral settlement of bank transfers. In light of this result, the interbank payment system provided by the central bank can be regarded as an implicit interbank settlement contract to save liquidity. The central bank is necessary for this contract as the custodian of collateral. Bank reserves can be characterized as the balances of liquid collateral submitted by banks to participate into this contract. This result explains the rate-of-return dominance puzzle and the need for substitution between bank reserves and other liquid assets simultaneously. The optimal contract is the floor system, not only because it pays interest on bank reserves, but also because it eliminates the over-the-counter interbank money market. The model indicates it is efficient if all banks share the same custodian of collateral, which justifies the current practice that a public institution provides the interbank payment system.

Keywords: bank reserves; large value payment system; interbank money market; clearing house; collateral; legal tender.
JEL codes: E41, E42.

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1 Introduction

Base money consists of currency and bank reserves. Banks hold bank reserves not merely to satisfy a reserve requirement, but also to make interbank payments to settle bank transfers between their depositors. In fact, the daily transfer of bank reserves in a country tends to be as large as a sizable fraction of annual GDP.[1] Also, several countries have abandoned a reserve requirement.[2] Banks in these countries still use bank reserves to settle bank transfers.

Banks, however, obtain bank reserves from the central bank in exchange for liquid assets through open market operations. The representative examples of these assets are Treasury bills and bonds, which are the liquid stores of wealth for institutional investors. In theory, any liquid asset should be able to serve as a payment instrument. Thus, the active use of bank reserves by banks poses two questions: Why do banks need payment instruments separately from other liquid assets? Why does the central bank need to replace liquid assets with liquid assets?

These questions are related to the legal restrictions theory of the demand for money by Wallace (1983), which discusses why currency can co-exist with Treasury securities despite having a dominated rate of return.[3] In this paper, I bring similar questions to bank reserves. Given the fact that banks can easily transfer Treasury securities among them, this paper presents a parsimonious model featuring a pair of banks that settle bank transfers with safe liquid bonds.[4] The model implies that introducing bank reserves improves the efficiency of the interbank payment system, even if banks can pay other liquid assets that are transferable at no physical transaction cost.

[1] For example, the average daily transfer of bank reserves in the U.S. was 20.2% of annual GDP in 2014.

[2] They include Australia, Canada, Denmark, Mexico, New Zealand, Norway, and Sweden.

[3] Wallace (1983) argues that even though the large denominations and nonnegotiablity of Treasury securities are inconvenient for payers and payees, private financial institutions should be able to issue small-denomination bearer notes backed by Treasury securities to circumvent the problems. He concludes that it is necessary to consider legal restrictions on such intermediation to explain the co-existence between currency issued by the central bank and Treasury securities.

[4] For example, the Federal Reserve System offers the Fedwire Securities service, which allows participating financial institutions to transfer securities among them in book-entry form.

The key reason for this result is a hold-up problem. Without bank reserves, each pair of banks must settle bank transfers between them bilaterally. In bilateral bargaining over the terms of settlement, a bank is required to pay a premium to send bank transfers to the other bank, because depositors' demand for timely payments weakens its bargaining position. The presence of this premium increases the amount of liquid assets necessary for interbank payments. This premium corresponds to interest in an over-the-counter (OTC) interbank money market transaction, in which banks essentially cancel out credit and debit positions in bank transfers. This interpretation is based on the literature on the OTC interbank money market, such as Ennis and Weinberg (2013), Afonso and Lagos (2015), and Bech and Monnet (forthcoming), which characterizes each transaction in the market as bilateral bargaining.

In light of this result, the interbank payment system operated by the central bank, a so-called large value payment system, can be regarded as an implicit interbank settlement contract to prevent a hold-up problem. In this system, a bank can send bank transfers unilaterally by remitting the equivalent nominal balance of bank reserves. This feature of the system can be replicated by an implicit contract, as a contract can determine the terms of settlement in advance. Furthermore, given limited commitment underlying banks' need for payment instruments, banks have to collateralize the contract. A mutual exchange of collateral, however, does not work due to ex-ante symmetry between banks. Thus, banks need a third party acting as the custodian of collateral, which is the role of the central bank in the contract. This result implies that an open market operation is a channel through which banks submit liquid collateral to the central bank. Bank reserves correspond to the balances of liquid collateral under the custody of the central bank. Banks have incentive to hold bank reserves through the contract even if the central bank does not return all the accrued interest on collateral, because they can avoid a hold-up problem due to ex-post bargaining over the settlement of bank transfers. Hence, the model explains the rate-of-return dominance puzzle as well as the need for substitution between bank reserves and other liquid assets by characterizing bank reserves as part of an implicit interbank settlement contract.

This result adds to the payment-system literature that analyzes the effects of elastic money supply to replace illiquid IOUs, such as Freeman (1996), Fujiki (2003, 2006), Mills (2004), Gu, Guzman and Haslag (2011), and Chapman and Martin (2013). There also exists a search-theoretic literature on money and illiquid collateral, such as Shi (1996), Ferraris and Watanabe (2008), and Andolfatto, Berentsen, and Waller (2013). In the business-cycle literature, Kiyotaki and Moore (2012) investigate the effect of money supply that replaces illiquid private securities. While these papers characterize money as an independent liquid asset, this paper analyzes the effect of substitution between money and liquid assets by characterizing money as part of an implicit contract.

Furthermore, Allen, Carletti, and Gale (2009) analyze the role of the central bank's liquidity supply when there is an aggregate deposit withdrawal from the banking system. This paper shows that the supply of bank reserves is beneficial even without such an event. Characterizing the central bank as a custodian of collateral also adds to a recent literature on the custodians of collateral in financial contracts, such as Monnet and Nellen (2014).[5]

The optimal contract replicates the floor system in practice: the central bank pays interest on bank reserves; and it supplies a sufficiently large amount of bank reserves in advance to eliminate the interbank money market. Simply supplying interest-bearing central-bank liabilities is insufficient for the optimality. This result adds to the paper by Martin and McAndrews (2010), which questions the need for an overnight interbank money market. It is also related to the work by Berentsen, Marchesiani and Waller (2014), which compares the channel and the floor system given a financial constraint on the central bank. This paper analyzes the case in which the central bank supplies bank reserves in exchange for other assets, following Keister, Martin, and McAndrews (2015).

In addition, the model implies that a liquidity-saving effect can explain not only the

[5] Monnet and Nellen (2014) analyze the role of a clearing house in segregating collateral for a future contract between risk-averse investors.

current use of bank reserves, but also the historical use of legal tender, such as specie and legal tender notes, for interbank payments. The modern interbank payment system can have a greater liquidity-saving effect than historical systems, as it can use liquid assets in general as collateral without relying on legal tender. In this regard, the convertibility between bank reserves and legal tender in practice is unnecessary for the liquidity-saving effect of the modern interbank payment system.

Finally, this paper discusses whether the interbank payment system must be provided by a public institution like the central bank. This question is related to the paper by Green (1997), which argues that private banks can provide the discount window on behalf of the central bank in Freeman's (1996) model. Also, Kahn (2013) analyzes the competition between a public and a private interbank payment system. This paper's model implies that the provider of the large value payment system does not have to be the issuer of currency, as currency plays no role in the model. Any institution can provide the optimal contract if it can act as the custodian of collateral. The model, however, also indicates that it is efficient if all banks share the same custodian of collateral. Thus, the provider of the interbank payment system is a natural monopoly. This implication of the model justifies the current practice that a public institution provides the interbank payment system.

The remainder of the paper is organized as follows. The daily routine in the interbank payment system in practice is briefly reviewed in section 2. The baseline model is presented in section 3. The interbank payment system provided by the central bank is introduced into the model in section 4. Section 5 discusses related issues. Section 6 concludes.

2 Daily routine in the interbank payment system

This section briefly summarizes the daily routine in the interbank payment system in practice. In each day, banks receive depositors' instructions to send bank transfers to other banks. Small-valued bank transfers are offset at an electronic system called an automated clearing house (ACH) to calculate the net amount of bank transfers to be sent by each bank. Large-valued bank transfers are not offset at an ACH, but remain gross bank transfers to be sent. After the netting at an ACH, banks settle net and gross bank transfers from their depositors by remitting the corresponding nominal amounts of current-account balances at the central bank, i.e., bank reserves, to each other. The remittance of bank reserves is processed electronically by a system called a large value payment system.[6]

Banks can obtain bank reserves in advance by selling liquid securities, such as Treasury securities and high-quality private securities, to the central bank through open market operations. In each day, however, there can be some banks running short of bank reserves, because of an imbalance between incoming and outgoing bank transfers for each bank. These banks can borrow bank reserves from other banks in the interbank money market.

While banks can also obtain bank reserves by depositing banknotes at the central bank, it is no longer a channel for new reserve supply these days. Currently, banks must withdraw bank reserves to obtain new banknotes, as the central bank supplies banknotes to the public only via banks. Thus, banks regain only previously supplied bank reserves by depositing banknotes received from depositors.[7]

In the following section, this paper presents a baseline model in which banks settle all the bank transfers in an interbank market (see Figure 1). Given the fact that the interbank money market is an OTC market, an interbank transaction is assumed to be bilateral. Also, given the presence of Treasury securities and other liquid assets in practice, banks can pay to each other safe liquid bonds that are transferable at no physical transaction cost in the model. Bank reserves and the large value payment system will be introduced later into the

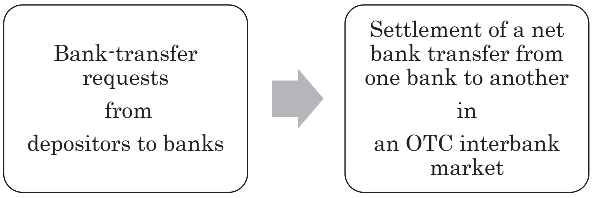
[6]For example, this system is called Fedwire in the U.S., TARGET2 in the Eurozone, CHAPS in the U.K., and BoJ-NET in Japan.

[7]Historically, the central bank issued banknotes to the public in return for the deposits of gold and silver and through government expenditures it financed. Thus, commercial banks could obtain new bank reserves by depositing gold, silver, and banknotes received from the public at the central bank. This channel for new reserve supply, however, has ceased to exist, as gold and silver are no longer circulating as currency and as the government makes payments to the public mostly by bank transfers nowadays.

model to clarify their role in interbank payments.

Figure 1: Comparison between the interbank payment system in practice and the baseline model

(a) Interbank payment system in practice



(b) Baseline model

3 Baseline model of a decentralized interbank payment system

Time is discrete and indexed by $t = 0, 1, 2$. There are two banks indexed by $i = A, B$. Each bank receives a unit amount of goods from its depositors in period 0. Goods can be interpreted as physical cash. For simplicity, assume that the deposit interest rate is zero.[8]

Each bank can transform deposited goods into bank loans and bonds. Bank loans generate an amount R_L of goods in period 2 per invested good. Similarly, the gross rate of return on bonds in period 2 is R_B. Assume that

$$R_L > R_B > 1, \tag{1}$$

[8] It is implicitly assumed that depositors can store goods with no interest by themselves, and also that each bank can set the deposit interest rate for its depositors monopolistically. Also, depositors can be committed to punishing or suing a defaulting bank so that each bank can be committed to repaying deposits in period 2. See Appendix A for the formal assumption about depositors.

in which one equals the gross rate of return on deposits. Banks cannot produce any good by terminating bank loans or bonds in period 1.

In period 1, bank i for $i = A, B$ receives depositors' orders to remit a fraction λ_i of its total deposits to the other bank. The joint probability distribution of λ_A and λ_B is

$$(\lambda_A,\ \lambda_B) = \begin{cases} (\eta,\ 0) & \text{with probability } 0.5, \\ (0,\ \eta) & \text{with probability } 0.5, \end{cases} \tag{2}$$

where $\eta \in (0, 1)$. Thus, banks are symmetric ex-ante. Here, it is implicitly assumed that overlapping gross flows of bank transfers between banks are automatically canceled out at an ACH. Thus, only one of the banks must send net bank transfers to the other bank. For $i = A, B$, call bank i an "originating bank" if $\lambda_i = \eta$, and a "receiving bank" if $\lambda_i = 0$.

In reality, information about bank transfers is confidential, as depositors usually prefer to keep private their daily cash flows.[9] Accordingly, assume that depositors require banks to settle bank transfers without revealing bank-transfer requests, $(\lambda_A,\ \lambda_B)$, to the public; thus, banks cannot write a verifiable contingent contract to transfer deposit liabilities, η.[10]

As a result, banks have to settle bank transfers by transferring bank loans and bonds between them after the realization of λ_A and λ_B in period 1.[11] Bonds are transferable at no physical transaction cost. In contrast, if a bank transfers its bank loans to the other bank in period 1, then the gross rate of return on the transferred bank loans becomes δ ($\in (0, R_L]$). The difference between R_L and δ can be interpreted as a loan monitoring cost per loan incurred by a bank purchasing bank loans.

Assume that the interbank market is an OTC market; so banks determine the terms of settlement through bilateral bargaining. The outcome of bargaining is determined by

[9] For example, information on a manufacturer's payments to suppliers can reveal the manufacturer's production cost to competitors. Also, traders may short-sell assets held by an institutional investor with a weak cash position. Even within the central bank, it is usually the case that only a limited number of staff can observe the information about bank-reserve transfers for each bank.

[10] This assumption does not contradict a depositor's ability to sue a defaulting bank to enforce a deposit contract in period 2, as it is a depositor's will to reveal the deposit balance at the court in such a case.

[11] Banks cannot arrange a pledgeable implicit contract in period 0 even if they swap some assets as collateral between them, because a defaulting bank can cancel out the collateral taken by, and from, the other bank.

Nash bargaining in which each bank has equal bargaining power. If banks do not reach any agreement, then no bank transfer is made. In this case, the originating bank must incur a cost $\gamma\eta$ ($\gamma > 0$) in period 2. This cost can be interpreted as a long-term cost due to a loss of depositors, or a cost payable in period 2 due to a litigation filed by depositors for failed payments. For normalization, assume that the receiving bank does not face any penalty. The results shown below go through as long as the penalty on the receiving bank is smaller than that on the originating bank. This assumption reflects the fact that a deposit contract allows a depositor to send a bank transfer on demand, for which an originating bank is liable, but a receiving bank is not.

In period 2, each bank receives returns on its bank loans and bonds, repays deposits given a zero deposit interest rate, and consumes the residual as profit. Banks are risk-neutral; thus each bank chooses its portfolio of bank loans and bonds in period 0 to maximize the expected profit in period 2. See Table 1 for the summary of events in the model. Given the values of parameters, $(R_L,\ R_B,\ \delta,\ \eta,\ \gamma)$, an equilibrium is characterized by: (i) the solution to the Nash bargaining problem for each realization of $(\lambda_A,\ \lambda_B)$ in period 1, given each bank's holding of bank loans and bonds; and (ii) the solution to each bank's profit-maximization problem on the allocation of deposited goods into bank loans and bonds in period 0, given each bank's rational expectation of (i).

3.1 Parametric assumptions

Throughout the paper, assume that

Assumption 1. $R_B > (1+\gamma)\,\eta$.

This assumption ensures banks can settle bank transfers if they invest into a sufficiently large amount of bonds. Also, assume that the penalty on failed bank transfers, γ, is sufficiently high that banks choose to settle bank transfers in any equilibrium considered below:

Assumption 2. $\gamma > 4\left(\dfrac{R_L}{R_B} - 1\right)$.

Table 1: Summary of events in the baseline model

Period 0	Period 1	Period 2
There are two banks; each bank receives a unit amount of goods from depositors, given a zero deposit interest rate.	One of the banks has an outflow of bank transfers, η, to the other bank. The probability to be the originating bank is 0.5 for each bank.	Banks receive returns on bank loans and bonds, repay deposits, and consume the residual.
Banks invest deposited goods into bank loans and bonds.	Banks bargain over how much amounts of bank loans and bonds the originating bank must pay to the receiving bank to settle bank transfers.	The return of goods per loan equals R_L if bank loans are not transferred in period 1, and δ ($\leq R_L$) if bank loans are transferred in the period.
	The originating bank must incur a penalty, $\gamma\eta$, if it fails to send bank transfers within period 1.	The return of goods per bond always equals R_B ($< R_L$).

3.2 Hold-up problem in a decentralized interbank payment system

Let us start from the case in which bank loans are transferable at no transaction cost:

Assumption 3. $\delta = R_L$.

Solve the model backward. Under Assumption 3, the bargaining problem between the originating and the receiving bank in period 1 takes the following form:

$$\max_{\{l\in[0,k],\ b\in[0,a]\}} [-(R_L l + R_B b - \eta) - (-\gamma\eta)]^{0.5}(R_L l + R_B b - \eta)^{0.5}, \tag{3}$$

where: k and a are the amounts of bank loans and bonds, respectively, held by the originating bank at the beginning of period 1; l and b denote the amounts of bank loans and bonds, respectively, that the originating bank pays to the receiving bank; and η is the face value of bank transfers in period 1.

The left square bracket is the trade surplus for the originating bank, and the right parenthesis is that for the receiving bank. The first term in the left square bracket, $-(R_L l + R_B b -$

η), is the net payoff for the originating bank in case of the settlement of bank transfers.[12] The second term in the left square bracket, $-\gamma\eta$, is the penalty for failed settlement of bank transfers.[13] This penalty determines the threat point for the originating bank. In contrast, the trade surplus for the receiving bank equals the profit from receiving bank transfers, $R_L l + R_B b - \eta$, given no penalty on the receiving bank for failed settlement of bank transfers.

The solution to the bargaining problem is

$$R_L l + R_B b = \eta + \frac{\gamma\eta}{2}, \tag{4}$$

which is feasible under Assumption 1. Thus, the originating bank must pay an extra value of assets, $\gamma\eta/2$, besides the face value of bank transfers, η.

This result is due to the bilateral settlement of bank transfers. Because of depositors' demand for timely payments, the originating bank must send bank transfers within period 1. This time constraint causes a hold-up problem, allowing the receiving bank to charge the originating bank a premium for the settlement of bank transfers. Hence, banks need extra liquidity for interbank payments if they settle bank transfers bilaterally.

3.3 Efficiency of a decentralized interbank payment system in case of liquid bank loans

Now move back to period 0. The profit maximization problem for each bank in the period is:

$$\begin{aligned} &\max_{\{k\geq 0,\, a\geq 0\}} \; R_L k + R_B a - 1 + \frac{1}{2}\left(-\frac{\gamma\eta}{2}\right) + \frac{1}{2}\frac{\gamma\eta}{2}, \\ &\qquad \text{s.t. } k + a = 1, \end{aligned} \tag{5}$$

where the constraint is a flow of funds constraint that the sum of investments into bank loans, k, and bonds, a, must equal the total amount of deposits, 1, at each bank in period

[12] In this case, the originating bank pays assets worth $R_L l + R_B b$, while its deposit liabilities declines by η.

[13] For simplicity, it is assumed that banks can settle either all the bank transfers, η, or none of them. The result of the model does not change even if banks can choose to settle only part of requested bank transfers, η. See Appendix D for more details.

0. The first two terms in the objective function are the returns on bank loans and bonds in period 2. The third term is the face value of deposit liabilities issued in period 0, given a zero deposit interest rate. The last two terms are the expected net loss and gain due to outgoing and incoming bank transfers, i.e., $\pm(R_L l + R_B b - \eta)$, as implied by (4).

Given $R_L > R_B > 1$ as assumed in (1), the solution to this problem is

$$(k,\ a) = (1,\ 0). \tag{6}$$

Thus, each bank invests only into the assets with the highest rate of return:

Proposition 1. Suppose Assumption 1 holds. Under Assumption 3, each bank chooses the efficient resource allocation, (6), in period 0.

3.4 Inefficiency of a decentralized interbank payment system in case of illiquid bank loans

The efficiency result described above is overturned if bank loans are illiquid. Now suppose that the cost of liquidating bank loans, $R_L - \delta$, is sufficiently high:

Assumption 4. $\delta < \dfrac{R_L}{1+\gamma}$.

This assumption implies $\delta < R_B$ given Assumption 2; thus, the rate of return on bank loans becomes smaller than that on bonds if transferred.

Under Assumption 4, the bargaining problem for the settlement of bank transfers in period 1 takes the following form:

$$\max_{\{l\in[0,k],\ b\in[0,a]\}} [-(R_L l + R_B b - \eta) - (-\gamma\eta)]^{0.5}(\delta l + R_B b - \eta)^{0.5}. \tag{7}$$

The left square bracket and the right parenthesis contain the trade surpluses for the originating and the receiving bank, respectively. Note that the gross rate of return on transferred bank loans in the right parenthesis is changed from R_L to δ.

Denote the changes in profit for the originating and the receiving bank as a result of this bargaining by $\theta(a)$ and $\phi(a)$, respectively. Both $\theta(a)$ and $\phi(a)$ are the functions of the amount of bonds held by the originating bank, a, given that the bank invests the rest of deposits into bank loans in period 0, i.e., $k = 1 - a$. The solution to the bargaining problem implies that the following result holds for all $\delta \in (0, R_L)$ under Assumption 1:

$$
\begin{aligned}
&(\theta(a),\ \phi(a)) \\
&= \begin{cases} (-\gamma\eta,\ 0), & \text{if } R_B a - \eta < -\frac{\delta\gamma\eta}{R_L - \delta}, \\ (-[R_L l(a) + R_B b(a) - \eta],\ \delta l(a) + R_B b(a) - \eta), & \text{otherwise,} \end{cases}
\end{aligned}
\tag{8}
$$

where $l(a)$ and $b(a)$ denote the solutions for l and b, respectively, given a:

$$
(l(a),\ b(a)) = \begin{cases} \left(\frac{\delta\gamma\eta - (R_L+\delta)(R_B a - \eta)}{2R_L\delta},\ a\right), & \text{if } R_B a - \eta \in \left[-\frac{\delta\gamma\eta}{R_L-\delta},\ \frac{\delta\gamma\eta}{R_L+\delta}\right], \\ (0,\ a), & \text{if } R_B a - \eta \in \left[\frac{\delta\gamma\eta}{R_L+\delta},\ \frac{\gamma\eta}{2}\right], \\ \left(0,\ \frac{1}{R_B}\left(\eta + \frac{\gamma\eta}{2}\right)\right), & \text{if } R_B a - \eta \geq \frac{\gamma\eta}{2}. \end{cases}
\tag{9}
$$

See Appendix B for the proof.

These equations imply that banks fail to agree on the settlement of bank transfers (i.e., $\theta(a) = -\gamma\eta$), if a is too small. In this case, the amount of bank loans that must be liquidated to settle bank transfers is too large, given the loan liquidation cost, $R_L - \delta$. Otherwise, the originating bank settles bank transfers by paying assets worth more than the face value of bank transfers, η. This result holds because the receiving bank can charge the originating bank a premium, given the time constraint that the originating bank must complete bank-transfer requests within period 1. This hold-up problem is as same as the reason behind the second term on the right-hand side of (4).

In period 0, the profit maximization problem for each bank can be written as

$$
\begin{aligned}
&\max_{\{k \geq 0,\ a \geq 0\}} R_L k + R_B a - 1 + \frac{1}{2}\theta(a) + \frac{1}{2}\phi(a'), \\
&\qquad \text{s.t. } k + a = 1,
\end{aligned}
\tag{10}
$$

where a' denotes the amount of bonds held by the other bank at the end of the period, which is taken as given by each bank. Under Assumptions 2 and 4, each bank invests into the just enough amount of bonds in period 0 to avoid the liquidation of bank loans in period 1:

Proposition 2. Suppose Assumptions 1, 2 and 4 hold. Each bank chooses

$$(k,\ a) = \left(1 - a,\ \frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L + \delta}\right)\right), \tag{11}$$

in period 0. Given this value of a, the originating bank pays only bonds in period 1:

$$(l,\ b) = \left(0,\ \frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L + \delta}\right)\right). \tag{12}$$

Proof. See Appendix C. □

Thus, banks must prepare an extra amount of bonds besides the face value of bank transfers ex-ante, if bank loans are illiquid.[14]

3.5 Interpreting bilateral bargaining as an OTC interbank money market transaction

Bilateral bargaining in the baseline model can be interpreted as an OTC interbank money market transaction. In practice, an originating bank borrows bank reserves from other banks when it runs short of bank reserves to send bank transfers. In aggregate, the movement of bank reserves is such that bank reserves are transferred from receiving banks with excess reserves to originating banks in short of bank reserves, and then returned into the opposite direction when originating banks remit borrowed bank reserves to settle bank transfers. Thus, under the veil of a round-trip transfer of bank reserves, originating and receiving banks essentially negotiate a premium to settle bank transfers in the form of interest in the interbank money market. In the model, banks simply bargain over a premium to cancel out credit and debit positions in bank transfers without a round-trip transfer of bank reserves. If they fails to agree on settlement, bank transfers are canceled, as in deferred net settlement.

[14]This result does not change even if banks can choose to settle only part of requested bank transfers, η, in period 1. See Appendix D for more details.

From this perspective, the bond transfer in period 1 can be interpreted as a repo transaction, as it makes no difference whether the originating bank pays its assets in period 1, or is committed to paying the return on the assets in period 2, given that the assets mature in period 2. In the latter interpretation, the originating bank pledges bonds in period 1 to repay the debit position in bank transfers, η, with interest, $\delta\gamma\eta/(R_L + \delta)$, in period 2.[15]

The finite time horizon of the model precludes reputation among banks, and thus prohibits banks from transferring deposit liabilities without an ex-post compensation. Such an arrangement corresponds to unsecured interbank credit. While unsecured interbank loans occupy an important part of the interbank money market in practice, not all pairs of banks can rely on them fully, as evidenced by the active use of bank reserves for interbank payments and repos in the interbank money market.[16] The model features a pair of banks that need payment instruments and collateral due to limited commitment between them.

4 Large value payment system as an implicit interbank settlement contract

In practice, an originating bank can settle bank transfers unilaterally by remitting the equivalent nominal balance of bank reserves to a receiving bank through the large value payment system. To introduce this feature of the system into the model, consider an implicit contract that allows the unilateral settlement of bank transfers without ex-post bargaining.

Because of the non-verifiability of bank-transfer requests, each bank can bail out of a contract any time. If a contract is arranged without a third party, then each bank's payoff in a pledgeable interbank settlement contract must be as same as that in bilateral bargaining in period 1. Otherwise, one of the banks would be necessarily better off by bailing out of the contract, as the outcome of Nash bargaining is Pareto-efficient. Even if banks exchange

[15] Assume that the receiving bank must incur a monitoring cost, $R_L - \delta$, per bank loan submitted as collateral, so that it cannot change the monitoring cost by switching from a spot transaction to a repo.

[16] See Demiralp, Preslopsky, and Whitesell (2004) for the role of a repo in the interbank money market.

collateral between them in period 0, they take an equal amount of collateral from each other given their ex-ante symmetry in period 0. Thus, a bank does not lose anything by reneging on a contract, as it can cancel out the collateral taken by, and from, the other bank.

This result implies that a custodian of collateral is necessary to implement a contract. Assume that the central bank can offer an implicit contract such that: (i) it receives amounts $\hat{l}$ and $\hat{b}$ of bank loans and bonds, respectively, as collateral from each bank in period 0. (ii) Then, each bank reports bank-transfer requests from its depositors to the central bank in period 1. The reports can be false as the central bank cannot see bank-transfer requests to each bank directly. Denote bank i's report by $\hat{\lambda}_i$ ($\in \{0, \eta, \emptyset\}$) for $i = A, B$, in which $\hat{\lambda}_i = \emptyset$ implies that bank i bails out of the contract. (iii) For each bank i reporting $\hat{\lambda}_i = \eta$, the central bank records a transfer of the bank's collateral to the other bank on its book in exchange for an increase in deposit liabilities by η at the other bank, unless the other bank reports $\hat{\lambda}_{-i} = \emptyset$.[17] (iv) If $\hat{\lambda}_i = \emptyset$ for $i = A$ or B in period 1, then no bank loans or bonds are transferred between banks. (v) For any realization of $(\hat{\lambda}_A, \hat{\lambda}_B)$, the central bank returns the resulting balance of collateral for each bank in period 2. In this implicit contract, the central bank does not reveal reported bank-transfer requests to the public, as in practice; thus, the truthful revelation of bank-transfer requests to the central bank does not contradict the non-verifiability of bank-transfer requests assumed in the baseline model.

At the end of period 1, banks can also bargain over the settlement of bank-transfer requests that are not settled by the contract, if any, as in the baseline model.

4.1 Optimal contract

Given the involvement of the central bank into a contract, guess and verify that each bank can be committed to the truthful revelation of bank-transfer requests and no ex-post opt-out

[17] Assume that the central bank can certify an increase in deposit liabilities at the other bank, if the other bank tries to renege on the redemption of deposits to depositors in period 2. This assumption can be interpreted as so-called finalty in the large value payment system, in which the central bank guarantees the irrevocable settlement of bank transfers in the system.

of a contract (i.e., $\hat{\lambda}_i = \lambda_i$ for $i = A, B$). Even in this case, the receiving bank bails out of a contract if it suffers an ex-post loss from the contract in period 1. Thus, the participation constraint for the receiving bank to remain in a contract is

$$\delta\hat{l} + R_B\hat{b} \geq \eta, \tag{13}$$

where the left-hand side is the value of asset transfer to the receiving bank in the contract, and the right-hand side is an increase in its deposit liabilities under the contract.[18]

Given the participation constraint, the optimal contract problem in period 0 is:

$$\begin{aligned} \max_{\{k\geq 0,\, a\geq 0,\, \hat{l}\geq 0,\, \hat{b}\geq 0\}} \quad & R_L k + R_B a - 1 - \frac{1}{2}(R_L\hat{l} + R_B\hat{b} - \eta) + \frac{1}{2}(\delta\hat{l} + R_B\hat{b} - \eta), \\ \text{s.t.} \quad & k + a = 1, \\ & \delta\hat{l} + R_B\hat{b} \geq \eta, \\ & k \geq \hat{l},\ a \geq \hat{b}, \end{aligned} \tag{14}$$

where the first constraint is the flow of funds constraint for each bank in period 0, the second one is the participation constraint for the receiving bank in period 1, and the third constraint is a feasibility constraint such that each bank invests into sufficient amounts of assets to submit amounts $\hat{l}$ and $\hat{b}$ of bank loans and bonds, respectively, in period 0.[19]

Under Assumptions 2 and 4, the solution to this problem is

$$(\hat{l},\ \hat{b},\ k,\ a) = \left(0,\ \frac{\eta}{R_B},\ 1-a,\ \hat{b}\right). \tag{15}$$

[18]If the receiving bank receives a penalty for failed bank transfers, then the penalty is subtracted from the right-hand side of (13).

[19]If the value of bank transfers, η, is a random variable, then the amount of collateral transferred from the originating bank to the receiving bank, $\hat{l}$ and $\hat{b}$, becomes contingent on the realization of η, even though each bank must submit a fixed amount of bank loans and bonds in period 0. The optimal contract minimizes the amounts of bank loans and bonds to be transferred, as transferring bank loans and investing into bonds are both costly for banks. Therefore, the second constraint is binding for all the realizations of η, as long as the central bank holds enough collateral to satisfy the constraint. By continuity, it can be shown that if the range of possible values of η is bounded and sufficiently narrow, then banks submit enough bonds to settle any realization of bank-transfer requests through the contract. Otherwise, the contract may leave requested bank transfers unsettled if the realized value of η is too large. In reality, the central bank can provide bank reserves through unsecured loans if the supplied amount of bank reserves is not sufficient to settle bank transfers. It is beyond the scope of this paper to analyze such an emergency unsecured lending by the central bank, given the cost of the lending, such as monitoring cost and moral hazard among banks.

This contract reduces the amount of bonds that each bank must invest into in period 0, as implied by comparison between (11) and (15). This result obtains because the use of an implicit contract eliminates a hold-up problem due to ex-post bargaining by specifying the terms of settlement of bank transfers in advance.

4.2 Pledgeability of the optimal contract

Now verify the truthful revelation of bank-transfer requests to the central bank and no opt-out of the optimal contract by each bank. First, suppose that banks enter into the optimal contract in period 0 and then either bank bails out of the contract in period 1. In this case, banks can transfer only bank loans between them at the end of period 1, because the central bank keeps their entire bond holdings until period 2, given $a = \hat{b}$. Thus, the bilateral bargaining problem in this case is

$$\max_{\tilde{l} \in [0,k]} [-(R_L \tilde{l} - \eta) - (-\gamma\eta)]^{0.5} (\delta \tilde{l} - \eta)^{0.5}, \tag{16}$$

where the left square bracket and the right parenthesis are the trade surpluses for the originating and the receiving bank, respectively, and $\tilde{l}$ is the amount of bank loans transferred from the originating bank to the receiving bank.

Under Assumption 4, the total trade surplus, $(1 + \gamma)\eta/R_L - \eta/\delta$, is negative due to a high loan liquidation cost. Thus, banks do not settle bank transfers outside the contract. Also, each bank in period 1 has a weakly higher payoff from the contract than no settlement of bank transfers, because the contract characterized by (15) leaves each bank break-even, while the originating and the receiving bank's payoff are $-\gamma\eta$ and 0, respectively, in case of no settlement of bank transfers. Hence, no bank has incentive to bail out of the contract in period 1.

Next, given no opt-out of the contract, if the originating bank makes a false report of no bank-transfer request, then it must incur a penalty for failed bank transfers, $\gamma\eta$. This payoff is inferior to a zero net payoff in the truthful revelation of bank-transfer requests in the contract. Also, the receiving bank is break-even whether it misreports or truthfully reveals its bank-transfer requests. Thus, each bank can be committed to the truthful revelation of bank-transfer requests to the central bank in the optimal contract. Hence, the following proposition holds:

Proposition 3. Under Assumptions 1, 2, and 4, banks participate into the implicit interbank settlement contract characterized by (15) in period 0. They can be committed to $\hat{\lambda}_i = \lambda_i$ for $i = A, B$.

Note that the central bank cannot offer a better contract than (15), because this contract is the second-best contract under (13). The central bank cannot remove (13), as the receiving bank can always choose to leave bank transfers unsettled.

4.3 Characterizing the large value payment system as an implicit interbank settlement contract

The optimal contract allows the originating bank to settle bank transfers unilaterally by reporting bank-transfer requests to the central bank, as in the large value payment system in practice. In light of this result, an open market operation can be interpreted as a channel through which banks submit liquid collateral, i.e., bonds in the model, to the central bank. Bank reserves correspond to the balance of collateral for each bank under the custody of the central bank. The remittance of bank reserves from one bank to another is equivalent to the reallocation of the balance of collateral by the central bank to settle bank transfers under the terms of the contract. Also, the large value payment system in practice does not force banks to remain in the system by a threat of confiscating bank reserves. Instead, the central bank just does not release its assets to absorb bank reserves on demand from banks. The model shows that the retention of liquid assets by the central bank is sufficient to eliminate incentive for banks to settle bank transfers outside the large value payment system.[20]

[20] In this regard, it is true that banks in reality hold an inventory of liquid securities besides bank reserves. Such an inventory, however, is likely for dealing with a bank's clients, as it is inefficient if a bank regularly

Thus, the large value payment system can be characterized as an implicit interbank settlement contract in which the central bank acts as the custodian of collateral. The use of an implicit contract improves the efficiency of the interbank payment system by eliminating ex-post bargaining over the settlement of bank transfers. This result explains why the central bank needs to replace liquid assets with bank reserves, and also why a third party like the central bank must provide the large value payment system.

4.4 Optimality of the floor system

The optimal contract corresponds to the floor system in practice. In this system, the central bank supplies a sufficiently large amount of bank reserves for interbank payments in advance, so that banks do not need to borrow bank reserves in the interbank money market. To give banks incentive to hold the supplied amount of bank reserves, the central bank pays interest on bank reserves. Consequently, the interest rate anchors the short-term nominal interest rate in the financial market. This system has been adopted by New Zealand since July 2006.

The optimal contract replicates the two features of the floor system. First, it requires banks to pledge to the central bank an enough amount of bonds to settle possible bank transfers in the future, so that banks do not have to settle any bank transfer bilaterally in period 1. This feature of the contract is equivalent to the elimination of the OTC interbank money market, because the bilateral settlement of bank transfers outside the contract can be interpreted as an OTC interbank money market transaction, as described in section 3.5.[21]

Second, the optimal contract requires the central bank to pass on the whole accrued interest on collateral to banks. This feature of the contract corresponds to interest payments on bank reserves. It must be part of the optimal contract because if the central bank takes some of accrued interest on collateral as profit, then banks have to submit more bonds in

maintains an enough amount of liquidity to settle bank transfers outside the large value payment system. In this paper, it is implicitly assumed that the opportunity cost to divert the inventory of liquid assets spared for dealing is too high.

[21]The floor system in practice obviates the need for both secured and unsecured interbank overnight loan transactions. In light of the model, it is sufficient for efficiency if only secured transactions are eliminated.

period 0 to make the receiving bank break-even by transferring a larger amount of bonds in period 1.

The second feature of the optimal contract is equivalent to the Friedman rule. To confirm that the Friedman rule is not sufficient for the optimality, suppose that the central bank just issue interest-bearing liabilities to banks in exchange for bonds in period 0, by promising to repay the whole return on the received bonds to the holders of its liabilities in period 2. In this case, central-bank liabilities and bonds are identical assets; thus, no change occurs to the baseline model. Thus, the optimality of the floor system is not solely due to interest payments on bank reserves, but also rests on the elimination of the OTC interbank money market.

This result provides an answer to the question posed by Martin and McAndrews (2010) as to whether the overnight interbank money market is necessary, given no intraday interbank money market in practice and the continuity between an intraday and an overnight transaction. This paper shows that it is optimal to eliminate the overnight interbank money market by the floor system, because of a hold-up problem in an OTC interbank money market transaction.

4.5 Comparison between the floor system and other bank-reserve supply policies

It is possible to compare the floor system and the conventional reserve supply policy within the model, in the latter of which the central bank supplies only a partial amount of bank reserves and leaves active transactions in the OTC interbank money market. To feature the conventional reserve supply policy in the model, suppose that the central bank accepts bonds from each bank as collateral only up to an amount smaller than η/R_B in period 0. In this case, banks can settle only a fraction of bank transfers through the interbank settlement contract, and must leave the rest of bank transfers to be settled through bilateral bargaining in period 1.[22] This policy requires each bank to prepare a larger amount of bonds in period 0 than the optimal contract, because of a premium for the bilateral settlement of bank transfers in period 1.[23]

It is also possible to extend the model to incorporate another contract featuring the channel system. In this system, the central bank supplies bank reserves to banks through daylight overdrafts. It pays interest if a bank holds an overnight credit position in bank reserves, but charges a higher interest rate if a bank runs an overnight debit position in bank reserves. Given the threat point created by the two central-bank overnight interest rates, banks cancel out intraday credit and debit positions in bank reserves due to bank transfers by the end of each day by arranging interbank loans in the OTC interbank money market. It can be shown that a contract featuring the channel system requires banks to prepare a larger amount of bonds in period 0 than the optimal contract, because of the use of the bilateral settlement of bank transfers in period 1, i.e., an OTC interbank money market transaction. Thus, the floor system dominates the channel system. The channel system converges to the floor system as the spread between the two central-bank overnight interest rates goes to zero. See appendix E for more details.

4.6 Explaining the rate-of-return dominance puzzle under the conventional reserve supply policy

The central bank pays no interest on bank reserves under the conventional reserve supply policy. To replicate this feature of the policy, consider a contract such that the central bank does not return accrued interest on bonds submitted as collateral, i.e., $R_B - 1$ per bond, to banks in period 2, along with a cap on the amount of bonds that the central bank accepts as collateral from each bank in period 0. If $R_B - 1$—that is, the real interest margin between

[22] It can be shown that the originating bank needs to pay a premium for the settlement of bank transfers even if banks can choose to settle only part of requested bank transfers, η, in period 1 in the baseline model. Thus, the large value payment system is necessary even in this case. See Appendix D for more details.

[23] It is straightforward to confirm this result, as the bargaining problem, (7), can be normalized by the face value of bank transfers, η.

bank reserves and bonds —is sufficiently small, then it holds by continuity that banks find it optimal to participate into the contract to save liquidity even at the expense of the interest margin paid to the central bank. This result explains the rate-of-return dominance puzzle regarding bank reserves under the conventional reserve supply policy.

5 Discussion

5.1 Implication for historical interbank payment systems

Historically, private banks issued banknotes that promised to pay out specie, sparing note holders from a burden to carry coins physically. Early examples are goldsmiths in England in the 17th century. Afterwards, banks started accepting each other's notes, or issuing checks for the remittance of deposit balances. In these cases, they transferred legal tender, such as specie and legal tender notes, to settle mutual claims at first.[24] Subsequently, they came to use current-account balances at the central bank, i.e., bank reserves, for interbank settlement, as in the present time. This arrangement dates back to the nineteenth century.[25]

To feature a historical interbank payment system using legal tender in the baseline model, suppose that each bank can store goods between periods 0 and 1 to pay off an amount η of goods to the other bank when it becomes the originating bank in period 1. This payment corresponds to the settlement of banknotes or checks by legal tender, as goods can be interpreted as physical currency, that is, specie and legal tender notes. The payment discharges the originating bank from contractual obligations to send bank transfers for depositors unilaterally, because the definition of legal tender is such that a creditor cannot sue a debtor for

[24]See Cannon (1900), Cheque and Credit Clearing Company (2009), and Norman, Shaw, and Speight (2011) for examples across countries.

[25]Commercial banks in the City of London formed a clearing house in the late 18th century. They started using current-account balances at the Bank of England for the means of interbank settlement in 1854. They reallocated current-account balances among them through discounting of commercial bills in the London Discount Market. The Bank of England also discounted commercial bills in this market through interbank dealers called bill brokers. See Bisschop (1910), Cheque and Credit Clearing Company (2009), and Sowerbutts, Schneebalg, and Hubert (2016) for more details.

non-payment if the debtor pays into court in legal tender.[26] In period 0, each bank chooses whether to store goods or invest into bonds to settle bank transfers in period 1. If banks settle bank transfers by a transfer of bonds, then they must determine the terms of settlement through bilateral bargaining in period 1, as in the baseline model. This set-up reflects the historical observation that banks settled mutual claims by specie and legal tender notes when they could transfer commercial and government bills among them.[27]

Banks choose to store goods in period 0 if the gross rate of return on bonds, R_B, is sufficiently close to that on storage, i.e., one:

$$R_L(1-\eta)+\eta-1 > R_L\left\{1-\frac{1}{R_B}\left(\eta+\frac{\delta\gamma\eta}{R_L+\delta}\right)\right\}+\eta+\frac{\delta\gamma\eta}{R_L+\delta}-1, \tag{17}$$

where the left-hand and the right-hand side are the expected profits for a bank when a bank stores an amount η of goods in period 0 and when it invests into bonds in period 0 as in the baseline model, i.e., (11), respectively.[28] In this case, even though banks must forgo a higher rate of return on bonds, they choose to avoid a hold-up problem due to ex-post bargaining over the settlement of bank transfers. Thus, a liquidity-saving effect can explain the use of legal tender in historical interbank payment systems. Note that the parameter restriction on R_B for this result is consistent with the rate-of-return dominance puzzle under the conventional reserve supply policy.

The use of legal tender for interbank payments, however, is dominated by the optimal contract described above. In the optimal contract, each bank needs to spare only an amount η/R_B of goods in period 0 for the settlement of bank transfers, rather than an amount η of

[26]Unlike a note, a check is not a liability to the one who presents a check, but is an instruction from a depositor to repay a specific amount of his or her deposit to a payee specified on the check on behalf of the depositor. Thus, the settlement of a check by a payment of legal tender is the fulfillment of a debt obligation to a depositor.

[27]For example, banks in England exchanged currency and commercial bills in the London Discount Market in the nineteenth century. Also, banks in Scotland transferred exchequer bills at par to settle each other's banknotes at the note exchange in Edinburgh between 1846 and 1864. See Sowerbutts, Schneebalg, and Hubert (2016), Lawson (1845), and Kerr (1908) for more details.

[28]Note that no deadweight loss occurs in the optimal contract, as no bank loan is transferred between banks. Thus, the gain and the loss in period 1 are canceled out, and the expected profit for each bank in period 0 equals the expected return on the initial portfolio choice.

goods. This result implies that the large value payment system at the present time can have a greater liquidity-saving effect than historical systems as it can use liquid assets in general as collateral without relying on legal tender.[29]

5.2 No role of currency or legal tender in the liquidity-saving effect of the large value payment system

The model abstracts from the fact that banks can convert bank reserves into currency, or legal tender, at par any time. The model implies the convertibility is unnecessary for the liquidity-saving effect of the current large value payment system, as legal tender plays no role in the optimal contract.

Similarly, even though depositors can withdraw currency from banks to make payments, currency payments are imperfect substitutes to bank transfers because of a higher physical transaction cost. Thus, the convertibility of bank reserves into currency does not eliminate a penalty that depositors would impose on banks for failed bank transfers. The transaction cost would be also high if banks withdrew currency from the central bank and settled bank transfers between them by currency outside the large value payment system. It is implicitly assumed that banks do not have incentive to pay such a high transaction cost. Hence, the convertibility of bank reserves into currency does not alter the result of the model.

5.3 Does the interbank payment system have to be provided by a public institution?

A related question is whether the issuer of currency must run the large value payment system. The central bank plays both roles in most countries. In Singapore, however, the government agency operating the large value payment system had been separated from the issuer of currency until 2002.[30] Also, the Hongkong Shanghai Banking Corporation (HSBC), a large

[29] Bank reserves are usually not designated as legal tender. See Section 31 U.S.C. §5103 for example.

[30] The issuer of Singapore dollar notes used to be the Board of Commissioners of Currency until it was merged with the Monetary Authority of Singapore in October 2002. The Monetary Authority of Singapore manages the large value payment system for the country. The two institutions were merged for operational

commercial bank, had been the provider of the large value payment system for the Hong Kong dollar until 1997, while Hong Kong dollar notes were issued by authorized commercial banks, including HSBC, under a currency board pegged to the U.S. dollar.[31] Thus, the provider of the large value payment system does not have to be the issuer of currency, which is consistent with no role of currency or the convertibility of bank reserves into currency in the model.

Then, can a private institution run the large value payment system on behalf of the central bank? On one hand, the model implies that any institution can provide an implicit interbank settlement contract if it can act as the custodian of collateral. Historically, the New York Clearing House, which is the first private clearing house in the U.S. preceding the Federal Reserve System, issued clearing-house gold certificates in exchange for the deposits of specie, so that member banks could settle checks by those certificates rather than a physical transfer of specie (see Gorton 1984).[32] Thus, there is a historical precedent that a private clearing house played the role of the custodian of collateral for interbank payments.[33]

On the other hand, the model also implies that it is efficient if all banks in a country share one interbank settlement contract. Even though the model features only two banks, it can be interpreted as capturing each pair of banks that need payment instruments to send bank transfers to each other. If there are more than two banks, it is inefficient if a bank uses different custodians of collateral for different banks, because collateral submitted to a custodian of collateral cannot be utilized to send bank transfers through a contract

efficiency.

[31]In April 1997, the Hong Kong Clearing Interbank Clearing Limited, a private company jointly owned by the Hong Kong Monetary Authority and the Hong Kong Association of Banks, took over the role of providing the large value payment system.

[32]Clearing-house gold certificates were the primary means of settlement at the New York Clearing House. The other means of settlement were United States gold certificates issued by the federal government, specie, and legal tender notes. See Cannon (1900).

[33]Currently, the New York Clearing House is called the Clearing House, running a private interbank payment system, CHIPS, in the U.S. The collateral in CHIPS is bank reserves in the Fedwire; thus, the Clearing House does not act as the custodian of collateral, but relies on the custodian service by the Federal Reserve System currently.

with another custodian of collateral. Thus, the custodian of collateral for the large value payment system is a natural monopoly. This result justifies the current practice that a public institution runs the large value payment system, which is consistent with the assumption in the model that the central bank provides an implicit interbank settlement contract.

5.4 Implication for tiering

While the model implies that it is efficient if all banks participate into one contract, there exists a case in which only large banks participate into the large value payment system in practice.[34] In such a case, small banks hold deposit accounts at large banks, and large banks send bank transfers in the large value payment system on behalf of them as their agents. This type of arrangement within a country is called tiering. Also, such an arrangement is common for international bank transfers, in which small banks send payments oversea via intermediary banks in the same country.

The model suggests that a small bank does not participate into the large value payment system directly if it is too costly for the bank to handle wholesale securities used as collateral in the system. In such a case, a larger bank can intermediate a smaller bank's access to the large value payment system through its book, if it can accept a wider range of assets from the smaller bank to provide deposit balances. This implication adds another perspective to the literature on tiering, such as Kahn and Roberds (2009) and Chapman and Martin (2013), which implies that tiering occurs if the central bank needs to delegate the monitoring of small banks through tiering.

6 Conclusions

This paper shows that banks need extra liquidity if they settle bank transfers through bilateral bargaining, which corresponds to an OTC interbank money market transaction. In light of this result, the large value payment system can be regarded as an implicit interbank

[34]An example is Canada.

settlement contract to save liquidity, in which bank reserves are the balances of collateral under the custody of the central bank. The optimal contract corresponds to the floor system, as it eliminates the OTC interbank money market. The liquidity-saving effect of the contract explains the rate-of-return dominance puzzle under the conventional reserve supply policy.

To illustrate these results in a simple set-up, this paper abstracts from related issues, such as the role of the large value payment system in the central bank's ability to control the interest rate and the optimal interbank payment system from the view of the implementation of monetary policy. Also, the model focuses on a pair of banks that need payment instruments and collateral to send bank transfers between them. It remains an issue to incorporate the co-existence of unsecured interbank credit and the need for payment instruments among banks in a richer model. In addition, this paper abstracts from the operational cost for the central bank to hold a large amount of assets to implement the floor system, which would be an important issue when comparing the efficiencies of the floor system and other reserve supply policies in practice. A further investigation into these issues is left for future research.

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Appendices

A Baseline model with a formal assumption about depositors

A.1 Preference and technology

Time is discrete and indexed by $t = 0, 1, 2$. There are two banks indexed by $i = A, B$. Each bank has a fixed customer base consisting of a unit continuum of risk-neutral depositors. Each depositor is endowed with a unit of goods in period 0. A depositor can save its good in two ways. One is storage technology, in which a depositor can store its good without depreciation or appreciation between consecutive periods. The other is a bank deposit. If a depositor deposits its good in period 0, then the depositor's bank can transform the good into a bank loan or a bond in that period. A bank loan generates an amount R_L of goods in period 2 per invested good. Similarly, the gross rate of return on a bond in period 2 is R_B. Assume that

$$R_L > R_B > 1, \tag{A.1}$$

in which the last term is the gross rate of return on storage.

Each depositor becomes a buyer or a seller due to an idiosyncratic preference shock in period 1. A buyer can consume goods produced by sellers at the other bank in period 1, but cannot consume goods in period 2. A seller can produce goods at a unit utility cost per good in period 1, and can consume goods in periods 1 and 2. Each depositor maximizes the following expected utility:

$$U = p_1 \rho c_{b,1} + (1 - p_1)(c_{s,1} - h_{s,1} + c_{s,2}). \tag{A.2}$$

where: p_1 is the unconditional probability to be a buyer in period 1 for each depositor in period 0; $c_{b,1}$ and $c_{s,1}$ are the amounts of consumption in period 1 in case of becoming a buyer and a seller, respectively; $h_{s,1}$ and $c_{s,2}$ are the production in period 1 and the consumption in period 2, respectively, in case of becoming a seller; and ρ (> 0) is the weight on utility in case of becoming a buyer. Assume that ρ is arbitrarily large, and also that the preference shock is private information for each depositor.

A.2 Deposit contracts

There exists a competitive goods market so that buyers can buy goods from sellers in period 1. Depositors, however, are anonymous to each other; thus, buyers cannot buy goods on credit. Banks can offer a deposit contract in which buyers can order their bank to remit deposit balances from their accounts to the sellers' bank accounts in period 1. Hence, buyers can pay the price of goods in period 1 by bank transfers. Deposits are redeemed in period 2, as neither depositor or bank can generate goods by terminating bank loans or bonds in period 1.

For simplicity, assume that depositors can sue or punish a defaulting bank, so that each bank can be committed to repaying the outstanding balance of deposits in period 2. In case of punishment, the penalty can be interpreted implicitly as a cost for a bank to lose depositors in the long run. Each bank can set the deposit interest rate for its depositors monopolistically in period 0, given the assumption that the customer base for each bank is fixed. To satisfy the participation constraint for depositors, a bank cannot set a deposit interest rate lower than zero, because in such a case, depositors would be better off by storing goods by themselves. Furthermore, a bank does not gain from setting a positive bank-transfer fee, because in such a case, a bank must offer an arbitrarily high deposit interest rate to keep depositors indifferent between a deposit contract and storage in period 0, given the assumption that ρ is arbitrarily large. Also, a bank does not offer a negative bank-transfer fee to set the deposit interest rate below zero, because a bank cannot observe the preference shock to each depositor. Note that if a bank offered a negative bank-transfer fee with a deposit interest rate below zero, then all depositors would choose to send bank transfers to buy goods in period 1, which would cause a loss to the bank. Thus, banks offer a deposit contract with a zero deposit interest rate and a zero bank-transfer fee in period 0.

A.3 Probability distribution of bank-transfer requests

The buyer fraction of depositors at each bank is stochastic. For $i = A, B$, λ_i denotes the buyer fraction of depositors at bank i. The joint probability distribution of λ_A and λ_B is

$$(\lambda_A,\ \lambda_B) = \begin{cases} (\eta,\ 0) & \text{with probability } 0.5, \\ (0,\ \eta) & \text{with probability } 0.5, \end{cases} \tag{A.3}$$

where $\eta \in (0, 1)$. Given (A.3), the unconditional probability for each depositor to be a buyer, i.e., p_1, is

$$p_1 = 0.5\eta. \tag{A.4}$$

The depositor's utility function, (A.2), implies that the relative price of goods in period 1 in terms of goods in period 2 equals one. It also implies that buyers spend all of their deposits to buy goods from sellers in period 1. Thus, η equals the fraction of deposits to be sent to the other bank. Here, assume that sellers do not buy goods in period 1, as they are indifferent between consumption in periods 1 and 2.

The rest of assumptions, i.e., those on the settlement of bank transfers, each bank's objective function, and the definition of an equilibrium, are as same as defined in section 3.

B Proof for (8) and (9)

Given $\delta < R_L$, the first-order conditions for the bargaining problem, (7), with respect to l and b are:

$$-\frac{R_L}{-[R_L l + R_B b - \eta] + \gamma\eta} + \frac{\delta}{\delta l + R_B b - \eta} + \underline{\theta}_l - \overline{\theta}_l = 0, \tag{A.5}$$

$$-\frac{R_B}{-[R_L l + R_B b - \eta] + \gamma\eta} + \frac{R_B}{\delta l + R_B b - \eta} - \overline{\theta}_b = 0, \tag{A.6}$$

where $\underline{\theta}_l$, $\overline{\theta}_l$, and $\overline{\theta}_b$ are proportional to the non-negative Lagrange multipliers for $0 \leq l$, $l \leq k$, and $b \leq a$, respectively. The Lagrange multiplier for the other constraint, $0 \leq b$, is always zero, because it is positive only if $\underline{\theta}_l > \overline{\theta}_l$. Note that if $\underline{\theta}_l > \overline{\theta}_l$, then $b = l = 0$, under which $\delta l + R_B b - \eta$ is negative.

Given that (A.5) and (A.6) have the same denominators in the first two terms on their left-hand sides and also given the assumption that $R_L \geq \delta$, $\underline{\theta}_l = \overline{\theta}_l = \overline{\theta}_b = 0$ cannot hold. Thus, there are four cases to consider: $\{l = 1 - a,\ b = a\}$; $\{l \in (0,\ 1 - a),\ b = a\}$; $\{l = 0,\ b = a\}$; and $\{l = 0,\ b \in (0, a)\}$.

In the first case, $\underline{\theta}_l = 0$ and $\overline{\theta}_l \geq 0$. For this case to happen, it must hold that

$$\frac{R_L}{-[R_L(1-a) + R_B a - \eta] + \gamma\eta} \leq \frac{\delta}{\delta(1-a) + R_B a - \eta}. \tag{A.7}$$

Given (A.6) and the assumption that $R_L \geq \delta$, $\overline{\theta}_b > 0$.

In the second case, $\underline{\theta}_l = \overline{\theta}_l = 0$. In this case, (A.5) implies that

$$\exists l \in (0,\ 1-a),\ \text{s.t.}\ \frac{R_L}{-[R_L l + R_B a - \eta] + \gamma\eta} = \frac{\delta}{\delta l + R_B a - \eta}. \tag{A.8}$$

Given (A.6) and the assumption that $R_L \geq \delta$, $\overline{\theta}_b > 0$.

In the third case, $\underline{\theta}_l \geq 0$, $\overline{\theta}_l = 0$, and $\underline{\theta}_b \geq 0$. Thus, (A.6) implies

$$\frac{R_B}{-[R_B a - \eta] + \gamma\eta} \leq \frac{R_B}{R_B a - \eta}. \tag{A.9}$$

Also, (A.5) implies

$$\frac{R_L}{-[R_B a - \eta] + \gamma\eta} \geq \frac{\delta}{R_B a - \eta}. \tag{A.10}$$

In the fourth case, $\underline{\theta}_l \geq 0$, $\overline{\theta}_l = 0$, and $\underline{\theta}_b = 0$. Hence,

$$\exists b \in (0,\ a),\ \text{s.t.}\ \frac{R_B}{-[R_B b - \eta] + \gamma\eta} = \frac{R_B}{R_B b - \eta}. \tag{A.11}$$

This condition is sufficient for (A.5) under $l = 0$ and $\underline{\theta}_l \geq \overline{\theta}_l = 0$, given the assumption that $R_L \geq \delta$.

Summarizing the four cases, the solutions for l and b under $\delta < R_L$ take the following form:

$$(l(a),\ b(a)) = \begin{cases} (1-a,\ a)\,, & \text{if } R_B a - \eta \le \frac{\delta\gamma\eta - 2R_L\delta(1-a)}{R_L+\delta}, \\ \left(\frac{\delta\gamma\eta-(R_L+\delta)[R_B a-\eta]}{2R_L\delta},\ a\right), & \text{if } R_B a - \eta \in \left(\frac{\delta\gamma\eta-2R_L\delta(1-a)}{R_L+\delta},\ \frac{\delta\gamma\eta}{R_L+\delta}\right), \\ (0,\ a)\,, & \text{if } R_B a - \eta \in \left[\frac{\delta\gamma\eta}{R_L+\delta},\ \frac{\gamma\eta}{2}\right], \\ \left(0,\ \frac{1}{R_B}\left[\eta + \frac{\gamma\eta}{2}\right]\right), & \text{if } R_B a - \eta > \frac{\gamma\eta}{2}, \end{cases} \tag{A.12}$$

if both banks have non-negative trade surpluses.

In the third and the fourth case, it is immediate that both banks have non-negative trade surpluses. In the second case, the necessary and sufficient condition for non-negative trade surpluses for both banks is

$$\delta\gamma\eta + (R_L - \delta)[R_B a - \eta] \ge 0. \tag{A.13}$$

In the first case, the necessary and sufficient conditions for non-negative trade surpluses are:

$$\gamma\eta \ge R_L(1-a) + R_B a - \eta, \tag{A.14}$$

$$\delta(1-a) + R_B a - \eta \ge 0. \tag{A.15}$$

If (A.13) and (A.14)-(A.15) are not satisfied in the second and the first case, respectively, then banks do not settle bank transfers in period 1 in each case.

Now show the following lemma:

Lemma 1. Under Assumption 1, $(l(a),\ b(a)) = (1-a, a)$ never occurs in an equilibrium.

Proof. This lemma is equivalent to say that the first case of (A.12) does not exist for any $a \in [0,1]$, or violates (A.14) or (A.15). First, a necessary condition for the existence of the first case of (A.12) is that there exists $a \in [0,1]$ such that

$$R_B a - \eta \le \frac{\delta\gamma\eta - 2R_L\delta(1-a)}{R_L + \delta}, \tag{A.16}$$

as implied by (A.12). Note that both sides of this condition are increasing linear functions of a and also that the left-hand side is higher than the right-hand side at $a = 1$ under

Assumption 1. Thus, there exists $a \in [0,1]$ satisfying (A.16) if and only if the intercept of the left-hand side is lower than that of the right-hand side:

$$-\eta < \frac{\delta\gamma\eta - 2R_L\delta}{R_L + \delta}. \tag{A.17}$$

If this condition is violated, then the first case of (A.12) does not exist for any $a \in [0,1]$.

Suppose that (A.17) holds. This condition is equivalent to

$$(\eta - \delta)R_L > \delta[R_L - (1+\gamma)\eta]. \tag{A.18}$$

Thus, $\eta > \delta$, and hence $R_B > \delta$, must hold given Assumption 1. Given $R_B > \delta$, (A.14) and (A.15) can be written as

$$a \geq \max\left\{\frac{R_L - (1+\gamma)\eta}{R_L - R_B}, \frac{\eta - \delta}{R_B - \delta}\right\}, \tag{A.19}$$

where the first and the second term in the max operator are derived from (A.14) and (A.15), respectively. Under (A.17) and Assumption 1, it can be shown that:

$$\begin{aligned}
&\frac{R_L - (1+\gamma)\eta}{R_L - R_B} - \frac{\eta - \delta}{R_B - \delta} \\
&\propto R_L(R_B - \delta) - (1+\gamma)\eta(R_B - \delta) - \eta(R_L - R_B) + \delta(R_L - R_B) \\
&= R_L(R_B - \eta) + (\eta - \delta)R_B - (1+\gamma)\eta(R_B - \delta) \\
&= R_L(R_B - \eta) + (\eta - \delta)R_B - (1+\gamma)\eta(R_B - \eta + \eta - \delta) \\
&= [R_L - (1+\gamma)\eta](R_B - \eta) + (\eta - \delta)[R_B - (1+\gamma)\eta] > 0.
\end{aligned} \tag{A.20}$$

The inequality holds due to $\eta > \delta$ under (A.17). Thus, (A.14) is sufficient for (A.15) under (A.17) and Assumption 1.

Finally, show that (A.14) is violated in the first case of (A.12), if (A.17) and Assumption 1 hold. In this case, the first case of (A.12) can exist only for $a \in [0, a^*]$ such that

$$R_B a^* - \eta = \frac{\delta\gamma\eta - 2R_L\delta(1 - a^*)}{R_L + \delta}. \tag{A.21}$$

The root for this equation can be explicitly solved as

$$a^* = \frac{R_L(\eta - \delta) - \delta[R_L - (1+\gamma)\eta]}{R_L(R_B - \delta) - \delta(R_L - R_B)}. \tag{A.22}$$

It can be shown that

$$\begin{aligned}
a^* - \frac{R_L - (1+\gamma)\eta}{R_L - R_B} & \\
\propto\ & R_L(\eta - \delta)(R_L - R_B) - \delta[R_L - (1+\gamma)\eta](R_L - R_B) \\
& - R_L(R_B - \delta)[R_L - (1+\gamma)\eta] + \delta(R_L - R_B)[R_L - (1+\gamma)\eta] \\
=\ & R_L(\eta - \delta)(R_L - R_B) - R_L(R_B - \delta)[R_L - (1+\gamma)\eta] \\
\propto\ & \frac{\eta - \delta}{R_B - \delta} - \frac{R_L - (1+\gamma)\eta}{R_L - R_B} < 0, \qquad \text{(A.23)}
\end{aligned}$$

where the last inequality is implied by (A.20). Thus, a^* is below the lower bound for a that satisfies (A.14). Hence, banks cannot have non-negative trade surpluses in the first case of (A.12), if (A.17) holds. □

It remains to pin down the range of a for the second case of (A.12). The necessary and sufficient condition for non-negative trade surpluses in the second case of (A.12), i.e., (A.13), implies

$$a \geq \frac{1}{R_B}\left(\eta - \frac{\delta\gamma\eta}{R_L - \delta}\right). \tag{A.24}$$

Now show that the right-hand side of this condition is higher than the lower bound for a satisfying the range of $R_B a - \eta$ in the second case of (A.12).

If (A.17) is violated, then (A.16) never holds for $a \in [0, 1]$ as shown in the proof for Lemma 1. In this case, the range of $R_B a - \eta$ in the second case of (A.12) does not have a non-negative lower bound for a, so (A.24) becomes the lower bound for a in the second case of (A.12).

If (A.17) is satisfied, it can be shown that the right-hand side of (A.24) is greater than

a^* in (A.22), that is, the root for (A.21):

$$
\begin{aligned}
&\frac{1}{R_B}\left(\eta - \frac{\delta\gamma\eta}{R_L-\delta}\right) - a^* \\
&\quad \propto \eta[R_L-(1+\gamma)\delta][(R_L+\delta)R_B - 2\delta R_L] - \{\eta[R_L+(1+\gamma)\delta] - 2\delta R_L\}(R_L-\delta)R_B \\
&\quad = \eta R_L(2\delta R_B - 2\delta R_L) - \eta(1+\gamma)\delta(2R_LR_B - 2\delta R_L) + 2\delta R_L(R_L-\delta)R_B \\
&\quad = 2\delta R_L[-\eta(R_L-R_B) - \eta(1+\gamma)(R_B-\delta) + R_B(R_L-\delta)] \\
&\quad \propto -\eta(R_L-R_B) - \eta(1+\gamma)(R_B-\delta) + R_B(R_L-R_B+R_B-\delta) \\
&\quad = (R_L-R_B)(R_B-\eta) + (R_B-\delta)[R_B-(1+\gamma)\eta] > 0. \qquad \text{(A.25)}
\end{aligned}
$$

The last inequality follows from $R_B > \delta$ under Assumption 1 and (A.17).

Thus, (A.24) is the lower bound for a in the second case of (A.12) regardless of whether (A.17) is satisfied. Banks do not make any deal in period 1 if the value of a is lower than the right-hand side of (A.24).

C Proof for Proposition 2

If $l(a) = 0$ at the optimum of the bargaining problem (10), then each bank chooses the lower bound for a such that $l(a) = 0$, because a further increase in the bond holdings only results in a transfer of more bonds to the receiving bank given $l(a) = 0$, as implied by (8). Hereafter, denote the lower bound for a such that $l(a) = 0$ by $\hat{a}$.

Next, compare $\hat{a}$ and the value of a such that $l(a) > 0$. For the range of a such that $l(a) > 0$, the objective function in the profit maximization problem for a bank in period 0, (10), can be written as

$$
\begin{aligned}
\Pi(a,\ a') \equiv{}& R_L(1-a) + R_Ba - 1 \\
&+ \frac{1}{2}\left\{-R_L\frac{\delta\gamma\eta - (R_L+\delta)(R_Ba-\eta)}{2R_L\delta} - R_Ba + \eta\right\} + \frac{1}{2}\phi(a'). \qquad \text{(A.26)}
\end{aligned}
$$

The derivative of this function with respect to a is

$$\begin{aligned}\frac{\partial \Pi(a,\ a')}{\partial a} &= -R_L + R_B + \frac{1}{2}\left[\frac{R_L(R_L+\delta)R_B}{2R_L\delta} - R_B\right] && \text{(A.27)}\\ &= -R_L + R_B + \frac{1}{2}\frac{R_L-\delta}{2\delta}R_B && \text{(A.28)}\\ &\propto R_L R_B - \delta(4R_L - 3R_B). && \text{(A.29)}\end{aligned}$$

Because the objective function in the profit maximization problem for a bank in period 0, (10), is continuous at both $\hat{a}$ and the upper bound for a such that $l(a) > 0$, choosing a value of a such that $l(a) > 0$ is dominated by choosing $a = \hat{a}$ in period 0 if

$$\delta < \frac{R_L R_B}{4R_L - 3R_B}. \tag{A.30}$$

Finally, find the condition under which choosing $a = \hat{a}$ in period 0 dominates no settlement of outgoing bank transfers. If no settlement of outgoing bank transfers is optimal for a bank, then each bank sets $a = 0$ in period 0 because it does not need any liquidity for interbank settlement in period 1. Thus, choosing $a = \hat{a}$ in period 0 dominates no settlement of outgoing bank transfers if and only if

$$\begin{aligned}R_L\left[1 - \frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L+\delta}\right)\right] + R_B\left[\frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L+\delta}\right)\right] - 1&\\ + \frac{1}{2}\left[-R_B\frac{1}{R_B}\left(\eta + \frac{\delta\gamma\eta}{R_L+\delta}\right) + \eta\right] + \frac{1}{2}\phi(a')&\\ > R_L - 1 + \frac{1}{2}(-\gamma\eta) + \frac{1}{2}\phi(a'),&\end{aligned} \tag{A.31}$$

where the left- and the right-hand side are the expected profit in period 2 for a bank with $a = \hat{a}$ and $a = 0$, respectively. This condition is equivalent to

$$\delta < \frac{R_L[(2+\gamma)R_B - 2R_L]}{2(R_L - R_B)(1+\gamma)}, \tag{A.32}$$

because

$$
\begin{aligned}
-\left(\frac{R_L}{R_B}-1\right)&\left(\eta+\frac{\delta\gamma\eta}{R_L+\delta}\right)+\frac{1}{2}\left(-\frac{\delta\gamma\eta}{R_L+\delta}\right)-\frac{1}{2}(-\gamma\eta)\\
&\propto -2\left(\frac{R_L}{R_B}-1\right)\eta[R_L+(1+\gamma)\delta]+\gamma\eta R_L\\
&= -2\left(\frac{R_L}{R_B}-1\right)\eta(1+\gamma)\delta+\eta R_L\left[\gamma-2\left(\frac{R_L}{R_B}-1\right)\right]. \quad \text{(A.33)}
\end{aligned}
$$

If both (A.30) and (A.32) hold, then it is optimal for a bank to choose $a=\hat{a}$ in period 0. Under Assumption 2, (A.30) is sufficient for (A.32) because

$$
\begin{aligned}
\frac{R_L[(2+\gamma)R_B-2R_L]}{2(R_L-R_B)(1+\gamma)}&-\frac{R_LR_B}{4R_L-3R_B}\\
&\propto [(2+\gamma)R_B-2R_L](4R_L-3R_B)-2(R_L-R_B)(1+\gamma)R_B\\
&= \gamma R_B(4R_L-3R_B)-2(R_L-R_B)(4R_L-3R_B)\\
&\quad -2(R_L-R_B)(1+\gamma)R_B\\
&= (2R_L-R_B)[\gamma R_B-4(R_L-R_B)]. \quad \text{(A.34)}
\end{aligned}
$$

Assumption 4, in turn, is sufficient for (A.30) under Assumption 2, because

$$
\begin{aligned}
\frac{R_LR_B}{4R_L-3R_B}-\frac{R_L}{1+\gamma} &\propto R_B(1+\gamma)-(4R_L-3R_B) \quad \text{(A.35)}\\
&= -4(R_L-R_B)+R_B\gamma. \quad \text{(A.36)}
\end{aligned}
$$

Thus, Assumptions 2 and 4 are sufficient for $a=\hat{a}$ in period 0.

D Solution to the baseline model when the partial settlement of bank transfers is possible

If banks can settle only part of requested bank transfers, η, the Nash bargaining problem between banks in period 1 is modified to

$$
\max_{\{l\in[0,k],\, b\in[0,a],\, \kappa\in[0,1]\}}[-(R_Ll+R_Bb-\kappa\eta)-\gamma(1-\kappa)\eta-(-\gamma\eta)]^{0.5}(\delta l+R_Bb-\kappa\eta)^{0.5}. \quad \text{(A.37)}
$$

for $\delta \in (0, R_L]$, where κ denotes the settled fraction of bank transfers. In the left square bracket, $-\gamma\eta$ is the originating bank's payoff at the threat point. Denote the trade surplus for each bank by

$$\Gamma(l,\ b,\ \kappa)\ \equiv -(R_L l + R_B b - \kappa\eta) - \gamma(1-\kappa)\eta - (-\gamma\eta), \tag{A.38}$$

$$\Theta(l,\ b,\ \kappa)\ \equiv \delta l + R_B b - \kappa\eta. \tag{A.39}$$

The first-order conditions imply

$$-\frac{R_L}{\Gamma(l,\ b,\ \kappa)} + \frac{\delta}{\Theta(l,\ b,\ \kappa)} \gtreqless 0\ \Leftrightarrow l \begin{cases} = k \\ \in [0, k] \\ = 0, \end{cases} \tag{A.40}$$

$$-\frac{R_B}{\Gamma(l,\ b,\ \kappa)} + \frac{R_B}{\Theta(l,\ b,\ \kappa)} \gtreqless 0\ \Leftrightarrow b \begin{cases} = a \\ \in [0, a] \\ = 0, \end{cases} \tag{A.41}$$

$$\frac{1+\gamma}{\Gamma(l,\ b,\ \kappa)} - \frac{1}{\Theta(l,\ b,\ \kappa)} \gtreqless 0\ \Leftrightarrow \kappa \begin{cases} = 1 \\ \in [0, 1] \\ = 0. \end{cases} \tag{A.42}$$

Under Assumption 3, (A.40) and (A.41) imply (4), under which (A.42) implies $\kappa = 1$. Thus, the solution to the baseline model does not change in case of liquid bank loans.

Under Assumption 4, (A.40) and (A.42) imply that $l > 0$ can never be part of the solution, as each bank's trade surplus must be non-negative. Given $l = 0$, (A.40)-(A.42) yield the following solution:

$$(l,\ b,\ \kappa) = \begin{cases} \left(0,\ \frac{1}{R_B}\left(\eta + \frac{\gamma\eta}{2}\right),\ 1\right), & \text{if } R_B a - \eta \geq \frac{\gamma\eta}{2}, \\ (0,\ a,\ 1), & \text{if } R_B a - \eta \in \left[\frac{\gamma\eta}{2+\gamma},\ \frac{\gamma\eta}{2}\right], \\ \left(0,\ a,\ \frac{(2+\gamma)R_B a}{2(1+\gamma)\eta}\right), & \text{if } R_B a - \eta \in \left[-\eta,\ \frac{\gamma\eta}{2+\gamma}\right]. \end{cases} \tag{A.43}$$

Note that the lower bound for a in the first case is less than one given (1), and also that the lower bound for a in the third case is zero.

The portfolio choice problem for each bank in period 0 is specified as (10), except that the definition of $\theta(a)$ and $\phi(a')$ are modified according to the solution to the Nash bargaining problem described above. Given (2), the solution to the portfolio choice problem implies

$$(k,\ a) = \left(1 - a, \frac{1}{R_B}\left(\eta + \frac{\gamma\eta}{2+\gamma}\right)\right), \tag{A.44}$$

which is feasible given (1). Thus, a bank chooses a to minimize a premium for the settlement of bank transfers while choosing $\kappa = 1$, i.e., settling all the bank transfers, as implied by (A.43). Nonetheless, the originating bank must pays a premium, as in the baseline model in the main text.

E Comparison between the channel system and the floor system

This section confirms the optimality of the floor system in comparison to an alternative modern reserve supply policy, the so-called channel system. This system has been adopted by Australia, Canada, the Euro area, Norway, and the U.K. In this system, the central bank supplies only a tiny, or even no, amount of bank reserves to banks overnight. For example, the targeted overnight balance of bank reserves in Canada was zero for March 2006 to May 2007. The central bank, however, allows a large volume of collateralized overdrafts of bank reserves during each day, so that banks can smoothly send bank reserves to each other to settle bank transfers among them. The settlement of bank transfers is final immediately after bank reserves are transferred to a receiving bank, because the central bank guarantees the settlement in any event during the course of the day.

At the end of each day, an imbalance between outgoing and incoming bank transfers for each bank results in a distribution of debit and credit positions in bank reserves across banks. If these positions are left overnight, then the central bank charges a higher interest rate on a debit position than the interest rate that it pays on a credit position. Typically, banks arrange overnight loans of bank reserves in the interbank money market, so that no bank has a debit position in bank reserves overnight. The interbank interest rate tends to fall between the two central-bank overnight interest rates on bank reserves.

E.1 Extension of the model to nest the channel system

To nest the channel system in the model, consider the following contract. In period 0, the central bank requires each bank to pledge an amount $(1 + f)\eta/R_B$ of bonds as collateral, where f is the central-bank interest rate on overnight credit positions in bank reserves, which will be defined below. This collateral corresponds to the collateral for an intraday overdraft of bank reserves at the central bank in reality. In period 1, the central bank transfers an amount η/R_B of collateral from the originating bank to the receiving bank on its book, guaranteeing the settlement of bank-transfer requests to each bank, $(\lambda_A,\ \lambda_B)$, immediately after they are realized. This assumption reflects the fact that the central bank provides intraday overdrafts of bank reserves and the finalty of bank-reserve transfers in the channel system. If no action is taken afterwards, then the central bank charges the originating bank an interest rate, f, on the face value of bank transfers, η. This interest charge is subtracted from collateral pledged by the originating bank in period 0; thus, no collateral is returned to the originating bank in this case. Also, the central bank adds an amount $d\eta/R_B$ of bonds to the receiving bank's collateral, where d is the overnight interest rate on a credit position in bank reserves. Assume $f \geq d \geq 0$, so that the central bank has enough bonds to pass to the receiving bank. The central bank returns the remaining balance of collateral for each bank in period 2.

If the receiving bank sends a reverse transfer of bank reserves, η, to the originating bank, then the net transfer of bank reserves becomes zero for each bank. In this case, the central bank neither charges or adds interest on any bank's collateral in period 1. Assume that after the realization of bank-transfer requests in period 1, banks can negotiate the terms of a reverse transfer of bank reserves through Nash bargaining with equal bargaining power for each bank. This transaction corresponds to a repo in the OTC interbank money market, as described in section 3.5. To minimize the friction associated with the channel system, assume that the originating bank can receive a reverse transfer of bank reserves to pay off an overdraft at the central bank at the same time as it takes back collateral from the central bank and transfers it to the receiving bank. The central bank typically allows such a transaction for a bank running an overdraft in practice.

E.2 Equilibrium with the channel system

Now solve the bargaining problem for a reverse transfer of bank reserves in period 1. This problem can be written as:

$$\max_{b'\geq 0}\{-(R_B b' - \eta) - (-f\eta)\}^{0.5}[R_B b' - \eta - d\eta]^{0.5}, \tag{A.45}$$

where b' is the amount of bonds that the originating bank pays to the receiving bank for a reverse transfer of bank reserves. The left curly and the right square bracket in (A.45) are the trade surpluses for the originating and the receiving bank, respectively. Note that the threat point is determined by interest rates on overnight debit and credit positions in bank reserves, f and d, because bank transfers requested by depositors are settled before the bargaining under the central bank's guarantee.

The solution to this problem is

$$b' = \left(1 + \frac{f+d}{2}\right)\frac{\eta}{R_B}. \tag{A.46}$$

Thus, the interbank overnight interest rate, $(f + d)/2$, falls between the two central-bank overnight interest rates, f and d. Note that paying b' is feasible for a bank, as it is less than the amount of bonds pledged as collateral to the central bank, $(1 + f)\eta/R_B$.

E.3 Liquidity costs associated with the channel system

Overall, each bank in this contract must invest into an amount $(1 + f)\eta/R_B$ of bonds in period 0. To minimize the amount of bonds necessary, the central bank must set $f = d = 0$. Note that this policy eliminates incentive for banks to trade in period 1, and makes the contract equivalent to the optimal contract shown in section 4, which corresponds to the floor system. Thus, the floor system dominates the channel system, as long as the channel system involves the active use of the OTC interbank money market.

This result illustrates two liquidity costs associated with the channel system. First, allowing banks to have an intraday debit position in bank reserves raises a collateral requirement for banks, because the central bank must make sure that an overnight debit position in bank reserves is secured if a bank does not repay an intraday debit position in bank reserves by the end of the day.

Second, even if the central bank can induce banks to pay off an intraday debit position in bank reserves without collateral, banks still need to hold the amount of bonds necessary to settle bank transfers in the OTC interbank money market in period 1, i.e., b'. If f and d are set positive as is usual in the channel system, then the value of b' exceeds the amount of bonds necessary for the floor system, i.e., η/R_B. This result holds because raising f and d strengthens the bargaining position of the receiving bank against the originating bank. Thus, the active use of the OTC interbank money market makes the liquidity-saving effect of the channel system smaller than that of the floor system.