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## **Reputation and Liquidity Traps**

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# Reputation and Liquidity Traps\*

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## Abstract

Can the central bank credibly commit to keeping the nominal interest rate low for an extended period of time in the aftermath of a deep recession? By analyzing credible plans in a sticky-price economy with occasionally binding zero lower bound constraints, I find that the answer is yes if contractionary shocks hit the economy with sufficient frequency. In the best credible plan, if the central bank reneges on the promise of low policy rates, it will lose *reputation* and the private sector will not believe such promises in future recessions. When the shock hits the economy sufficiently frequently, the incentive to maintain reputation outweighs the short-run incentive to close consumption and inflation gaps, keeping the central bank on the originally announced path of low nominal interest rates.

JEL: E32, E52, E61, E62, E63

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# 1 Introduction

Statements about the period during which the short-term nominal interest rate is expected to remain near zero have been an important feature of recent monetary policy in the United States. The FOMC has stated that a highly accommodative stance of monetary policy will remain appropriate for a considerable time *after* the economic recovery strengthens.<sup>1</sup> With the current policy rate at its effective lower bound, the expected path of short-term rates is a prominent determinant of the long-term interest rates, which affects the decisions of households and businesses. Thus, the statement expressing the FOMC's intention to keep the policy rate low for a considerable period has likely done much to keep the long-term nominal rates low and thereby stimulating economic activities.

Some policymakers and economists have debated whether these statements should be interpreted as a commitment to optimal time-inconsistent policy.<sup>2</sup> In the New Keynesian model—a widely-used model of monetary policy at central banks—in response to a large contractionary shock, the central bank equipped with commitment technology promises to keep the nominal interest rate low even after the contractionary shock disappears. Such a promise reduces the long-term real interest rate and stimulates household spending.<sup>3</sup> However, in the model, if the central bank were to re-optimize again after the shock disappears, it would renege on the promise and raise the rate to close consumption and inflation gaps. In other words, the policy of an extended period of low nominal interest rates is *time-inconsistent*. In reality, no central bank has an explicit commitment device to bind its future policy decisions. Thus, while the theory of optimal commitment policy can explain why the central bank should promise an extended period of low policy rates, it can neither explain why the central bank should fulfill such a promise, nor why the private sector should believe it.

This paper provides a theory that explains why the central bank may want to fulfill the promise of keeping the nominal interest rate low even after the economic recovery strengthens. The theory is based on credible plans in a stochastic New Keynesian economy in which the nominal interest rate is subject to the zero lower bound constraint and contractionary shocks hit the economy occasionally. Credible plans can capture rich interactions between the government action and the private sector's belief. I use this equilibrium concept to ask under what conditions, if any, the policy of keeping the nominal interest rate low even after the economic recovery strengthens is time-consistent.<sup>4</sup>

I find that the policy of keeping the nominal interest rate low for long is time-consistent if the frequency of contractionary shocks is sufficiently high. The force that keeps the central bank from raising the nominal interest rate is *reputation*. In the best credible equilibrium, if the central bank reneges on its promise to keep the nominal interest rate low, it will lose reputation and the

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<sup>1</sup>See the FOMC statements since September 2012.

<sup>2</sup>For alternative perspectives on the degree of commitment implied by the FOMC's statement, see Bullard (2013), Dudley (2013), and Woodford (2012).

<sup>3</sup>See, for example, Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Adam and Billi (2006), and Werning (2012).

<sup>4</sup>The equilibrium concept has been referred to as many different names, including sustainable plans, reputational equilibria, sequential equilibria, and subgame Markov-Perfect equilibria.

private sector will never believe such promises in the face of future contractionary shocks. If the private sector does not believe the promise of an extended period of low nominal interest rates, the contractionary shock will cause large declines in consumption and inflation. Large consumption collapse and deflation in the future liquidity traps reduce welfare even during normal times because the central bank cares about the discounted sum of future utility flows. Thus, the potential loss of reputation gives the central bank an incentive to fulfill the promise. When the frequency of shocks is sufficiently high, this incentive to maintain reputation outweighs the short-run incentive to raise the rate to close consumption and inflation gaps, keeping the central bank on the originally announced path of low nominal interest rates.

I arrive at this result in two steps. First, I construct a plan—a pair of government and private sector strategies—that induces the outcome that would prevail under the discretionary government. I will refer to this plan as *the discretionary plan* and show that this plan is time-consistent. Second, I propose a plan that guides the government to adhere to the Ramsey policy and the private sector to act accordingly, but instructs the private sector to believe that the government is following the discretionary outcome if the government ever deviates from the Ramsey policy. I will refer to this plan as *the revert-to-discretion plan*. By construction, this plan induces the Ramsey outcome. I then show by numerical simulations that the revert-to-discretion plan is time-consistent if the contractionary shock hits the economy sufficiently frequently.

The threshold frequency of the crisis above which the Ramsey outcome is time-consistent is very small when the model is calibrated so that the declines in inflation and output in the crisis state under the discretionary outcome are broadly in line with those during the Great Recession. Under the “Great Recession” parameterization, the threshold crisis frequency is 0.015 percentage points, implying that the revert-to-discretion plan is time-consistent if the crisis on average occurs at least once every 1,700 years. Even when the reversion to the discretionary outcome is assumed to last for finite periods, the threshold frequency remains small. For example, when the punishment regime lasts for 10 years, the threshold crisis frequency is 0.3 percentage points, implying that the revert-to-discretion plan is time-consistent if the crisis on average occurs at least once every 80 years.

The recent recession has shown that the zero lower bound can be a binding constraint in many advanced economies. Some argue that the zero lower bound is likely to bind more frequently in the future than in the past.<sup>5</sup> Accordingly, developing effective strategies to mitigate the adverse consequences of the zero lower bound is an important task for macroeconomists. While many researchers have shown theoretically the value of optimal commitment policy in limiting the adverse consequences of the zero lower bound constraints, there are concerns about the effectiveness of this policy in reality, partly based on the notion that this policy is time-inconsistent.<sup>6</sup> The result of this paper—reputational force, combined with a very small crisis probability, can make this policy

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<sup>5</sup>See, for example, IMF (2014)

<sup>6</sup>See Adam and Billi (2007), Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005) among others for the value of commitment policy at the ZLB. See Plosser (2013) and Sheard (2013) among others for this view.

time-consistent—can be interpreted as alleviating such concerns.

This paper is related to the literature examining whether reputation can make the Ramsey equilibrium time-consistent in various macroeconomic models. In early contributions, Barro and Gordon (1983) and Rogoff (1987) asked whether reputation can overcome inflation bias in models with a short-run trade-off between inflation and output. Chari and Kehoe (1990), Phelan and Stacchetti (2001), and Stokey (1991) have studied the time-consistency of the Ramsey policy in models of fiscal policy, while Chang (1998), Ireland (1997), and Orlik and Presno (2010) have studied the time-consistency of the Friedman rule in monetary models. More recently, Kurozumi (2008), Loisel (2008) and Sunakawa (2013) studied the time-consistency of the Ramsey policy in New Keynesian models with cost-push shocks but without the zero lower bound constraint.

The paper is closely related to other works examining how the government can improve allocations at the zero lower bound in the absence of commitment technology. Eggertsson (2006) and Bhattarai, Eggertsson, and Gafarov (2013) have showed that, if the government has access to nominal debt, it chooses to issue nominal debt during the period of contractionary shocks so as to give the future government an incentive to lower the nominal interest rate and create inflation, and that this goes a long way toward achieving the Ramsey allocations. Jeanne and Svensson (2007) demonstrated that, if the government has concerns about its balance sheet, it can attain the Ramsey allocations by managing the balance sheet so as to give the future government an incentive to depreciate its currency and thus create inflation. This paper contributes to this body of work by proposing a new mechanism by which the central bank can attain the Ramsey allocations without a commitment technology. The proposed mechanism in this paper is novel in that it does not involve any additional policy instruments.

The paper is also related to Bodenstein, Hebden, and Nunes (2012) who study the consequence of imperfect credibility in the context of a New Keynesian economy with occasionally binding zero lower bound constraints. While both papers are motivated by the idea that credibility of the central bank may be a key factor in understanding the effectiveness of the forward guidance policy, our approaches and the questions we ask are different. Their analysis is positive. They model imperfect credibility in a specific way—randomizing the timing of central bank’s optimization in a way reminiscent of the Calvo-pricing model—and ask how imperfect credibility affects output and inflation at the zero lower bound. On the other hand, my analysis is normative. I ask why the central bank may want to fulfill the promise and under what conditions the Ramsey outcome can prevail.

The rest of the paper is organized as follows. Section 2 describes the model and defines the competitive equilibria. Section 3 defines the discretionary and the Ramsey outcomes and discusses their key features. Section 4 defines a plan and credibility, and section 5 constructs the revert-to-discretionary plan that induces the Ramsey outcome. Section 6 presents the main results on the credibility of the Ramsey outcome, and section 7 explores their quantitative importance in a calibrated model. Section 8 discusses additional results and a final section concludes.

## 2 Model and competitive outcomes

The model is given by a standard New Keynesian economy. The environment features a representative household, monopolistic competition among a continuum of intermediate-goods producers, and sticky prices. The model abstracts from capital. Since this is a workhorse model, I will start with the well-known equilibrium conditions of the economy.<sup>7</sup> Following the majority of the previous literature on the zero lower bound, I will conduct the analysis in a partially log-linearized version of the model, in which the equilibrium conditions are log-linearized except for the zero lower bound constraint on the nominal interest rate.<sup>8</sup>

The only exogenous variable of the model is  $s_t$ , interpreted as “the natural rate of interest.” I will also refer to  $s_t$  as the contractionary shock, the crisis shock, or the state.  $s_t$  takes two values, H and L. H will be set to the steady-state real interest rate, and L will be assigned to a negative value so that the nominal interest rate that would keep inflation and consumption at the steady-state level is negative. When  $s_t = H$ , the economy is said to be in the high state or the normal state. When  $s_t = L$ , I will say that the economy is in the low state, or the economy is hit by the contractionary or crisis shock. A lower L will be interpreted as the shock being more severe. I will use  $s^t$  to denote a history of states up to period t (i.e.  $s^t := \{s_k\}_{k=1}^t$ ) and  $\mathbb{S}$  to denote the set of values  $s_t$  can take, i.e.,  $\mathbb{S} := \{L, H\}$ .

The natural rate of interest rate evolves according to a two-state Markov process. Transition probabilities are given by

$$\text{Prob}(s_{t+1} = L | s_t = H) = p_H \tag{1}$$

$$\text{Prob}(s_{t+1} = L | s_t = L) = p_L \tag{2}$$

$p_H$  is the probability of moving to the low state next period when the economy is in the normal state today, and will be referred to as the *frequency* of the contractionary shocks.  $p_L$  is the probability of staying in the low state when the economy is in the low state today, and will be referred to as the *persistence* of the contractionary shocks. One key exercise of this paper will be to examine the credibility of the Ramsey policy in various economies with different values of  $p_H$  and  $p_L$ .

I refer to the state-contingent sequence of consumption, inflation, and the nominal interest rate,  $\{c_t(s^t), \pi_t(s^t), r_t(s^t)\}_{t=1}^{\infty}$ , as *an outcome*. Given a process of  $s_t$ , an outcome is said to be *competitive* if, for all  $t \geq 1$  and  $s^t \in \mathbb{S}^t$ ,  $c_t(s^t) \in \mathbb{C} := [c_{min}, c_{max}]$ ,  $\pi_t(s^t) \in \Pi := [\pi_{min}, \pi_{max}]$ ,  $r_t(s^t) \in \mathbb{R} := [r_{min}, r_{max}]$  and

$$\chi_c c_t(s^t) = \chi_c E_t c_{t+1}(s^{t+1}) - [r_t - E_t \pi_{t+1}(s^{t+1})] + s_t \tag{3}$$

$$\pi_t = \kappa c_t + \beta E_t \pi_{t+1}(s^{t+1}) \tag{4}$$

<sup>7</sup>See, for example, Clarida, Gali, and Gertler (1999)

<sup>8</sup>See, for example, Eggertsson and Woodford (2003) and Werning (2012).

where  $c_t(s^t)$  and  $\pi_t(s^t)$  are consumption and inflation expressed as the log deviation from the deterministic steady-state and  $r_t(s^t)$  is the nominal interest rate.<sup>9</sup>  $\chi_c$  is the inverse intertemporal elasticity of substitution of the representative household, and  $\kappa$  is the slope of the Phillips curve. The zero lower bound constraint on the nominal interest rate is imposed by setting

$$r_{min} = 0 \tag{5}$$

$c_{max}$  is motivated by the fact that the economy has a finite amount of labor.  $c_{min}$  is motivated by the fact that consumption cannot fall below zero.  $\pi_{max}$  and  $\pi_{min}$  will be implied by the quadratic price adjustment cost function if the original nonlinear model is given by the Rotemberg-pricing model.<sup>10</sup> I assume that there exists an upper bound on the nominal interest rate,  $r_{max}$ , and that  $c_{min}$  is sufficiently small given  $r_{max}$  so that I can abstract from corner solutions in the consumer's problem where the consumption Euler equation does not hold with equality. This assumption considerably eases the exposition and is made without loss of generality.<sup>11</sup>

The government's objective function at period  $t$  is given by

$$w_t(s^t) := E_t \sum_{j=0}^{\infty} \beta^j u(\pi_{t+j}(s^{t+j}), c_{t+j}(s^{t+j})) \tag{6}$$

where the utility flow at each period is given by the following function.

$$u(\pi, c) := -\frac{1}{2} [\pi^2 + \lambda c^2] \tag{7}$$

For any outcome, there is an associated state-contingent sequence of values,  $\{w_t(s^t)\}_{t=1}^{\infty}$ , which will be referred to as a *value sequence*.

## Notations

Throughout the paper, I will use the following notations. For any variable  $x$ , its state-contingent sequence is denoted by  $\mathbf{x}$ . In other words,

$$\mathbf{x} := \{x_k(s^k)\}_{k=1}^{\infty}$$

A *state-contingent sequence up to time  $t$*  and a *continuation state-contingent sequence starting at time  $t$*  are respectively denoted by  $\mathbf{x}^t$  and  $\mathbf{x}_t$ . In other words,

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<sup>9</sup>In the model without investment and government spending, consumption equals output. It is common in the New Keynesian literature to replace consumption with output in the Euler equation. However, I will depart from the common practice in presenting the model. Formulating credible plans requires us to specify who chooses what and when, and it is more natural to think of the household as choosing consumption, instead of output.

<sup>10</sup>If the original nonlinear model is given by the Calvo model, then  $\pi_{min}$  is given by the fraction of firms allowed to reset their prices each period. When optimizing firms decide to reduce prices to arbitrarily close zero, the aggregate price declines by the fraction of optimizing firms. In the Calvo model, there is no force to bound inflation rate from above. The results of the paper do not depend on the bounds on inflation nor consumption.

<sup>11</sup>Otherwise, the definition of competitive outcomes needs to be modified to allow for the possibility that  $\chi_c E_t c_{t+1}(s^{t+1}) - [r_t - E_t \pi_{t+1}(s^{t+1})] + s_t < c_{min}$ .

$$\mathbf{x}^t := \{x_k(s^k)\}_{k=1}^t, \quad \mathbf{x}_t := \{x_k(s^k)\}_{k=t}^\infty$$

$x^t$  (non-bold font) is used to denote a particular realization of  $\mathbf{x}^t$ , and should not be confused with  $\mathbf{x}^t$  (bold font).

For any variable  $x$  with a range  $\mathbb{X}$ ,  $x(s)$  denotes a state-contingent sequence with  $s_1 = s$ , which is defined by a sequence of functions mapping a history of states with  $s_1 = s$  to  $\mathbb{X}$ . In other words,

$$\begin{aligned} x_1 &: s \rightarrow \mathbb{X} \\ x_t &: s \times \mathbb{S}^{t-1} \rightarrow \mathbb{X} \end{aligned}$$

$x^t(s)$  denotes a state-contingent sequence with  $s_1 = s$  up to time  $t$ .  $\mathbf{x}_t(s)$  denotes a continuation state-contingent sequence starting at time  $t$  with  $s_t = s$ , which is formally given by the following sequence of functions.

$$\begin{aligned} x_t &: s \rightarrow \mathbb{X} \\ x_{t+k} &: s \times \mathbb{S}^{k-1} \rightarrow \mathbb{X} \end{aligned}$$

$CE(s)$  denotes the set of all competitive outcomes with  $s_1 = s$ . That is, for each  $s \in \mathbb{S}$ ,

$$\begin{aligned} CE(s) &:= \{(\mathbf{c}(s), \boldsymbol{\pi}(s), \mathbf{r}(s)) \in \mathbb{C}^\infty \times \Pi^\infty \times \mathbb{R}^\infty \\ &\quad | \text{Equations (3) and (4) hold for all } t \geq 1 \text{ and for all } s^t \in \mathbb{S}^t \text{ with } s_1 = s\} \end{aligned}$$

$CE_k(s)$  denotes the set of continuation competitive outcomes starting at period  $k$  with  $s_k = s$ . That is, for each  $s \in \mathbb{S}$ ,

$$\begin{aligned} CE_k(s) &:= \{(\mathbf{c}_k(s), \boldsymbol{\pi}_k(s), \mathbf{r}_k(s)) \in \mathbb{C}^\infty \times \Pi^\infty \times \mathbb{R}^\infty \\ &\quad | \text{Equations (3) and (4) hold for all } t \geq k \text{ and for all } s^t \in \mathbb{S}^t \text{ with } s_k = s\} \end{aligned}$$

### 3 The discretionary outcome and the Ramsey outcome

This section defines the discretionary and Ramsey outcomes, and discusses their key features. These outcomes will play a major role in the analysis of credible policies in later sections.



### 3.1 The discretionary outcome

At each time  $t \geq 1$ , the discretionary government chooses today's consumption, inflation, and nominal interest rate in order to maximize its objective function, taking as given the value function and policy functions for consumption, inflation, and the nominal interest rate in the next period.<sup>12</sup> The Bellman equation of the government's problem is given by

$$w_t(s_t) = \max_{\{c_t \in \mathbb{C}, \pi_t \in \Pi, r_t \in \mathbb{R}\}} u(c_t, \pi_t) + \beta E_t w_{t+1}(s_{t+1}) \quad (8)$$

where the optimization is subject to the equations characterizing the competitive equilibria (i.e., equations (3) and (4)). Let  $\{w_d(\cdot), c_d(\cdot), \pi_d(\cdot), r_d(\cdot)\}$  be the set of time-invariant value function and policy functions for consumption, inflation, and the nominal interest rate that solves this problem in which the ZLB only binds in the low state.<sup>13</sup> *The discretionary outcome* is defined as, and denoted by, the state-contingent sequence of consumption, inflation, and the nominal interest rate,  $\{c_{d,t}(s^t), \pi_{d,t}(s^t), r_{d,t}(s^t)\}_{t=1}^{\infty}$  such that  $c_{d,t}(s^t) := c_d(s_t)$ ,  $\pi_{d,t}(s^t) := \pi_d(s_t)$ , and  $r_{d,t}(s^t) := r_d(s_t)$  and *the discretionary value sequence* is defined as, and denoted by,  $\{w_{d,t}(s^t)\}_{t=1}^{\infty}$  such that  $w_{d,t}(s^t) := w_d(s_t)$ .

Figure 1 shows the discretionary outcome and value sequence for a particular realization of  $s^{10} \in \mathbb{S}^{10}$  in which  $s_1 = L$ , and  $s_t = H$  for  $2 \leq t \leq 10$ . It also plots the sequence of contemporaneous utility,  $\{u(c_{d,t}, \pi_{d,t})\}_{t=1}^{\infty}$  associated with the consumption and inflation sequence. In each panel, solid black and dashed red lines are for the economy with  $p_H = 0.01$  and  $p_H = 0$ . Values for other parameters are the same in both black and red lines, and are listed in Table 1.

In the model without commitment, as soon as the contractionary shock disappears, the government raises the nominal interest rate in order to stabilize consumption and inflation. In the model where the high state is an absorbing state (i.e.,  $p_H=0$ ), the government raises the nominal interest rate to H, and consumption and inflation are fully stabilized at zero at time 2. Accordingly, the contemporaneous utility is zero as well. In the model with a positive  $p_H$ , the household and firms will have an incentive to lower consumption and prices in the normal period, as they expect that consumption and inflation will decline in some states tomorrow. The government tries to prevent those declines by reducing the nominal interest rates from the deterministic steady-state level, and in equilibrium, consumption and inflation are respectively slightly positive and negative. As a result, the contemporaneous utility flows are slightly negative.

One key feature of the model with recurring shocks is that the discretionary value remain negative even after the shock disappears, as captured in the the dashed red line bottom-right panel. The discretionary value stays negative even during the normal times for two reasons. First, consumption and inflation are slightly positive and negative due to the anticipation effects described

<sup>12</sup>Following the literature, I assume that the discretionary government acts as a planner and chooses the policy instrument and allocations without being explicit about the within-period timing assumption of the government and the private sector. While it is not important here, the within-period timing will be crucial in analyzing credible plans in later sections.

<sup>13</sup>In the Appendix, I demonstrate the existence of a time-invariant solution to this discretionary government's problem in which the ZLB binds in both states.

above, pushing down contemporaneous utility flow below zero in the high state. Second, and more quantitatively importantly, the possibility that consumption and inflation will decline in response to the future contractionary shock tomorrow lowers the discretionary value by reducing the continuation value of the government. This is in a sharp contrast to the economy in which the contractionary shock never hits after the initial shock. In such a model, the discretionary value becomes zero after the shock disappears, as shown in the red line in the bottom-right panel. This feature of the economy with recurring shocks—that the discretionary values remain negative even after the shock disappears—will be important in understanding the reputational force present in the credible plans.

### 3.2 The Ramsey outcome

The Ramsey planner chooses a state-contingent sequence of consumption, inflation, and the nominal interest rate in order to maximize the expected discounted sum of future utility flows at time one. For each  $s_1 \in \mathbb{S}$ , the Ramsey planner's problem is given by

$$\max_{(c(s_1), \pi(s_1), r(s_1)) \in CE(s_1)} w_1(s_1) \quad (9)$$

where the optimization is subject to the equations characterizing the competitive equilibria (i.e., equations (3) and (4)). *The Ramsey outcome* is defined as the state-contingent sequence of consumption, inflation, the nominal interest rate that solves the problem above. In other words, the Ramsey outcome is a competitive outcome with the highest time-one value. I will denote the Ramsey outcome by  $\{c_{ram,t}(s^t), \pi_{ram,t}(s^t), r_{ram,t}(s^t)\}_{t=1}^{\infty}$ . At each period  $t$  and for each  $s^t \in \mathbb{S}^t$ , the value associated with the Ramsey outcome is given by

$$w_{ram,t}(s^t) := E_t \sum_{j=0}^{\infty} \beta^j u(\pi_{ram,t+j}(s^{t+j}), c_{ram,t+j}(s^{t+j}))$$

I will refer to  $\{w_{ram,t}(s^t)\}_{t=1}^{\infty}$  as *the Ramsey value sequence*.

Solid black lines in Figure 2 shows the Ramsey outcome and value sequence in the economy with  $p_H = 0.01$  for a particular realization of  $s^{10} \in \mathbb{S}^{10}$ , together with the sequence of contemporaneous utility,  $\{u(c_{ram,t}(s^t), \pi_{ram,t}(s^t))\}_{t=1}^{\infty}$ , associated with the outcome sequence. The figure shows that the Ramsey planner keeps the nominal interest rate at zero even after the contractionary shock disappears. An extended period of low nominal interest rates, together with consumption boom and above-trend inflation at time 2, mitigates the declines in consumption and inflation during the period of the contractionary shock.

Since the contemporaneous utility flow is maximized when consumption and inflation are stabilized at zero, the consumption boom and above-trend inflation are undesirable *ex post*. Thus, if the Ramsey planner was hypothetically given an opportunity to re-optimize again after the shock disappears, the planner would choose to stabilize consumption and inflation. This is captured in the dashed red lines which show the sequence of consumption, inflation and the nominal interest

rate the Ramsey planner would choose in the hypothetical reoptimization at time 2. The planner would renege on the promise of the low nominal interest rate and raise the rate in order to stabilize consumption and inflation. This discrepancy between the pre-announced policy path (solid black lines) and the policy path the government would like to choose in the future (dashed red lines) captures the *time-inconsistency* of the Ramsey policy.

In the rest of the paper, we will study credible plans, which allow the private sector's belief to shift if the government reneges on the promise it has made in the past. By allowing the private sector's belief to depend on the history of policy actions, credible plans can give the government an incentive to fulfill the promise of the low nominal interest rate, and can make the Ramsey policy time-consistent.

## 4 Definition of a plan and credibility

This section defines a plan, credibility, and related concepts. The definitions closely follow Chang (1998).

### 4.1 Plan

A *government strategy*, denoted by  $\sigma_g := \{\sigma_{g,t}\}_{t=1}^{\infty}$ , is a sequence of functions that maps a history of the nominal interest rates up to the previous period and a history of states up to today into today's nominal interest rate. Formally,  $\sigma_{g,t}$  is given by

$$\begin{aligned}\sigma_{g,1} &: \mathbb{S} \rightarrow \mathbb{R} \\ \sigma_{g,t} &: \mathbb{R}^{t-1} \times \mathbb{S}^t \rightarrow \mathbb{R}\end{aligned}$$

The first period is a special case since there is no previous policy action. Given a particular realization of  $\{s_t\}_{t=1}^{\infty}$ , a sequence of nominal interest rates will be determined recursively by  $r_1 = \sigma_{g,1}$  and  $r_t = \sigma_{g,t}(r^{t-1}, s^t)$  for all  $t > 1$  and for all  $s^t \in \mathbb{S}^t$ . A government strategy is said to *induce* a sequence of the nominal interest rates.

A *private sector strategy*, denoted by  $\sigma_p := \{\sigma_{p,t}\}_{t=1}^{\infty}$ , is a sequence of functions mapping a history of nominal interest rates up to today and a history of states up to today into today's consumption and inflation. Formally,  $\sigma_{p,t}$  is given by

$$\sigma_{p,t} : \mathbb{R}^t \times \mathbb{S}^t \rightarrow (\mathbb{C}, \Pi)$$

Given a government and private-sector strategy, a sequence of consumption and inflation will be determined recursively by  $(c_t, \pi_t) = \sigma_{p,t}(r^t, s^t)$  for all  $t \geq 1$  and for all  $s^t \in \mathbb{S}^t$ . A private sector strategy, together with a government strategy, is said to *induce* a sequence of consumption and inflation.

Notice that, while the nominal interest rate today depends on the history of nominal interest rates up to the previous period, consumption and inflation today depend on the history of nominal interest rates up to today. The implicit within-period-timing protocol behind this setup is that the government moves before the private sector does.

**Definition of a plan:** A *plan* is defined as a pair of government and private sector strategies,  $(\sigma_g, \sigma_p)$ .

Notice that a plan induces an outcome—a state-contingent sequence of consumption, inflation, and the nominal interest rate. As discussed earlier, there is a value sequence  $\{w_t(s^t)\}_{t=1}^\infty$ , associated with any outcome. A plan is said to *imply* a value sequence.

## 4.2 Credibility

A few more concepts and notations need to be introduced before defining credibility. Let us use  $CE_t^R(s)$  to denote a set of state-contingent sequences of the nominal interest rate consistent with the existence of a competitive equilibrium when  $s_t = s$ . In other words, for each  $s \in \mathbb{S}$

$$CE_t^R(s) \equiv \{\mathbf{r}_t(s) \in \mathbb{R}^\infty \mid \exists (\mathbf{c}_t(s), \boldsymbol{\pi}_t(s)) \text{ such that } (\mathbf{c}_t(s), \boldsymbol{\pi}_t(s), \mathbf{r}_t(s)) \in CE_t(s)\}$$

**Definition of admissibility:**  $\sigma_g$  is said to be *admissible* if, after any history of policy actions,  $r^{t-1}$ , and any history of states,  $s^t$ ,  $\mathbf{r}_t(s)$  induced by the continuation of  $\sigma_g$  belongs to  $CE_t^R(s_t)$ .

**Definition of credibility:** A plan,  $(\sigma_g, \sigma_p)$ , is *credible* if

- $\sigma_g$  is admissible.
- After any history of policy actions,  $r^t$ , and any history of states,  $s^t$ , the continuation of  $\sigma_p$  and  $\sigma_g$  induce a  $(\mathbf{c}_t(s_t), \boldsymbol{\pi}_t(s_t), \mathbf{r}_t(s_t)) \in CE_t(s_t)$ .
- After any history  $r^{t-1}$  and  $s^t$ ,  $\mathbf{r}_t(s_t)$  induced by  $\sigma_g$  maximizes the government's objective over  $CE_t^R(s_t)$  given  $\sigma_p$ .

An outcome is said to be *credible* if there is a credible plan that induces it. When a certain plan A is credible and the plan A induces a certain outcome  $\alpha$ , we say that the outcome  $\alpha$  *can be made time-consistent by the plan A*.

## 5 The discretionary plan and the revert-to-discretion plan

In the first subsection, I will define the discretionary plan and demonstrate that it is credible. In the second subsection, I will define the revert-to-discretion plan and discuss a general condition under which it is credible.

## 5.1 The discretionary plan

The discretionary plan,  $(\sigma_g^d, \sigma_p^d)$ , consists of the following government strategy

- $\sigma_{g,1}^d = r_d(s_1)$  for any  $s_1 \in \mathbb{S}$
- $\sigma_{g,t}^d(r^{t-1}, s^t) = r_d(s_t)$  for any  $s^t \in \mathbb{S}^t$  and any  $r^{t-1} \in \mathbb{R}^{t-1}$

and the following private-sector strategy

- $\sigma_{p,t}^d(r^t, s^t) = (c_d(s_t), \pi_d(s_t))$  if  $r_t = r_d(s_t)$
- $\sigma_{p,t}^d(r^t, s^t) = (c_{br}(s_t, r_t), \pi_{br}(s_t, r_t))$  otherwise<sup>14</sup>

where

$$c_{br}(s_t, r_t) = E_t c_{d,t+1}(s^{t+1}) - \frac{1}{\chi_c} \left[ [r_t - E_t \pi_{d,t+1}(s^{t+1})] - s_t \right] \quad (10)$$

$$\pi_{br}(s_t, r_t) = \kappa c_{br}(s_t, r_t) + \beta E_t \pi_{d,t+1}(s^{t+1}) \quad (11)$$

The government strategy instructs the government to choose the nominal interest rate consistent with the discretionary outcome, regardless of the history of past nominal interest rates. The private sector strategy instructs the household and firms to choose consumption and inflation consistent with the discretionary outcome, as long as today's nominal interest rate chosen by the government is consistent with the discretionary outcome. If the government chooses an interest rate that is not consistent with the discretionary outcome, then the private sector strategy instructs the household and firms to optimally choose today's consumption under the belief that the government in the future will not deviate again.

By construction, the discretionary plan induces the discretionary outcome, and the value sequence implied by the discretionary plan is identical to the discretionary value sequence.

**Proposition 1:** The discretionary plan is credible.

See the Appendix for the proof. The discretionary plan will be a key ingredient in constructing the revert-to-discretion plan, which we will discuss now, and this proposition will be essential in analyzing the credibility of the revert-to-discretion plan.

## 5.2 The revert-to-discretion plan

The revert-to-discretion plan,  $(\sigma_g^{rtd}, \sigma_p^{rtd})$ , consists of the following government strategy

- $\sigma_{g,1}^{rtd} = r_{ram,1}(s_1)$  for any  $s_1 \in \mathbb{S}$

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<sup>14</sup>Subscript *br* stands for *best response*.

- $\sigma_{g,t}^{rtd}(r^{t-1}, s^t) = r_{ram,t}(s^t)$  if  $r_j = r_{ram,j}(s^j)$  for all  $j \leq t-1$
- $\sigma_{g,t}^{rtd}(r^{t-1}, s^t) = \sigma_{g,t}^d(r^{t-1}, s^t)$  otherwise.

and the following private-sector strategy

- $\sigma_{p,t}^{rtd}(r^t, s^t) = (c_{ram,t}(s^t), \pi_{ram,t}(s^t))$  if  $r_j = r_{ram,j}(s^j)$  for all  $j \leq t$
- $\sigma_{p,t}^{rtd}(r^t, s^t) = \sigma_{p,t}^d(r^t, s^t)$  otherwise.

The government strategy instructs the government to choose the nominal interest rate consistent with the Ramsey outcome, but chooses the interest rate consistent with the discretionary outcome if it has deviated from the Ramsey outcome at some point in the past. The private sector strategy instructs the household and firms to choose consumption and inflation consistent with the Ramsey outcome as long as the government has never deviated from the Ramsey outcome. If the government has ever deviated from the nominal interest rate consistent with the Ramsey outcome, the private sector strategy instructs the household and firms to choose consumption and inflation today based on the belief that the government in the future will choose the nominal interest rate consistent with the discretionary outcome.

By construction, the revert-to-discretion plan induces the Ramsey outcome, and the implied value sequence is identical to the Ramsey value sequence. The main exercise of the paper is to characterize the conditions under which the revert-to-discretion plan is credible. The following proposition will be useful in answering this question.

**Proposition 2:** The revert-to-discretion plan is credible if and only if  $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$  for all  $t \geq 1$  and all  $s^t \in \mathbb{S}^t$ ,

See the Appendix for proof. The condition that  $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$  for all  $t \geq 1$  and all  $s^t \in \mathbb{S}^t$  makes sure that the government does not have an incentive to deviate from the instruction given by the government strategy after any history  $r^{t-1}$  and  $s^t$  in which the Ramsey policy has been followed.

It is useful to decompose  $w_{ram,t}(s^t)$  and  $w_{d,t}(s^t)$  into two components in order to gain insights on this proposition. Notice that, after the history in which the Ramsey policy has been followed,  $w_{ram,t}(s^t)$  is the value of following the instruction given by the revert-to-discretionary plan and  $w_{d,t}(s^t)$  is the best value the government can attain if the government deviates from the instruction.  $w_{ram,t}(s^t)$  and  $w_{d,t}(s^t)$  can be both decomposed into today's utility flows ( $u(c_{ram,t}(s^t), \pi_{ram,t}(s^t))$ ,  $u(c_{d,t}(s^t), \pi_{d,t}(s^t))$ ) and the discounted continuation values ( $\beta E_t w_{ram,t+1}(s^{t+1})$ ,  $\beta E_t w_{d,t+1}(s^{t+1})$ ) as follows.

$$w_{ram,t}(s^t) = u(c_{ram,t}(s^t), \pi_{ram,t}(s^t)) + \beta E_t w_{ram,t+1}(s^{t+1})$$

$$w_{d,t}(s^t) = u(c_{d,t}(s^t), \pi_{d,t}(s^t)) + \beta E_t w_{d,t+1}(s^{t+1})$$

Thus, the restriction  $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$  can be written as

$$\begin{aligned} u(c_{ram,t}(s^t)) + \beta E_t w_{ram,t+1}(s^{t+1}) &\geq u(c_{d,t}(s^t), \pi_{d,t}(s^t)) + \beta E_t w_{d,t+1}(s^{t+1}) \\ \Leftrightarrow \beta E_t [w_{ram,t+1}(s^{t+1}) - w_{d,t+1}(s^{t+1})] &\geq [u(c_{d,t}(s^t), \pi_{d,t}(s^t)) - u(c_{ram,t}(s^t), \pi_{ram,t}(s^t))] \end{aligned}$$

The left-hand and right-hand sides of this last inequality constraint respectively capture *the loss in the continuation value* and *the gain in today's utility flow* if the government deviates from the Ramsey policy. Thus, the aforementioned proposition can be restated as “If the loss in the continuation value caused by the deviation from the Ramsey prescription is larger than the gain in today's utility flow, the revert-to-discretion plan is credible.”

According to this proposition, in order to check whether or not the revert-to-discretion plan is credible given a particular set of parameter values, it suffices to solve for the discretionary and Ramsey value sequences and check whether or not  $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$  for all  $t \geq 1$  and all  $s^t \in \mathbb{S}^t$ . While the partial log-linearization framework allows us to derive the discretionary value sequence in closed-form, the Ramsey value sequence cannot be characterized analytically.<sup>15</sup> Thus, it is not feasible to analytically characterize the conditions under which  $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$  for all  $t \geq 1$  and all  $s^t \in \mathbb{S}^t$ . In the next section, we will use numerical simulations to characterize the set of parameter values—particularly ones governing the natural rate process—for which the revert-to-discretion plan is credible.

## 6 Results

In this section, I solve the discretionary and Ramsey value sequences for various combinations of parameter values and characterize the circumstances under which  $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$  for all  $t \geq 1$  and all  $s^t \in \mathbb{S}^t$ , and thus the revert-to-discretionary plan is credible. I organize the results in the following way. I first describe the set of  $(p_H, p_L)$  under which the revert-to-discretionary plan is credible, given the baseline values for other parameters of the model (i.e.  $L, \beta, \chi_c, \kappa, \lambda$ ) as listed in Table 1. I then describe how this set varies when other parameters take alternative values.

First-order necessary conditions of the discretionary government's problem is given by a system of linear equations, and thus the discretionary outcome and values can be computed by linear algebra. The Ramsey outcomes and value sequence are solved globally by a time-iteration method of Coleman (1991). For each set of parameter values considered, I simulate the model until I observe one million episodes of contractionary shocks, and decides that “ $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$  for all  $t \geq 1$  and all  $s^t \in \mathbb{S}^t$ ” if the simulated Ramsey values are always above the simulated discretionary values.

Figure 3 show whether the revert-to-discretion plan is credible or not for the set of  $(p_H, p_L) \in \mathbb{P}_H \times \mathbb{P}_L$  where  $\mathbb{P}_H$  is 101 equally spaced grid points between  $[0, 0.01]$  and  $\mathbb{P}_L$  is 51 equally spaced

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<sup>15</sup>See Eggertsson and Woodford (2003).

grid points between  $[0, 1]$ . Blank areas indicate combinations of  $(p_H, p_L)$  for which the revert-to-discretion plan is credible. Blue dots indicate the combinations of  $(p_H, p_L)$  for which the revert-to-discretion plan is not credible. Black dots indicate the combinations of  $(p_H, p_L)$  for which the revert-to-discretion plan is not defined because the discretionary outcome does not exist.<sup>16</sup>

## 6.1 Frequency

**Result 1:** For any given  $p_L \in \mathbb{P}_L$ , there exists  $p_H^*$  such that the revert-to-discretion plan is credible if  $p_H \geq p_H^*$  and is not credible otherwise.

In other words, for any given  $p_L \in \mathbb{P}_L$ , the revert-to-discretionary plan is credible if and only if the contractionary shock hits the economy sufficiently frequently.

To gain insights on this result, Figure 4 compares particular realizations of the discretionary and Ramsey outcomes/value sequences for two economies—one with frequent shocks (i.e., a small  $p_H$ ) and the other with infrequent shock (i.e., a large  $p_H$ ). In this figure,  $s_1 = L$  and  $s_t = H$  for  $2 \leq t \leq 10$ . The left column shows the realization of the Ramsey and discretionary outcomes/value sequences in the model with infrequent shocks, while the right column shows the realization in the model with frequent shocks.

Top three rows show that the discretionary and Ramsey outcomes are very similar across two models with infrequent and frequent shocks. However, according to the bottom row, the discretionary and Ramsey value sequences behave differently when the shock frequencies are different. In particular, in the model with frequent shocks, the discretionary value stays below the Ramsey value at time 2 and remains so afterwards. In contrast, in the model with infrequent shocks, the discretionary value exceeds the Ramsey value at time 2. Thus, the revert-to-discretionary plan is not credible when the contractionary shock occurs infrequently.

To understand why the discretionary value stays below the Ramsey value in the model with frequent shocks, it is useful to examine how the loss in the continuation value and the gain in today's utility flow at time 2 vary with the frequency of the shock. The black and red lines in Figure 5 respectively depict these two objects for various values of  $p_H$ . Since the frequency of the shock does not substantially affects the discretionary and Ramsey outcomes at time 2, the gain in today's utility flow of deviating from the Ramsey policy are essentially unaltered by the shock frequency, as seen in the constant red line. However, the frequency of the shock does alter the loss in the continuation value associated with the deviation from the Ramsey prescription. In particular, the loss in the continuation value increases with frequency shocks. For sufficiently frequent shocks (i.e., sufficiently large  $p_H$ ), the losses in the continuation value becomes larger than the short-run gain, making the revert-to-discretionary plan credible.

To understand why the loss in the continuation value increases as the shock becomes more frequent, Figure 6 shows how the Ramsey continuation value and the continuation value in the case of deviation vary with the frequency at period 2. The panel shows that the discretionary

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<sup>16</sup>The Appendix explains in detail why the solution does not exist for certain combinations of  $(p_H, p_L)$ .



continuation value declines more rapidly as  $p_H$  gets larger than the Ramsey continuation value does. As seen in Figure 4, the contractionary shock leads to a larger decline in consumption and inflation in the discretionary outcome than in the Ramsey outcome in the face of contractionary shock. Thus, a higher probability of contractionary shocks reduce the expected discounted sum of future utility flows associated with the discretionary outcome by more than that associated with the Ramsey outcome, making the loss in the continuation value an increasing function of the frequency.

## 6.2 Persistence

**Result 2:** For a sufficiently high  $p_H \in \mathbb{P}_H$ , the revert-to-discretion plan is credible regardless of the value of  $p_L$ . For a sufficiently small  $p_H \in \mathbb{P}_H$ , there exists  $p_L^*$  such that the revert-to-discretion plan is credible if  $p_L \geq p_L^*$  and is not credible otherwise.

This result says that, even when the frequency of shock is small, the revert-to-discretion plan is credible if the contractionary shock is sufficiently persistent. For example, when  $p_H = 0.05$ , the revert-to-discretionary plan is credible regardless of the values of  $p_L$ . When  $p_H = 0.005$ , the revert-to-discretionary plan is credible if  $p_L > 0.5$ , but is not credible otherwise. Another way of phrasing this result is that the threshold value of  $p_H$  above which the revert-to-discretionary plan is credible is decreasing in  $p_L$ .<sup>17</sup>

To understand the mechanism behind this result, Figure 7 compares particular realizations of the discretionary and Ramsey outcomes/value sequences for two economies—one with transient shocks (i.e., a small  $p_L$ ) and the other with persistent shock (i.e., a large  $p_L$ ). In this figure,  $s_t = L$  for  $1 \leq t \leq 4$  and  $s_t = H$  for  $s_t = H$  for  $5 \leq t \leq 10$ . The left column shows the Ramsey and discretionary outcomes/value sequences in the model with transient shocks, while the right column shows those in the model with persistent shocks.

When the persistence is high, the household and firms expect to stay in the low state for long. Since marginal costs and inflation are low in the low state, such expectation implies lower expected marginal costs and higher expected real interest rates. Accordingly, the household and firms in the low state choose lower consumption and inflation. However, the Ramsey planner can mitigate this effect by promising a higher inflation, a larger consumption boom, and a longer period of zero nominal interest rates after the shock disappears. Thus, the declines in consumption and inflation from marginal increases in persistence is larger in the discretionary outcome than in the Ramsey outcome, as captured in the second and third rows in Figure 7. As a result, the continuation value of reverting back to the discretionary plan declines more rapidly with  $p_L$  than that of staying with the Ramsey outcome, as depicted in Figure 9. This implies that the long-run loss of reverting back to a discretionary plan is higher with more persistent shocks, as depicted by the solid black line in Figure 8. In the meantime, the promise of higher inflation and consumption increases in the

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<sup>17</sup>There is a discontinuity at  $p_L = 0$ . When the  $p_L$  is low, the marginal changes in  $p_L$  affects whether or not the discretionary value exceeds the Ramsey values only after a long-lasting spell of low states. When the probability of staying at the low state is zero, you never observe the low state lasting longer than one period.

economy with persistent shocks means that the short-run incentive to deviate from the promise is larger, as illustrated by Figure 8. Quantitatively, the long-run loss increases more rapidly than the short-run gain for large values of  $p_L$ , the former exceeds the latter for sufficiently large values of  $p_L$ .

### 6.3 Sensitivity Analysis

Figure 10 shows how alternative values of other parameters alter the set of  $(p_H, p_L)$  under which the revert-to-discretionary plan is credible. For the sake of brevity, I will discuss the results only casually in this section, and will delegate to the Appendix detailed analyses on how each parameter affects the outcomes and value sequences as well as the short-run gain and the long-run loss of deviating from the Ramsey policy.

#### Severity of the shock ( $L$ )

A larger shock (a larger  $|L|$ ) means larger declines in consumption and inflation under both discretionary and Ramsey outcomes. However, the Ramsey planner can promise a higher inflation and a larger consumption boom to mitigate the declines in consumption and inflation during the period of contractionary shocks. Thus, a marginal increase in the shock severity leads to larger marginal declines in low-state consumption and inflation under the discretionary outcome than under the Ramsey outcome, leading to larger marginal declines in the both high-state and low-state values. Accordingly, the long-run loss from renegeing on the Ramsey promise and reverting back to the discretionary outcome is larger in the economy with more severe shocks.

On the other hand, as the Ramsey promise entails a higher inflation and larger consumption boom, the short-run gain from renegeing on the promise is also larger with a larger shock. As such, the overall effects are mixed. According to the figure, while the threshold frequency is higher when the shock is larger in the economy with highly persistent shocks, the threshold frequency is lower when the shock is larger in the economy in which the shock persistence is low.

#### Discount rate ( $\beta$ )

With a higher  $\beta$ , the same difference between the discretionary and Ramsey continuation values translates into a larger difference between *discounted* continuation values. As a results, a high discount factor implies a larger long-run loss of renegeing on the promise. The discount rate also affects the short-run gain from renegeing on the promise as it alters inflation booms the Ramsey planner would promise, but this effect is quantitatively negligible. As a result, credible region expands with larger  $\beta$ . Figure 10 shows that the threshold  $p_H$  above which the revert-to-discretionary plan is credible is lower in the economy with a larger  $\beta$ . This result is consistent with the previous literature on credible plans which has shown that a sufficiently large  $\beta$  can make the Ramsey policy credible in various contexts.

### **Slope of the Phillips Curve ( $\kappa$ )**

When the slope of the Phillips curve is high (i.e. prices are flexible), declines in low-state consumption and inflation are exacerbated under both discretionary and Ramsey outcomes. While the Ramsey planner mitigates those declines by promising a higher inflation and consumption boom, the discretionary government cannot. Thus, a marginal increase in the slope parameter leads to larger marginal declines in consumption and inflation, and thus values, in the discretionary outcome than in the Ramsey outcome. Accordingly, the long-run loss from reverting back to the discretionary plan is larger in the economy with more flexible prices. On the other hand, the Ramsey promise of higher inflation and larger consumption booms means that short-run gain from renegeing on the promise once the shock disappears is higher under a more flexible price environment. Quantitatively, for the calibration considered in this paper, the second effects dominates the first effect. The threshold value of  $p_H$  above which the revert-to-discretionary plan is credible is lower for any given  $p_L$  as shown in Figure 10.<sup>18</sup>

### **Inverse IES ( $\chi_c$ )**

When the inverse IES is high, the household's consumption decision is more sensitive to the fluctuations in  $s_t$ . Since firms' pricing today depends on consumption today, inflation today is more sensitive to the fluctuations in  $s_t$  with a higher  $\chi_c$ . Thus, a higher  $\chi_c$  implies larger declines in consumption and inflation in the low state under both discretionary and Ramsey outcomes. While the Ramsey planner can mitigate those additional declines by future promises, the discretionary government has no tool to mitigate them. As a result, a marginal increase in the inverse IES leads to larger marginal declines in low-state consumption and inflation under the discretionary outcome than in the Ramsey outcome. Since these lower low-state consumption and inflation reduce values in both states. On the other hand, higher promised consumption and inflation with a larger  $\chi_c$  mean a larger short-run gain from renegeing on the promise. Thus, the effects are mixed. Similarly to the severity of shocks, while the threshold frequency is higher when the inverse IES is larger in the economy with highly persistent shocks, the threshold frequency is lower when the inverse IES is larger in the economy in which the shock persistence is low.

### **Weight on consumption volatility ( $\lambda$ )**

A larger  $\lambda$  means that the government cares more about consumption volatility relative to inflation volatility. Under the discretionary government, a greater concern for consumption volatility exacerbates the deflation bias in the high state, in turn magnifying deflation and consumption decline in the low state.<sup>19</sup> The Ramsey planner can mitigate this effect by promising a higher inflation and consumption boom in the future, and marginal increases in the weight on consumption volatility reduces the low-state consumption and inflation, and thus values in both states, by

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<sup>18</sup>Kurozumi (2008) and Sunakawa (2013) similarly find that the credible region increases with  $\kappa$  in the model with stabilization bias in the sense that the threshold  $\beta$  above which the Ramsey policy is credible decreases with  $\kappa$ .

<sup>19</sup>See Nakata and Schmidt (2014) for more detailed analyses

more under the discretionary outcome than under the Ramsey outcome. Accordingly, the long-run loss of reneging on the promise, and therefore accepting the continuation value associated with the discretionary outcome, is higher. On the other hand, promises of higher inflation and consumption hikes means that the short-run gain of deviating from the promise is larger. Quantitatively, the second effect dominates the first effect unless the persistence of the shock is very high, as shown in Figure 10. For most values of  $p_L$ , the threshold frequency above which the revert-to-discretionary plan is credible is higher when the central bank places a greater weight on consumption volatility in its objective function.<sup>20</sup>

## 7 Quantitative Analyses

Thus far, I have described how reputational force can make the policy of “low for long” credible under a textbook calibration. In this section, I parameterize the model so that the contractionary shock leads to declines in output and inflation that are in line with the Great Recession and the Great Depression and ask how frequently the crisis shock has to hit the economy in order for the revert-to-discretion plan to be credible.

The parameter values are chosen according to the parameterization of Denes, Eggertsson, and Gilbukh (2013). The Great Recession parameterization is chosen so that output and inflation decline by 10 and 2 percentage points respectively in the crisis state under the discretionary outcome with  $p_H = 0$  and the expected duration of the crisis is about 7 quarters. The Great Depression parameterization is chosen so that output and inflation decline by 30 and 5 percentage points respectively in the crisis state under the discretionary outcome with  $p_H = 0$  and the expected duration of the crisis is about 10 quarters. These values are listed in Table 2.<sup>21</sup>

With the Great Recession parameterization, the threshold frequency above which the revert-to-discretion plan is credible is 0.015 percent (see the first row in Table 3). This means that, if the crisis occurs on average once every 1,700 years, the central bank can credibly commit to the Ramsey promise. With the Great Depression parameterization, the threshold frequency is even lower, 0.003 percent. This means that, if the crisis occurs on average once every 10,000 years, the central bank can credibly commit to the Ramsey promise. In the U.S., two large shocks have hit the economy that has pushed the policy rate to zero over the past 100 years since the creation of the Federal Reserve System. Thus, the naïve estimate of the frequency parameter is 0.5 percent ( $= 2/400$ ) at quarterly frequency. The threshold frequency computed under either of the Great Recession or Great Depression is comfortably below this naïve estimate.

This exercise is not meant to be the final word on the power of reputation in the model with the zero lower bound. Future research may reveal that the threshold frequency is much higher in richer structural models. However, this exercise at least suggests how powerful reputation forces

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<sup>20</sup>Kurozumi (2008) and Sunakawa (2013) similarly find that the credible region decreases with  $\lambda$  in the model with stabilization bias in the sense that the threshold  $\beta$  above which the Ramsey policy is credible increases with  $\lambda$ .

<sup>21</sup>Denes, Eggertsson, and Gilbukh (2013), along with many other works using two-state Markov processes for the crisis shock, assume  $p_H = 0$  and focus on the dynamics of the economy at the zero lower bound.

can be in making the Ramsey policy time-consistent in this model.

## 8 Additional results and discussion

### 8.1 The revert-to-discretion(N) plan

One may feel that the private sector's punishment strategy of reverting to the discretionary outcome forever after the government's deviation may be too harsh and unrealistic. In reality, the household and firms do not live forever. The head of the central bank also changes at some frequencies. Even if the same central banker is in charge for an extended period of time, central bank doctrines can change over the course of his/her tenure.<sup>22</sup> Based on these considerations, I define and analyze the *revert-to-discretion(N)* plan in which the punishment regime lasts for a finite period of time (N) and the economy reverts back to the Ramsey outcome afterwards. Since a formal definition of this plan is involved, I relegate it to the Appendix for the sake of brevity. Here, I report the main results from the analysis.

Figure 11 shows how the credible regions vary with the number of punishment periods. Black and red lines are respectively the threshold frequencies above which the revert-to-discretion(N) plans are credible with  $N = 40$  and  $200$ , while the blue line depicts the threshold frequency for the standard revert-to-discretion plan. Not surprisingly, given  $p_L$ , the threshold frequency decreases with the number of punishment periods. A smaller punishment period is associated with a larger value for the government in the case of defection, and therefore with a smaller long-run loss from reneging on the Ramsey promise. Thus, with a less severe punishment, the contractionary shock needs to be more frequent in order to make the Ramsey policy credible.

While allowing for a finite-period punishment limits the power of reputation, the threshold frequency remains quantitatively small for both the Great Recession and the Great Depression scenarios considered in the previous section. Under the Great Recession parameterization, the threshold crisis frequencies are 0.312 and 0.037 percentage points when the discretionary regime lasts for 10 and 50 years. These numbers imply that the revert-to-discretion plan is credible if the crisis occurs on average at least once every 80 and 700 years with 10-year and 50-year punishment periods. In the Great Depression parameterization, the threshold crisis frequencies are 0.513 and 0.011 percentage points when the discretionary regime lasts for 10 and 50 years. These numbers imply that the revert-to-discretion plan is credible if the crisis occurs on average at least once every 50 and 10,000 years with 10-year and 50-year punishment periods.

### 8.2 The revert-to-deflation plan

Throughout the paper, I focus on the question of whether or not the Ramsey outcome can be made time-consistent by the revert-to-discretion plan. However, there is an alternative plan that induces the Ramsey outcome and that is credible under a different set of conditions than those for

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<sup>22</sup>For example, consider the gradual move toward transparency during the tenure of Alan Greenspan at the Federal Reserve.

credibility of the revert-to-discretion plan. In the Appendix, I construct a plan called the *revert-to-deflation* plan in which the government’s deviation from the Ramsey prescription is punished by reverting to the Markov-Perfect equilibrium in which the zero lower bound binds in both states, and show that it induces the Ramsey outcome and is credible regardless of the parameter values. Since an outcome is defined to be credible if there is a credible plan that induces it, the result that the revert-to-deflation plan is credible regardless of the parameter values means that the Ramsey outcome is credible regardless of the parameter values.

I focus on the revert-to-discretion plan, instead of the revert-to-deflation plan, for two reasons. First, in the revert-to-deflation plan, there is no short-run gain from reneging on the promise after the contractionary shock disappears. As the plan instructs the private sector to expect deflationary outcomes to persist in the future after the government’s deviation, the forward-looking household and firms would respond to the government’s deviation by lowering consumption and inflation immediately in the period of deviation. When one thinks about time-consistency of the Ramsey promise in this model, the premise is that the government can stabilize consumption and inflation in the period of defection by reneging on the Ramsey promise. If this short-run gain does not exist in the specified plan, then such a plan is not economically interesting.

The second reason, somewhat related to the first one, is that the private sector’s punishment strategy of the revert-to-deflation plan in which the nominal interest rate is zero and consumption and inflation are below steady-state even in the high state can be seen as too harsh and unrealistic. Of course, inside the model, there is nothing unrealistic about this punishment regime. A proposition in the Appendix indeed shows that no one would have incentives to deviate from their strategies in this regime with the permanently binding ZLB. Nevertheless, outside the model, a natural question emerges as to why the private sector and the government do not want to renegotiate to move to a better outcome. Within the theoretical literature on repeated games, the same concern regarding the plausibility of punishment strategy led to the development of appropriate concepts of the *renegotiation-proof* equilibrium in which players are allowed to renegotiate after the defection is detected.<sup>23</sup> Introducing this concept may render the revert-to-deflation plan incredible and could formally justify my focus on the revert-to-discretion plan. However, such analyses are beyond the scope of this paper.

### 8.3 Frequency, rather than the discount factor

While I analyze how each parameter of the model affects the credibility of the revert-to-discretion plan in details, I have placed a particular emphasis on the frequency parameter,  $p_H$ . This is in contrast to the majority of the existing literature that tends to focus on how the discount rate parameter affects the credibility of the Ramsey policy. Both parameters influence the credibility of the revert-to-discretion plan similarly by affecting the discounted continuation value in the case of reneging on the Ramsey promise. So, why did I focus on the frequency parameter?

I focus on the frequency parameter as opposed to the discount rate because the result that

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<sup>23</sup>See, for example, Abreu and Pearce (1991) and Farrell and Maskin (1989)

the Ramsey policy becomes credible with a sufficiently high  $\beta$  has been demonstrated in many different contexts and is regarded as a folk theorem of the reputational equilibria. Thus, while it is useful, confirming this result in this model would not necessarily generate insights about the specific model presented here. The frequency parameter for the crisis shock process is unique to this model relative to other models previously studied in the literature of sustainable plans. The crisis probability is also an economically interesting parameter in light of the recent global recession and has been studied empirically as of late. For example, Schularick and Taylor (2012) examine time variations in the probability of financial crises using panel data across countries. Nakamura, Steinsson, Barro, and Ursa (2013) estimate the probability of consumption disasters and explores its asset pricing implications.

I also focus on the frequency parameter as opposed to other structural parameters such as  $\chi_c$ ,  $\kappa$ , and  $\lambda$ . I do so mainly for pedagogical reasons. As stated previously, the frequency parameter affects the credibility of the Ramsey policy by affecting the discounted continuation value of renegeing on the Ramsey promise. This mechanism is very similar to the well-known mechanism in which the discount rate affects the credibility of the Ramsey policy, making it easier to digest the result. Other parameters affect the credibility of the Ramsey policy through both the short-run incentive to renege on the Ramsey promise and the long-run incentive to fulfill the promise. Those mechanisms are easier to digest once one understands a slightly simpler mechanism by which the frequency parameter affects the credibility of the revert-to-discretion plan.

## 8.4 Scope of the paper

This paper focuses on describing how reputational concern on the part of the central bank can make the Ramsey promise of keeping the policy rate low for long credible. To do so in a transparent way, I abstracted from two other widely studied frictions that render the Ramsey policy time-inconsistent. One such friction is the monopolistic competition in the product market that makes the steady-state output inefficiently low. In the model with this friction (often referred to as *the model with inflation bias*), the Ramsey planner promises low future inflation to achieve low inflation today while the discretionary central bank has incentives to create surprise inflation every period. The other friction is the presence of cost-push shocks. In the model with cost-push shocks (often referred to as *the model with stabilization bias*), the Ramsey planner promises to deviate from zero inflation in the future to improve the trade-off between inflation and output today. The discretionary central bank on the other hand cannot make such a promise and ends up with highly volatile inflation and output.

These two sources of time-inconsistency have been studied by many, and some have asked how reputational concerns can make the Ramsey promise credible in these contexts.<sup>24</sup> Once these other sources of inefficiency are introduced into the model analyzed in this paper, the value of commitment will increase. Thus, the set of parameter values under which the revert-to-discretion plan is credible

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<sup>24</sup>See Barro and Gordon (1983), Rogoff (1987), and Ireland (1997) for the model with inflation bias, and Kurozumi (2008), Loisel (2008), and Sunakawa (2013) for the model with stabilization bias.

is likely to increase. Analyzing an environment in which all these frictions are present would be an interesting venue for future research.

Also, for the sake of illustrating the key mechanism of the model in a transparent way, I (i) assumed that the crisis shock follows a two-state Markov process and (ii) worked with a semi-loglinear version of the sticky-price model. Some have recently argued that the quantitative prediction of the model is quite different across semi-loglinear and nonlinear versions.<sup>25</sup> In future research, it would be useful to extend the analysis for a continuous AR(1) shock on a fully nonlinear environment if one were to further explore the quantitative implications of the model.

## 9 Conclusion

Why should the central bank fulfill the promise of keeping the nominal interest rate low even after the economic recovery strengthens? What force will prevent the future central bank from renegeing on this promise? To shed light on these questions, this paper has analyzed credible plans in a stochastic New Keynesian economy in which the nominal interest rate is subject to the zero lower bound constraint and contractionary shocks hit the economy occasionally.

I have demonstrated that the policy of keeping the nominal interest rate low for long is credible if the contractionary shocks hit the economy sufficiently frequently. In the best credible plan, if the central bank reneges on its promise to keep the nominal interest rate low, it will lose reputation and the private sector will never believe such promises in the face of future contractionary shocks. If the private sector does not believe the promise of an extended period of low nominal interest rates, the contractionary shock will cause large declines in consumption and inflation. Large declines in consumption and inflation in the future recessions reduce welfare even during normal times since the agents care about the discounted sum of future utility flows. Thus, the potential loss of reputation gives the central bank an incentive to fulfill the promise. When the frequency or severity of shocks is sufficiently large, this incentive to maintain reputation outweighs the short-run incentive to raise the rate to close consumption and inflation gaps, and keeps the central bank on the originally announced path of low nominal interest rates.

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<sup>25</sup>See, for example, Braun, Korber, and Waki (2013).



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Table 1: Baseline Parameter Values

Parameter	Description	Parameter Value
$\beta$	Discount rate	$\frac{1}{1+0.0075} \approx 0.9925$
$\chi_c$	Inverse intertemporal elasticity of substitution	1
$\kappa$	The slope of the Phillips curve	0.024
$\lambda$	the relative weight on output volatility	0.003
$H$	the natural rate of interest in the high (normal) state	$\frac{1}{\beta} - 1 (=0.0075)$
$L$	the natural rate of interest in the low (contractionary) state	-0.0125

Table 2: Parameter Values for the Great Recession/Depression scenarios

Parameter	Great Recession (GR)	Great Depression (GD)
$\beta$	0.997	0.997
$\chi_c$	1.220	1.153
$\kappa$	0.0075	0.0091
$\lambda$	0.00057	0.00072
$H$	0.003	0.003
$L$	-0.0129	-0.0107
$p_L$	0.857	0.902

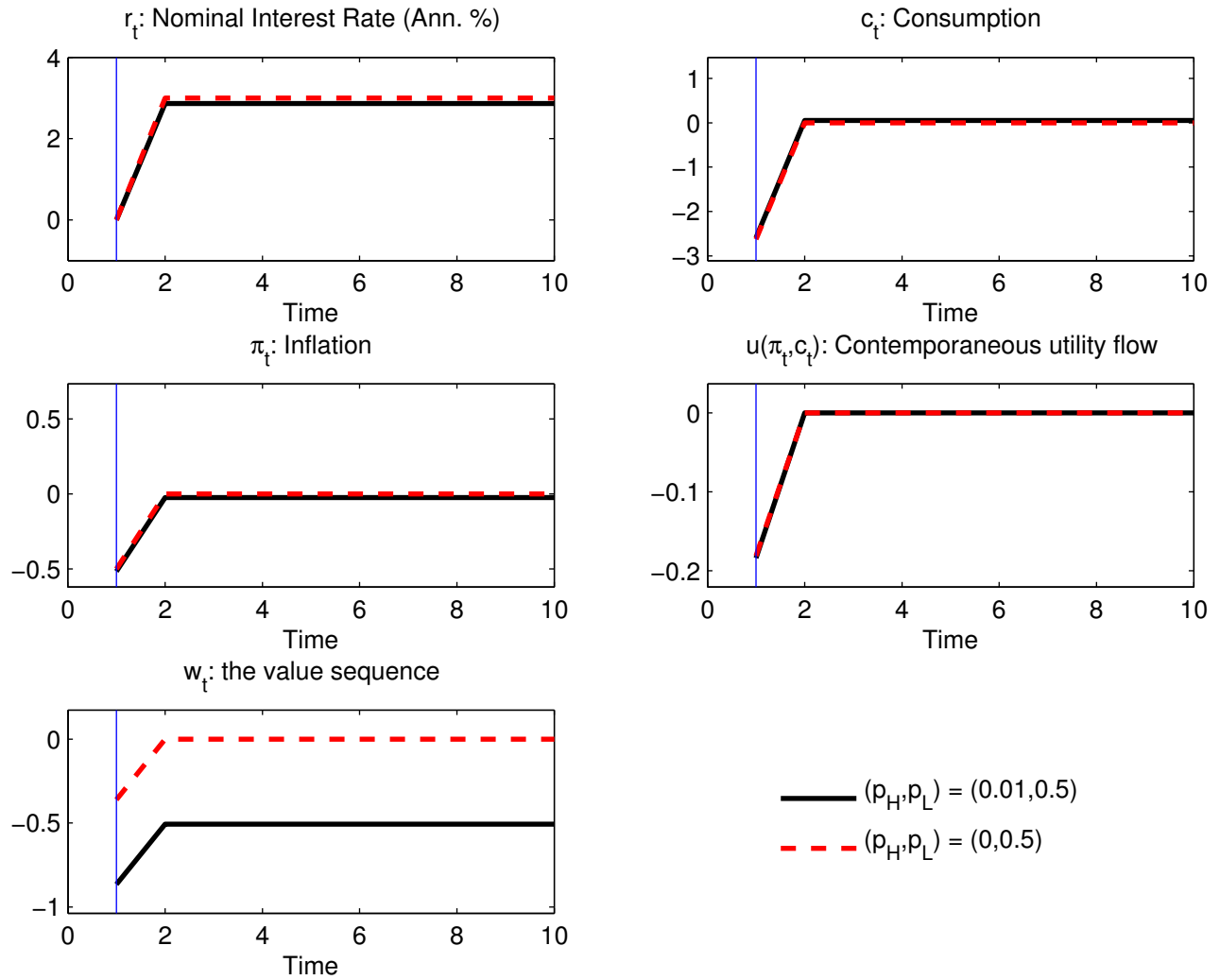
Table 3: Threshold Crisis Probabilities for the Great Recession/Depression scenarios

Punishment length	Minimum crisis prob.		Implied ave. non-crisis duration	
	$(100p_H)$		(in years)	
	GR	GD	GR	GD
$\infty$	0.015	0.003	1,689	9,737
50 years	0.037	0.011	678	2,209
25 years	0.078	0.045	322	553
10 years	0.312	0.513	80	49

\*This table shows the threshold crisis probabilities above which the revert-to-discretion plan is credible.

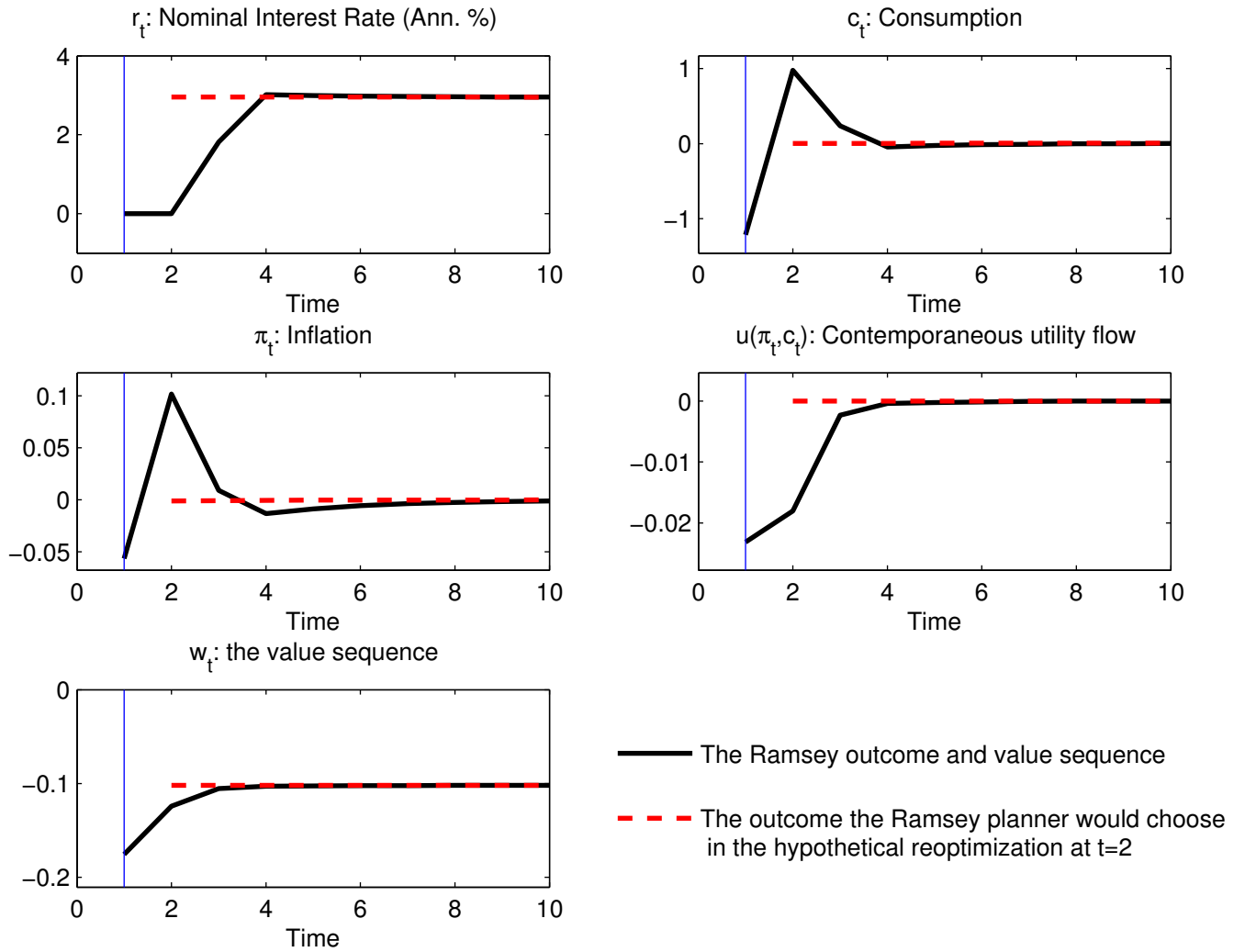
\*\*GR refers to the Great Recession calibration and GD refers to the Great Depression calibration.

Figure 1: The discretionary outcome and value sequence



\*Solid blue vertical lines show the period with contractionary shocks.

Figure 2: The Ramsey outcome and value sequence



\*Solid blue vertical lines show the period with contractionary shocks.

Figure 3: Credibility of the revert-to-discretion plan

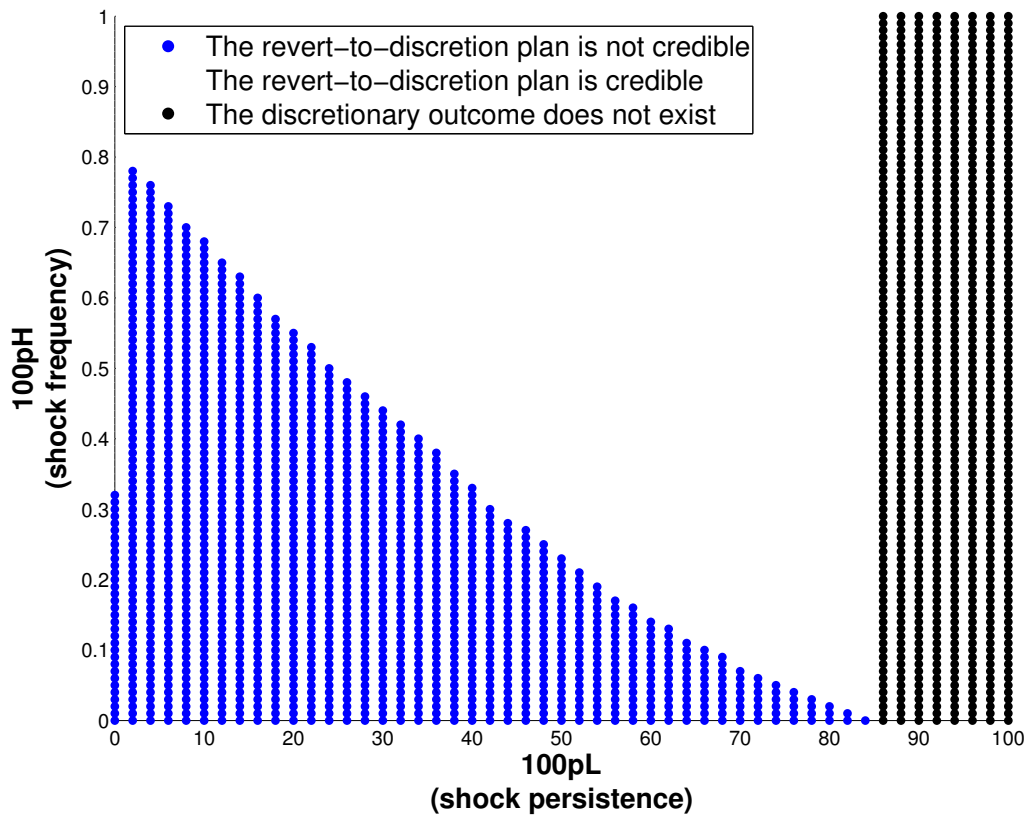
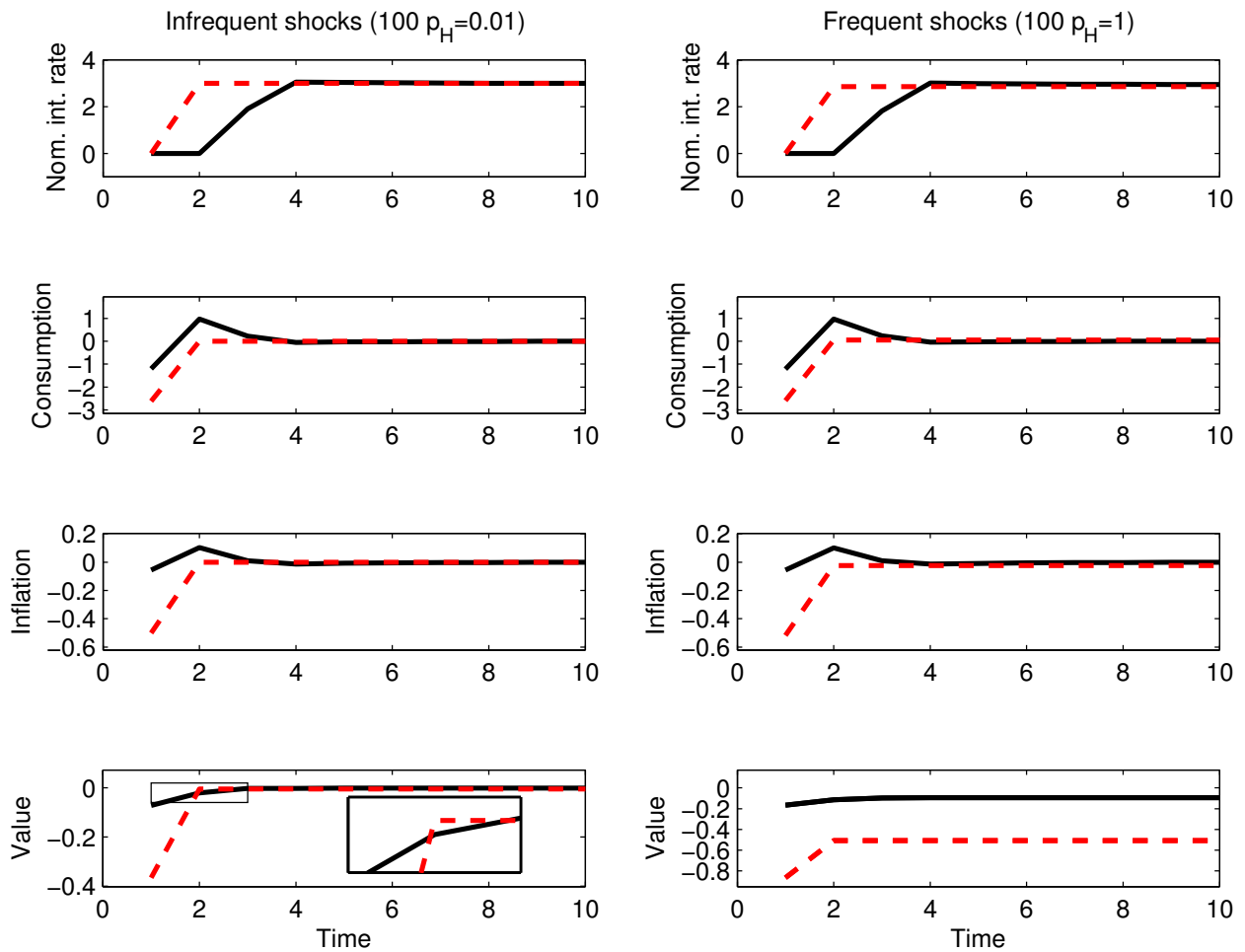
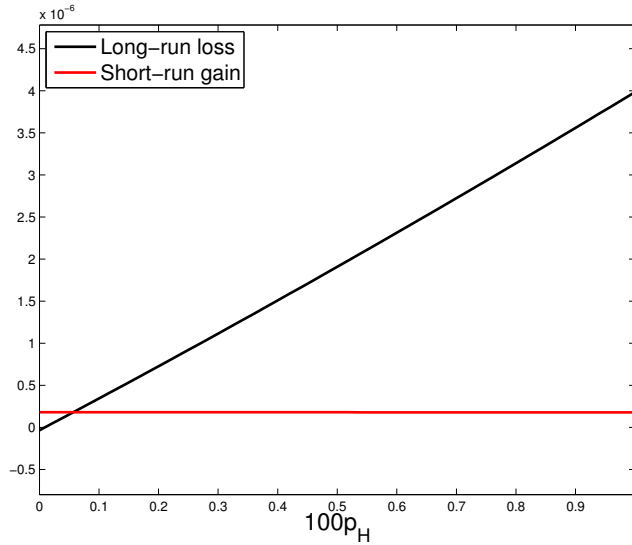


Figure 4: The discretionary and Ramsey outcomes/value sequences:  
Frequent vs. infrequent shocks



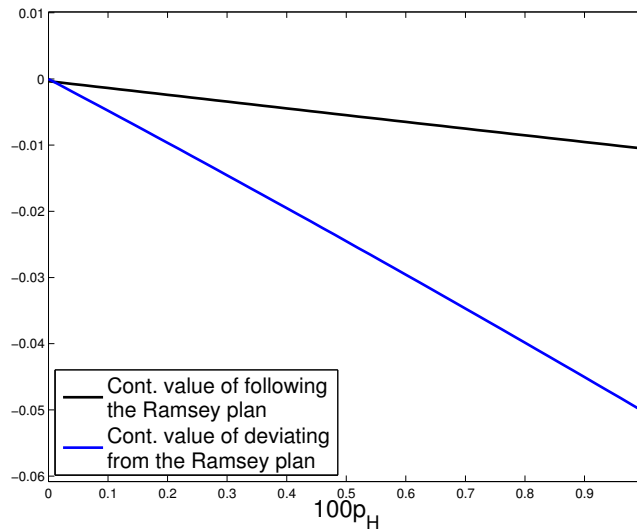
Solid black line: The Ramsey outcome and value sequence  
Dashed red line: The discretionary outcome and value sequence

Figure 5: The short-run gain and the long-run loss of deviating from the Ramsey policy (with alternative shock frequencies)



\*\*The long-run loss shows the loss in the continuation value if the government deviates from the Ramsey policy at  $t=2$ , given by  $\beta E_2[w_{ram,3}(s^3) - w_{d,3}(s^3)]$ , and the short-run gain shows the gain in today's utility flow if the government deviates from the Ramsey policy at  $t=2$ , given by  $[u(c_{d,2}(s^2), \pi_{d,2}(s^2)) - u(c_{ram,2}(s^2), \pi_{ram,2}(s^2))]$ , where  $s_1 = L$  and  $s_2 = H$ .

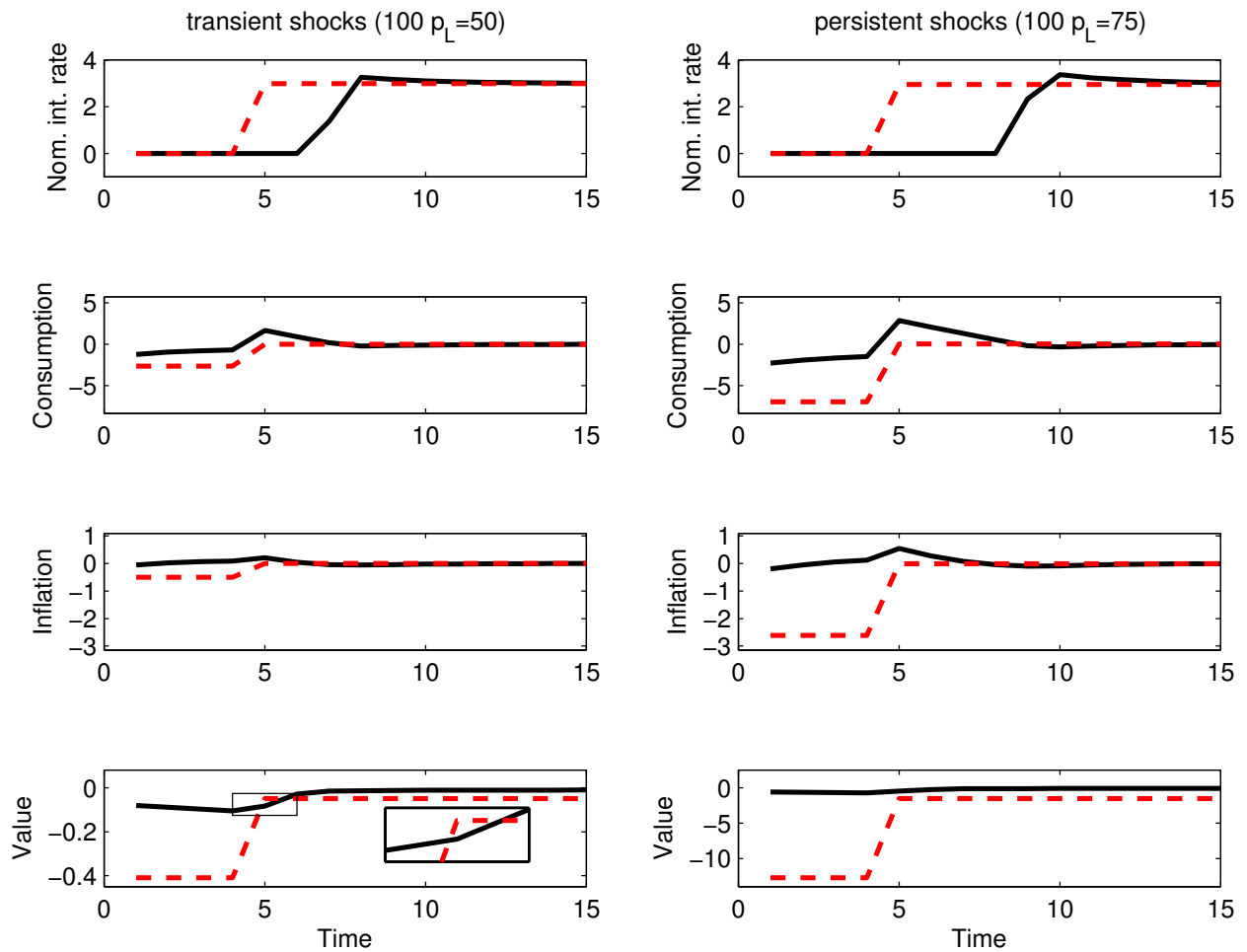
Figure 6: The continuation values of following versus deviating from the Ramsey policy (with alternative shock frequencies)



\*\*The continuation values of following and deviating from the Ramsey plan at  $t=2$  are respectively given by  $\beta E_2 w_{ram,3}(s^3)$  and  $\beta E_2 w_{d,3}(s^3)$  where  $s_1 = L$  and  $s_2 = H$ .

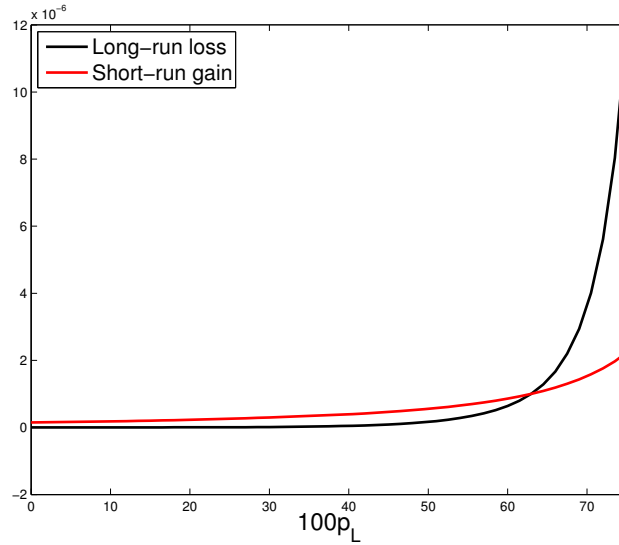


Figure 7: The discretionary and Ramsey outcomes/value sequences:  
 Transient vs. persistent shocks



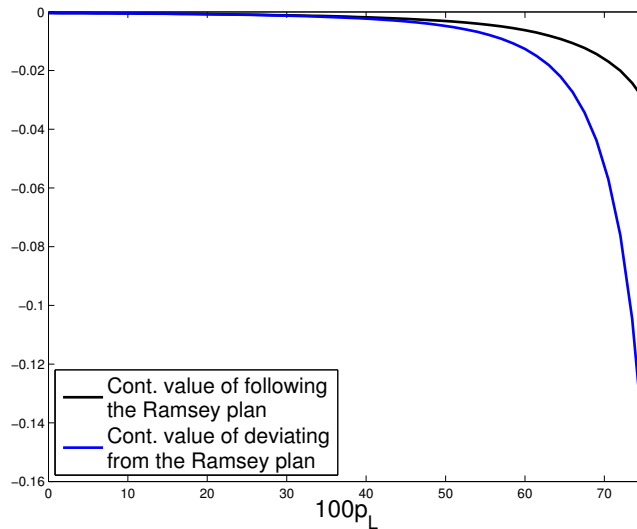
Solid black line: The Ramsey outcome and value sequence  
 Dashed red line: The discretionary outcome and value sequence

Figure 8: The short-run gain and the long-run loss of deviating from the Ramsey policy (with alternative shock persistence)



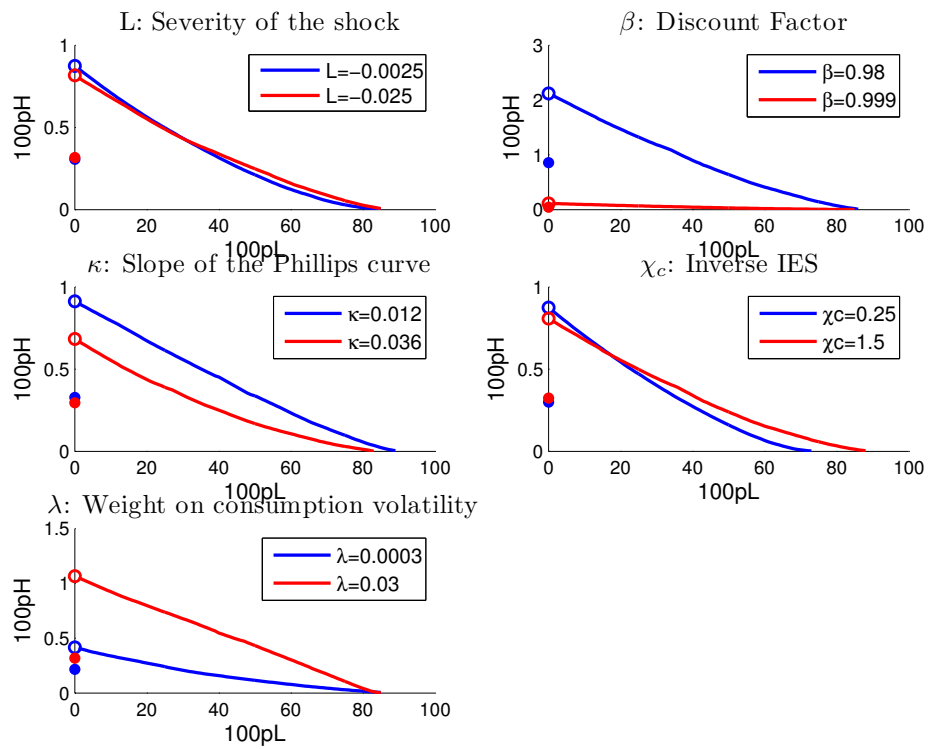
\*\*The long-run loss shows the loss in the continuation value if the government deviates from the Ramsey policy at  $t=5$ , given by  $\beta E_5[w_{ram,6}(s^6) - w_{d,6}(s^6)]$ , and the short-run gain shows the gain in today's utility flow if the government deviates from the Ramsey policy at  $t=5$ , given by  $[u(c_{d,5}(s^5), \pi_{d,5}(s^5)) - u(c_{ram,5}(s^5), \pi_{ram,5}(s^5))]$ , where  $s_t = L$  for  $1 \leq t \leq 4$  and  $s_5 = H$ .

Figure 9: The continuation values of following versus deviating from the Ramsey policy (with alternative shock persistence)



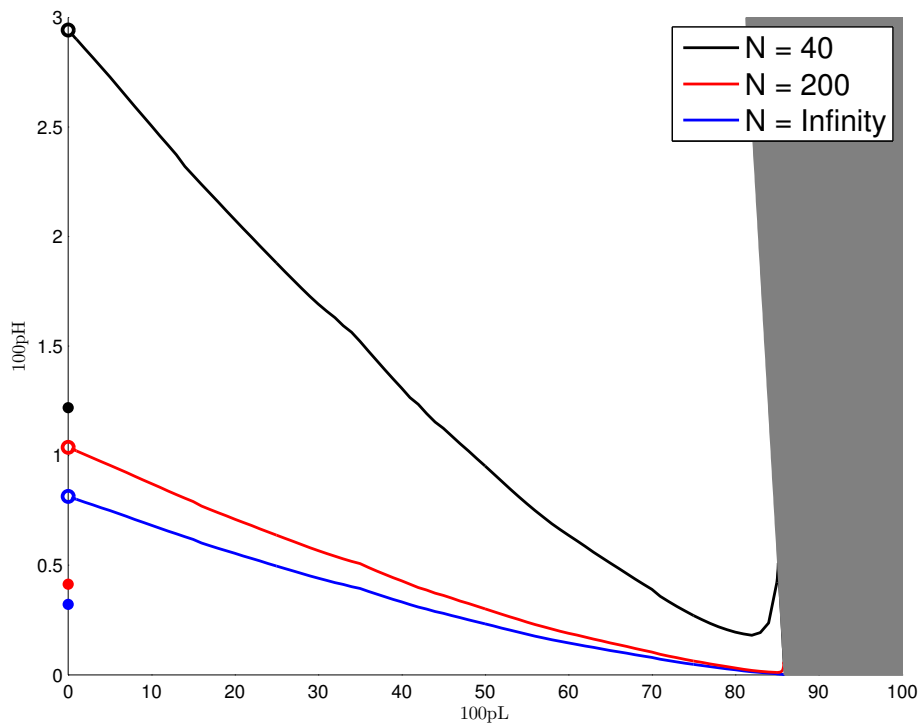
\*\*The continuation values of following and deviating from the Ramsey plan at  $t=5$  are respectively given by  $\beta E_5 w_{ram,6}(s^6)$  and  $\beta E_5 w_{d,6}(s^6)$  where  $s_t = L$  for  $1 \leq t \leq 4$  and  $s_5 = H$ .

Figure 10: Credibility of the revert-to-discretion plan: Sensitivity Analysis



\*In all charts, colored lines (and dots for the case with  $p_L = 0$ ) show the threshold frequency above which the revert-to-discretion plan is credible.

Figure 11: Credibility of the revert-to-discretion(N) plans  
(i.e., plans with finite-periods punishment)



\*Colored lines (and dots for the case with  $p_L = 0$ ) show the threshold frequency above which the revert-to-discretion plan is credible.  $N$  is the punishment periods. Grey areas represent combinations of  $p_H$  and  $p_L$  for which the discretionary outcome does not exist.

# Technical Appendix

Appendix A provides proofs of two propositions in the main text. Appendix B provides detailed analyses of Markov-perfect equilibria and demonstrates the existence of an equilibrium in which the government chooses to keep the nominal interest rate at zero in both high and low states. Appendix C constructs the *revert-to-deflation* plan and examines its credibility. Appendix D defines and analyzes the *revert-to-discretion(N)* plan in which the reversion to a regime in which the central bank cannot manipulate future expectations lasts for finite periods  $N$ . Appendix E provides detailed sensitivity analyses.

## A Proofs

### A.1 Credibility of the discretionary plan

**Proposition:** The discretionary plan is credible.

**Proof:** Let  $\sigma_g^d$  and  $\sigma_p^d$  be the government and private sector strategies associated with the discretionary plan. We need to show that (i)  $\sigma_g^d$  is admissible, (ii) after any history of  $r^t$  and  $s^t$ , the continuation of  $\sigma_p^d$  and  $\sigma_g^d$  induce a  $(\mathbf{c}_t(s_t), \boldsymbol{\pi}_t(s_t), \mathbf{r}_t(s_t)) \in CE_t(s_t)$ , and (iii) after any history  $r^t$  and  $s^t$ ,  $\mathbf{r}_t$  induced by  $\sigma_g^d$  maximizes the government's objective over  $CE_t^R(s_t)$  given  $\sigma_p^d$ .

**Proof of (i):** After *any* history  $r^t$  and  $s^t$ , the continuation of  $\sigma_g^d$  will induce  $\mathbf{r}_t(s_t)$  in which  $r_{t+k}(s^{t+k}) = r_{d,t+k}(s^{t+k})$  for all  $k \geq 0$  and all  $s^{t+k} \in \mathbb{S}^{t+k}$ . Call this sequence  $\mathbf{r}_{d,t}(s_t)$ . Take  $\mathbf{c}_{d,t}(s_t)$  and  $\boldsymbol{\pi}_{d,t}(s_t)$ . Clearly,  $(\mathbf{c}_{d,t}(s_t), \boldsymbol{\pi}_{d,t}(s_t), \mathbf{r}_{d,t}(s_t))$  belongs to  $CE_t(s_t)$ , meaning that  $\mathbf{r}_{d,t}(s_t)$  belongs to  $CE_t^R(s_t)$ . Thus,  $\sigma_g^d$  is admissible.

**Proof of (ii):** Consider a history  $r^t$  and  $s^t$  in which  $r_t = r_{d,t}(s^t)$ . The continuation of  $\sigma_g^d$  and  $\sigma_p^d$  will induce  $(\mathbf{c}_t(s_t), \boldsymbol{\pi}_t(s_t), \mathbf{r}_t(s_t))$  in which  $c_{t+k}(s^{t+k}) = c_{d,t+k}(s^{t+k})$ ,  $\pi_{t+k}(s^{t+k}) = \pi_{d,t+k}(s^{t+k})$  and  $r_{t+k}(s^{t+k}) = r_{d,t+k}(s^{t+k})$  for all  $k \geq 0$  and all  $s^{t+k} \in \mathbb{S}^{t+k}$ . Call them  $(\mathbf{c}_{d,t}(s_t), \boldsymbol{\pi}_{d,t}(s_t), \mathbf{r}_{d,t}(s_t))$ . Clearly,  $(\mathbf{c}_{d,t}(s_t), \boldsymbol{\pi}_{d,t}(s_t), \mathbf{r}_{d,t}(s_t))$  belongs to  $CE_t(s_t)$ .

Now, consider a history  $r^t$  and  $s^t$  in which  $r_t = r_{dev,t} \neq r_{d,t}$ . The continuation of  $\sigma_g^d$  and  $\sigma_p^d$  will induce  $(\mathbf{c}_t(s_t), \boldsymbol{\pi}_t(s_t), \mathbf{r}_t(s_t))$  in which

$$\begin{aligned} c_t(s^t) &= c_{br}(s_t, r_{dev,t}) \\ \pi_t(s^t) &= \pi_{br}(s_t, r_{dev,t}) \end{aligned}$$

where

$$c_{br}(s_t, r_t) := E_t c_{d,t+1}(s^{t+1}) - \frac{1}{\chi_c} \left[ [r_t - E_t \pi_{d,t+1}(s^{t+1})] - s_t \right]$$

$$\pi_{br}(s_t, r_t) := \kappa c_t(s^t) + \beta E_t \pi_{d,t+1}(s^{t+1})$$

and  $c_{t+k}(s^{t+k}) = c_{d,t+k}(s^{t+k})$  and  $\pi_{t+k} = \pi_{d,t+k}(s^{t+k})$  for all  $k \geq 1$  and all  $s^{t+k} \in \mathbb{S}^{t+k}$ , and (ii)  $r_{t+k}(s^{t+k}) = r_{d,t+k}(s^{t+k})$  for all  $k \geq 1$  and all  $s^{t+k} \in \mathbb{S}^{t+k}$ .

Call them  $(\mathbf{c}_{dev,t}(s_t), \boldsymbol{\pi}_{dev,t}(s_t), \mathbf{r}_{dev,t}(s_t))$ . The consumption Euler equation and the Phillips curve are satisfied for all  $t+k$  and all  $s^{t+k} \in \mathbb{S}^{t+k}$  with  $k \geq 1$  because the continuation outcome is given by the discretionary outcome. The Phillips curve is trivially satisfied at time  $t$ , and you can rearrange the expression for  $c_t(s^t)$  to confirm that the consumption Euler equation is also satisfied at period  $t$ .<sup>26</sup> Thus,  $(\mathbf{c}_{dev,t}(s_t), \boldsymbol{\pi}_{dev,t}(s_t), \mathbf{r}_{dev,t}(s_t))$  belongs to  $CE_t(s_t)$ .

These two cases cover all possible histories. Thus, for *any* history  $r^t$  and  $s^t$ , the continuation of  $\sigma_g^d$  and  $\sigma_p^d$  will induce  $(\mathbf{c}_t, \boldsymbol{\pi}_t, \mathbf{r}_t)$  that belongs to  $CE_t(s_t)$ .

**Proof of (iii):** Consider any history  $r^{t-1}$  and  $s^t$ . By one-shot deviation principle, it suffices to show that there is no profitable deviation today from the government strategy in order to prove (iii). That is, it suffices to show

$$u(c_{d,t}(s^t), \pi_{d,t}(s^t)) + \beta E_t w_{d,t+1}(s^{t+1}) \geq u(c_{br}(r_{dev,t}, s_t), \pi_{br}(r_{dev,t}, s_t)) + \beta E_t w_{d,t+1}(s^{t+1})$$

for all  $r_{dev,t} \in \mathbb{R}$ .

The left-hand side is the value of following the instruction given by the government strategy, and the right hand-side is the value of deviating from it. The continuation value in the case of the deviation is  $E_t w_{d,t+1}(s^{t+1})$  because the government and private sector strategies would induce the discretionary outcome from next period on even in the case of the government deviation today. Notice that, by the definition of the discretionary outcome,

$$u(c_{d,t}(s^t), \pi_{d,t}(s^t)) + \beta E_t w_{d,t+1}(s^{t+1}) = \max_{r_t \in \mathbb{R}} u(c_{br}(r_t, s_t), \pi_{br}(r_t, s_t)) + \beta E_t w_{d,t+1}(s^{t+1})$$

Thus, the aforementioned inequality holds, and there is no profitable one-shot deviation.

## A.2 Credibility of the revert-to-discretion plan

**Proposition:** The revert-to-discretion plan is credible if and only if  $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$  for all  $t \geq 1$  and all  $s^t \in \mathbb{S}^t$ .

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<sup>26</sup>Here, I am assuming that, for a given  $r_{max}$ ,  $c_{max}$  and  $\pi_{max}$  are sufficiently large and  $c_{min}$  and  $\pi_{min}$  are sufficiently small so that  $c_{br}(s_t, r_{dev,t}) \in \mathbb{C}$  and  $\pi_{br}(s_t, r_{dev,t}) \in \Pi$  for all  $r_t \in \mathbb{R}$ .

**Proof of “if”-part:** Let  $\sigma_g^{ram}$  and  $\sigma_p^{ram}$  be the government and private sector strategies associated with the revert-to-discretion plan. We need to show that, if  $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$  for all  $t \geq 1$  and all  $s^t \in \mathbb{S}^t$ , (i)  $\sigma_g^{ram}$  is admissible, (ii) given  $\sigma_g^{ram}$ , after any history  $r^t$  and  $s^t$ , the continuation of  $\sigma_p^{ram}$  and  $\sigma_g^{ram}$  induce a  $(\mathbf{c}_t(s_t), \boldsymbol{\pi}_t(s_t), \mathbf{r}_t(s_t)) \in CE_t(s_t)$ , and (iii) after any history  $r^{t-1}$  and  $s^t$ , the sequence  $\mathbf{r}_t(s_t)$  induced by  $\sigma_g^{ram}$  maximizes the government’s objective over  $CE_t^R(s_t)$  given  $\sigma_p^{ram}$ .

**Proof of (i):** Consider a history  $r^{t-1}$  and  $s^t$  where  $r_j = r_{ram,j}(s^j)$  for all  $j \leq t-1$  and all  $s^j \in \mathbb{S}^j$ . The continuation of  $\sigma_g^{ram}$  will induce  $\mathbf{r}_t(s_t)$  in which  $r_{t+k}(s^{t+k}) = r_{ram,t+k}(s^{t+k})$  for all  $k \geq 0$  and all  $s^{t+k} \in \mathbb{S}^{t+k}$ . Call this sequence  $\mathbf{r}_{ram,t}$ . Take  $\mathbf{c}_{ram,t}(s_t)$  and  $\boldsymbol{\pi}_{ram,t}(s_t)$ . Clearly,  $(\mathbf{c}_{ram,t}(s_t), \boldsymbol{\pi}_{ram,t}(s_t), \mathbf{r}_{ram,t}(s_t))$  belongs to  $CE_t(s_t)$ .

Now consider a history  $r^{t-1}$  and  $s^t$  in which  $r_j \neq r_{ram,j}(s^j)$  for some  $j \leq t-1$ . The continuation of  $\sigma_g^{ram}$  will induce  $\mathbf{r}_t$  in which  $r_{t+k}(s^{t+k}) = r_{d,t+k}(s^{t+k})$  for all  $k \geq 0$  and all  $s^{t+k} \in \mathbb{S}^{t+k}$ . Call this sequence  $\mathbf{r}_{d,t}$ . Take  $\mathbf{c}_{d,t}(s_t)$  and  $\boldsymbol{\pi}_{d,t}(s_t)$ . Clearly,  $(\mathbf{c}_{d,t}(s_t), \boldsymbol{\pi}_{d,t}(s_t), \mathbf{r}_{d,t}(s_t))$  belongs to  $CE_t(s_t)$ .

These two cases cover all possible histories. Thus, after *any* history  $r^{t-1}$  and  $s^t$ , the continuation of  $\sigma_g^{ram}$  will induce  $\mathbf{r}_t(s_t)$  that belongs to  $CE_t^R(s_t)$ .

**Proof of (ii):** Consider a history  $r^t$  and  $s^t$  where  $r_j = r_{ram,j}(s^j)$  for all  $j \leq t$ . The continuation of  $\sigma_g^{ram}$  and  $\sigma_p^{ram}$  induces  $(\mathbf{c}_t(s_t), \boldsymbol{\pi}_t(s_t), \mathbf{r}_t(s_t))$  where  $c_{t+k}(s^{t+k}) = c_{ram,t+k}(s^{t+k})$ ,  $\pi_{t+k}(s^{t+k}) = \pi_{ram,t+k}(s^{t+k})$ , and  $r_{t+k}(s^{t+k}) = r_{ram,t+k}(s^{t+k})$  for all  $k \geq 0$  and all  $s^{t+k} \in \mathbb{S}^{t+k}$ . Call this sequence  $(\mathbf{c}_{ram,t}(s_t), \boldsymbol{\pi}_{ram,t}(s_t), \mathbf{r}_{ram,t}(s_t))$ . Clearly,  $(\mathbf{c}_{ram,t}(s_t), \boldsymbol{\pi}_{ram,t}(s_t), \mathbf{r}_{ram,t}(s_t))$  belongs to  $CE_t(s_t)$ .

Consider a history  $r^t$  and  $s^t$  in which  $r_j \neq r_{ram,j}(s^j)$  for some  $j \leq t$ . The continuation of  $\sigma_g^{ram}$  and  $\sigma_p^{ram}$  induces  $(\mathbf{c}_t, \boldsymbol{\pi}_t, \mathbf{r}_t)$  where  $c_{t+k}(s^{t+k}) = c_{d,t+k}(s^{t+k})$ ,  $\pi_{t+k}(s^{t+k}) = \pi_{d,t+k}(s^{t+k})$ , and  $r_{t+k}(s^{t+k}) = r_{d,t+k}(s^{t+k})$  for all  $k \geq 1$  and all  $s^{t+k} \in \mathbb{S}^{t+k}$ . At time  $t$ ,  $c_t(s^t) = c_{br}(r_t, s^t)$  and  $\pi_t = \pi_{br,t}(r_t(s^t), s^t)$ .

Call this continuation sequence  $(\mathbf{c}_{dev,t}(s_t), \boldsymbol{\pi}_{dev,t}(s_t), \mathbf{r}_{dev,t}(s_t))$ . Clearly, the consumption Euler equation and the Phillips curve are satisfied for all  $k \geq 1$ . If  $r_t(s^t) = r_{d,t}(s^t)$ , it is clear that the consumption Euler equation and the Phillips curve is satisfied at  $k = 0$  as well. At time  $t$ , the Phillips curve is satisfied trivially, and you can rearrange the expression for  $c_t(s^t)$  to confirm that the consumption Euler equation is also satisfied. Thus,  $(\mathbf{c}_{dev,t}(s_t), \boldsymbol{\pi}_{dev,t}(s_t), \mathbf{r}_{dev,t}(s_t))$  belongs to  $CE_t(s_t)$ .

These two cases cover all possible histories. Thus, after any history  $r^t$  and  $s^t$ , the continuation of  $\sigma_g^{ram}$  and  $\sigma_p^{ram}$  will induce  $(\mathbf{c}_t(s_t), \boldsymbol{\pi}_t(s_t), \mathbf{r}_t(s_t))$  that belongs to  $CE_t(s_t)$ .

**Proof of (iii):** By one-shot deviation principle, it suffices to show that, after any history  $r^{t-1}$  and  $s^t$ , there is no profitable deviation today from the government strategy in order to prove (iii).

First, consider a history  $r^{t-1}$  in which  $r_j \neq r_{ram,j}(s^j)$  for some  $j \leq t-1$ . In this case, we want to show

$$u(c_{d,t}(s^t), \pi_{d,t}(s^t)) + \beta E_t w_{d,t+1}(s^{t+1}) \geq u(c_{br}(r_t, s_t), \pi_{br}(r_t, s_t)) + \beta E_t w_{d,t+1}(s^{t+1})$$

for all  $r_t \in \mathbb{R}$ . The left-hand side is the value of following the instruction given by the government strategy, and the right hand-side is the value of deviating from it. The same argument from the

proof of (iii) in the previous proposition applies so that there is no profitable one-shot deviation.

Now, consider a history  $r^t$  where  $r_j = r_{ram,j}(s^j)$  for all  $j \leq t$ . We want to show that

$$u(c_{ram,t}(s^t), \pi_{ram,t}(s^t)) + \beta E_t w_{ram,t+1}(s^{t+1}) \geq u(c_{br}(r_t, s_t), \pi_{br}(r_t, s_t)) + \beta E_t w_{d,t+1}(s^{t+1})$$

for all  $r_t \in \mathbb{R}$ . The left-hand side is the continuation value when the government chooses the nominal interest rate consistent with the Ramsey outcome (i.e.,  $r_t(s^t) = r_{ram,t}(s^t)$ ), and the right hand side is the possible continuation values in the case of deviation (i.e.,  $r_t(s^t) \neq r_{ram,t}(s^t)$ ). Notice that the left-hand side of the inequality is equal to  $w_{ram,t}(s^t)$ . If  $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$  for all  $t \geq 1$ , then

$$\begin{aligned} u(c_{ram,t}(s^t), \pi_{ram,t}(s^t)) + \beta E_t w_{ram,t+1}(s^{t+1}) &= w_{ram,t}(s^t) \\ &\geq w_{d,t}(s^t) \\ &= \max_{r_t \in \mathbb{R}} \left[ u(c_{br}(r_t, s_t), \pi_{br}(r_t, s_t)) + \beta E_t w_{d,t+1}(s^{t+1}) \right] \\ &\geq u(c_{br}(r_t), \pi_{br}(r_t)) + \beta E_t w_{d,t+1}(s^{t+1}) \end{aligned}$$

for all  $r_t \in \mathbb{R}$ . Thus, there is no profitable one-shot deviation in this case as well.

These two cases cover all possible histories. Thus, there is no profitable one-shot deviation after *any* history, and (iii) holds.

### Proof of “only if”-part:

We want to prove that, if a plan is credible, then  $w_{ram,t}(s^t) \geq w_{d,t}(s^t)$  for all  $t \geq 1$  and all  $s^t \in \mathbb{S}^t$ . We will do so by proving the contraposition, i.e., by showing that a plan is not credible if  $w_{ram,t}(s^t) < w_{d,t}(s^t)$  for some  $t \geq 1$  and some  $s^t \in \mathbb{S}^t$ .

Let  $t_v$  and  $s^{t_v}$  be such that  $w_{ram,t}(s^{t_v}) < w_{d,t}(s^{t_v})$ . Then,

$$u(c_{ram,t_v}(s^{t_v}), \pi_{ram,t_v}(s^{t_v})) + \beta E_{t_v} w_{ram,t_v+1}(s^{t_v+1}) < u(c_{br}(r_{dev,t}, s_t), \pi_{br}(r_{dev,t}, s_t)) + \beta E_t w_{d,t+1}(s^{t+1})$$

for some  $r_{dev,t} \in \mathbb{R}$ . Then, consider a government strategy that instructs the government to choose  $r_{dev,t}$  today and follow the discretionary outcome from tomorrow. The strategy delivers the better value today than the continuation of the revert-to-discretion plan, violating the third condition of credibility.



## B Analyses of Markov-Perfect Equilibria

In this section, I analyze the existence and multiplicity of Markov-Perfect equilibria—time-invariant solutions for the discretionary government’s problem described in the main text. In addition to the equilibrium in which the ZLB binds only in the low state (Type-I equilibrium), there are three other possible equilibria: one in which the ZLB binds in both states (Type-II equilibrium), one in which the ZLB binds in neither states (Type-III equilibrium), and the other in which the ZLB binds only in the high state (Type-IV equilibrium). This section aims to understand how frequency and persistent of the shock affect the existence of these four types of Markov-Perfect equilibria.

The first order necessary conditions for the discretionary government’s problem are given by

$$\chi_c c(H) = \chi_c [(1 - p_H)c(H) + p_H c(L)] + [(1 - p_H)\pi(H) + p_H \pi(L)] - r(H) + H \quad (12)$$

$$\pi(H) = \kappa c(H) + \beta [(1 - p_H)\pi(H) + p_H \pi(L)] \quad (13)$$

$$0 = \lambda c(H) - \phi_3(H) + \kappa \pi(H) \quad (14)$$

$$\chi_c c(L) = \chi_c [(1 - p_L)c(H) + p_L c(L)] + [(1 - p_L)\pi(H) + p_L \pi(L)] - r(L) + L \quad (15)$$

$$\pi(L) = \kappa c(L) + \beta [(1 - p_L)\pi(H) + p_L \pi(L)] \quad (16)$$

$$0 = \lambda c(L) - \phi_3(L) + \kappa \pi(L) \quad (17)$$

together with one of the following sets of conditions regarding the nominal interest rate and the Lagrange multiplier on the ZLB constraint,  $\phi_3(\cdot)$ .

$$r(H) \geq 0, r(L) = 0, \phi_3(H) = 0, \text{ and } \phi_3(L) \leq 0 \quad (\text{for type-I Equilibrium}) \quad (18)$$

$$r(H) = 0, r(L) = 0, \phi_3(H) \leq 0, \text{ and } \phi_3(L) \leq 0 \quad (\text{for type-II Equilibrium}) \quad (19)$$

$$r(H) \geq 0, r(L) \geq 0, \phi_3(H) = 0, \text{ and } \phi_3(L) = 0 \quad (\text{for type-III Equilibrium}) \quad (20)$$

$$r(H) = 0, r(L) \geq 0, \phi_3(H) \leq 0, \text{ and } \phi_3(L) = 0 \quad (\text{for type-IV Equilibrium}) \quad (21)$$

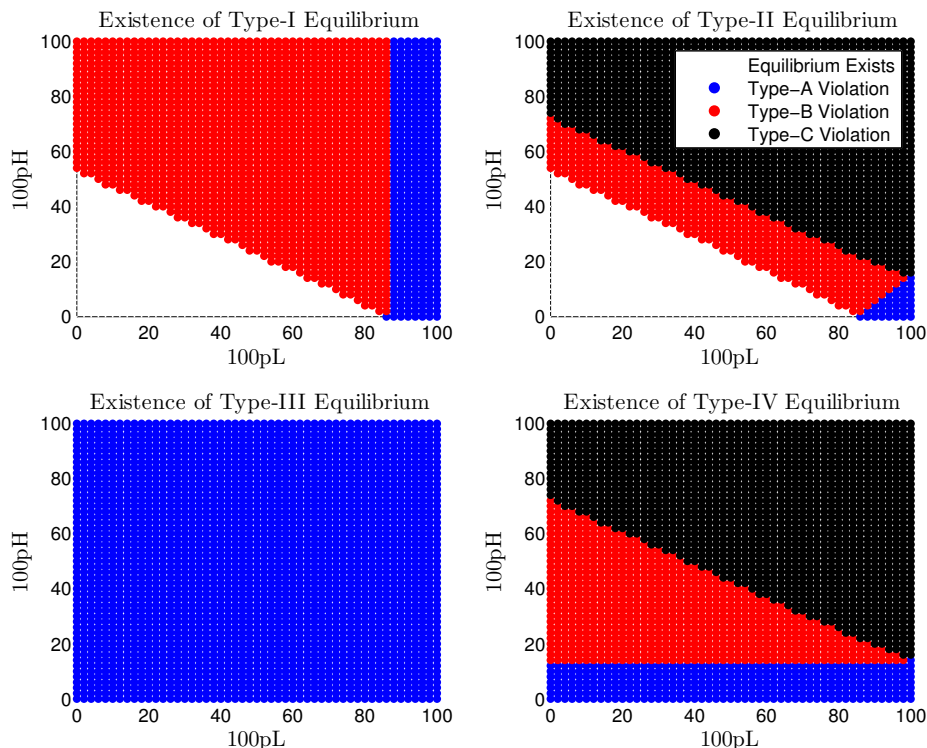
To check the existence of an equilibrium in which the ZLB binds only in the low state (Type-I), you need to solve the system of nonlinear equations above by assuming (i)  $\phi_3(H) = 0$  and (ii)  $r(L) = 0$  and then check whether or not (i)  $r(H) \geq 0$  and (ii)  $\phi_3(L) \leq 0$ . If either one or both of these two inequalities are violated, this means that there is no equilibrium in which the ZLB binds only in the low state.

The existence of other three equilibria can be checked in a similar way. For the equilibrium in which the ZLB binds in the both states (Type-II), you first solve the system of equations above by assuming (i)  $r(H) = 0$  and (ii)  $r(L) = 0$  and then check that (i)  $\phi_3(H) \leq 0$  and (ii)  $\phi_3(L) \leq 0$  to verify its existence. If either one or both of these two inequalities are violated, then this means that there is no equilibrium in which the ZLB binds in both states. For the equilibrium in which

the ZLB does not bind in both states (Type-III), you solve the system of linear equations above by assuming (i)  $\phi_3(H) = 0$  and (ii)  $\phi_3(L) = 0$  and then check that (i)  $r(H) \geq 0$  and (ii)  $r(L) \geq 0$ . Violation of either one or both of these two inequalities would mean that there is no equilibrium in which the ZLB does not bind in both states. Finally, for the equilibrium in which the ZLB binds only in the high state (Type-IV), you solve the system of equations above by assuming (i)  $r(H) = 0$  and (ii)  $\phi_3(L) = 0$  and then check that (i)  $\phi_3(H) \leq 0$  and (ii)  $r(L) \geq 0$ . Violation of either one or both of these two inequalities would mean that there is no equilibrium in which the ZLB binds only in the high state. In each of four alternative Markov-Perfect equilibria, I say the Type-A violation occurs if the first of the two inequalities alone is violated, Type-B violation if the second of the two inequalities alone is violated, and Type-C violation if both inequalities are violated.

Figure B.1 shows the existence of four possible equilibria for different combinations of  $p_L$  and  $p_H$ . In each panel, white areas show the combinations of frequency and persistence under which the equilibrium of a particular type exists. Colored dots indicate that either one or both of the relevant inequality constraints are violated and thus that the equilibrium does not exist. Different colors indicate different reasons for why the equilibrium does not exist. Red, blue and black dots respectively indicate Type-A, Type-B, and Type-C violations.

Figure B.1: Existence of Four Markov-Perfect Equilibria



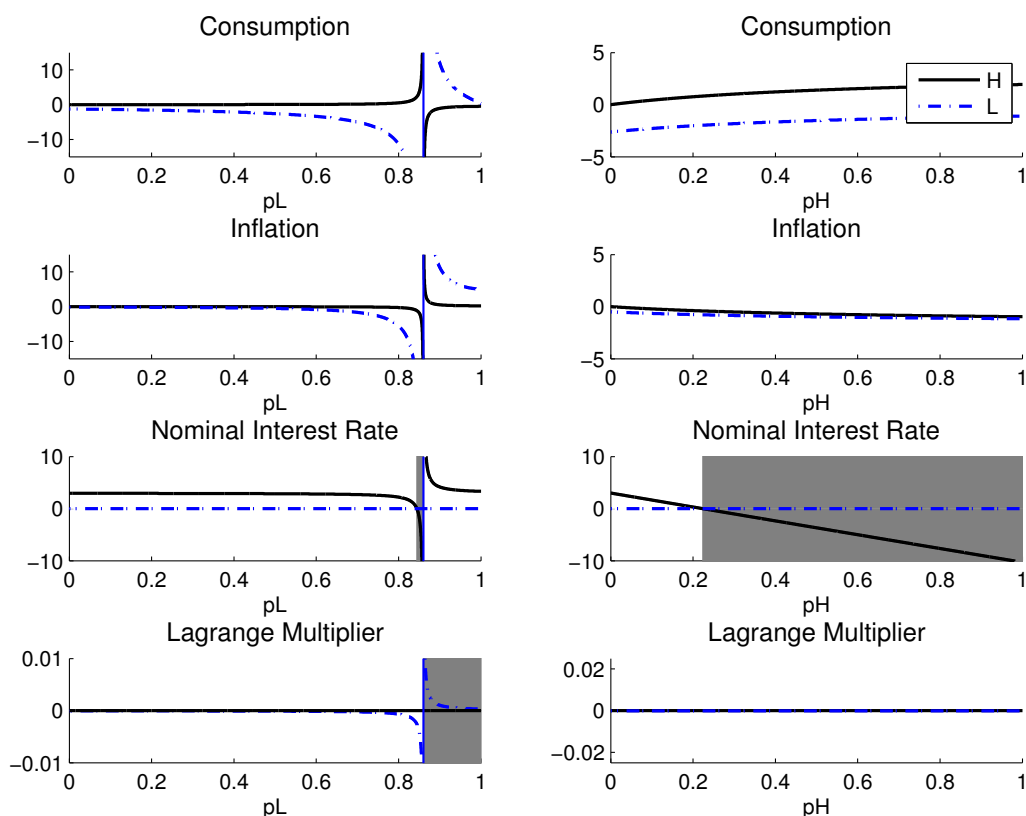
According to the top two panels, Type-I and Type-II Markov-Perfect equilibria exist when frequency and persistence of the shock are sufficiently low. According to the bottom two panels, Type-III and Type-IV Markov-Perfect equilibria do not exist regardless of frequency and persis-

tence. In what follows, I will take a closer look at each possible equilibrium to understand why an equilibrium of a particular type does and does not exist.

### Type-I: The ZLB binds only in the low state

Top-left panel in Figure B.1 shows that the equilibrium in which the ZLB binds only in the low state does not exist if either the frequency or the persistence of the shock is sufficiently high. To understand why, Figure B.2 plots how the solution to the system of linear equations above depends on  $p_L$  and  $p_H$ . The left panels show how consumption, inflation, nominal interest rate, and the Lagrange multiplier that solves the linear system vary with  $p_H$ , holding  $p_L$  constant at 0.5. The right panels show how they vary with  $p_L$  holding  $p_H$  constant at 0.01.

Figure B.2: Allocations in Type-I Markov-Perfect Equilibrium



\*Left panels show how allocations and the Lagrange multiplier vary with  $p_L$  holding  $p_H = 0.01$  while right panels show how they vary with  $p_H$  holding  $p_L = 0.5$ . In all panels, solid black and dash blue lines are for high and low states respectively. Shaded areas in the panels for the nominal interest rate show the parameter region where the nominal interest rate is below zero, and the shaded area in the Lagrange multiplier panels show the region where it is positive.

As described in the main text, the household and firms have incentives to reduce their consumption and prices even before the contractionary shock hits the economy since the anticipation of future shocks increases the expected real interest rate and reduces the marginal costs they face

in the high state. The government lowers the nominal interest rate to offset these effects. In equilibrium, consumption is positive, inflation is negative, and the nominal interest rate is below the deterministic steady-state in the high state. A higher frequency of shocks ( $p_H$ ) means this anticipation effect is stronger. The left panels in Figure B.2 indeed show that high-state consumption increases and high-state inflation and nominal interest rate decrease with  $p_H$ . When the frequency is sufficiently high, the nominal interest rate in the high state is negative, violating the ZLB constraint. Therefore, Type-I equilibrium does not exist with sufficiently large  $p_H$ .

The equilibrium also does not exist when persistence is sufficiently high. A more persistent shock means that the household and firms expect to be in the low state longer on average, which increases the expected real interest rate and decreases the expected marginal costs in the low state. Thus, the household and firms reduce consumption and prices in the low state by more in the economy with a higher  $p_L$ , as depicted in Figure B.2. However, there is a cut-off value  $p_L$  above which low-state consumption and inflation that solve the first order necessary conditions turn positive. With more persistence, low-state consumption and inflation are influenced more by the future low-state consumption and inflation, and positive low-state consumption and inflation can be self-fulfilling if persistence is sufficiently large. However, for such high persistence, the Lagrange multiplier on the ZLB constraint becomes positive as the government has the incentive to raise the nominal interest rate to lower consumption and inflation, violating one of the inequality constraints that needs to be satisfied for the equilibrium to exist.<sup>27</sup>

## Type-II: The ZLB binds in both states

According to the top-right panel in Figure B.1, the equilibrium in which the ZLB binds in both states exists when the frequency and persistence are sufficiently low. Figure B.3 shows that inflation and output are below their steady-states in both states. Declines in inflation and output in the low state are much larger in this equilibrium than those in the Type-I Markov-Perfect equilibrium. Similarly to the Type-I Equilibrium, an increase in the shock persistence leads to larger declines in consumption and output in the low state, and there is a cut-off value of  $p_L$  above which the low-state consumption and inflation that solve the system of linear equations turn positive. According to the right panels, consumption and inflation in both states increase with the frequency in this equilibrium. It can be shown analytically that the region of equilibrium existence for this case is identical to the existence region of the first Markov-Perfect equilibrium.<sup>28</sup>

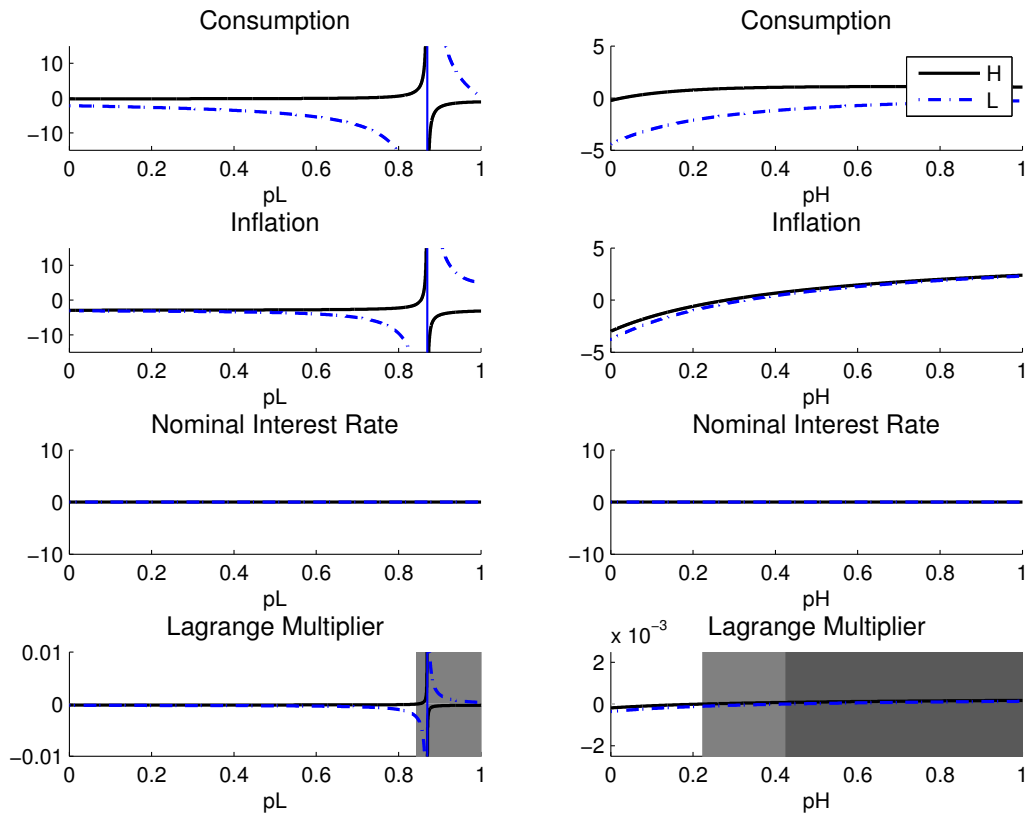
The existence of such an equilibrium with this Type-II Markov-Perfect equilibrium with permanently binding ZLB is at first surprising. If the government is optimizing, why can the government keep the economy out of a permanent liquidity trap? The key ingredient to understanding the existence of this equilibrium is the lack of commitment by the government, i.e. the government takes future policy functions as given. If the household and firms expect low consumption and

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<sup>27</sup>See Nakata and Schmidt (2014) for analytical results on the conditions on  $p_H$  and  $p_L$  guaranteeing the existence of this Type-I Markov-Perfect equilibrium.

<sup>28</sup>Analytical proof is available upon request.

Figure B.3: Allocations in Type-II Markov-Perfect Equilibrium



\*Left panels show how allocations and the Lagrange multiplier vary with  $p_L$  holding  $p_H = 0.01$  while right panels show how they vary with  $p_H$  holding  $p_L = 0.5$ . In all panels, solid black and dash blue lines are for high and low states respectively. Light gray areas in the panels for the Lagrange multiplier show the parameter region where one of low-state and high-state Lagrange multipliers is positive, while dark gray areas show the parameter region where the Lagrange multipliers are positive in both states.

inflation in the high state tomorrow, they would like to choose low consumption and inflation even in the high state today. The government would like to reduce the nominal interest rate in order to prevent the declines in inflation and consumption, but it cannot do so if the ZLB constraint is binding. Thus, below-trend consumption and deflation can be self-fulfilling in the high state.

### Type-III and Type-IV: The ZLB does not bind in the low state

According to the bottom two panels in Figure B.1, the equilibrium in which the ZLB does not bind in both states (Type-III) and the equilibrium in which the ZLB binds only in the high state (Type-IV) do not exist for any combinations of frequency and persistence. In the hypothetical Type-III equilibrium in which the low-state nominal interest rate is unconstrained, the government would like to lower the nominal interest rate below zero in the low state in order to stabilize consumption and output, which violates the inequality constraint that the nominal interest rate has to be positive (Type-B violations). In the hypothetical Type-IV equilibrium in which the low-state nominal interest rate is unconstrained, the reasons for non-existence are not always the same and depend on the frequency and persistence of the shock.

To summarize, given a pair of  $(p_H, p_L)$ , we either have two equilibria—one in which the ZLB binds only in the low state and the other in which the ZLB bind in both states—or do not have any equilibria. In the main text, I construct the *revert-to-discretion* plan—a plan in which the deviation from the Ramsey outcome would be punished by the first of these two Markov-Perfect equilibria—and analyze the conditions under which this plan can make the Ramsey outcome time-consistent. In the next section, I will construct a plan in which deviation from the Ramsey outcome is punished by the second Markov perfect equilibrium and study the conditions under which such a plan can make the Ramsey outcome time-consistent.

## C The revert-to-deflation plan and its credibility

In this section, I define the deflationary plan and the revert-to-deflation plan and examine their credibility.

### C.1 The deflationary plan

Let  $\{w_{def}(\cdot), c_{def}(\cdot), \pi_{def}(\cdot), r_{def}(\cdot)\}$  be the set of time-invariant value function and policy functions for consumption, inflation, and the nominal interest rate which solves the discretionary government's problem described in the main text and in which the ZLB binds in both states. *The deflationary outcome* is defined as, and denoted by, the state-contingent sequence of consumption, inflation, and the nominal interest rate,  $\{c_{def,t}(s^t), \pi_{def,t}(s^t), r_{def,t}(s^t)\}_{t=1}^{\infty}$  such that  $c_{def,t}(s^t) := c_{def}(s_t)$ ,  $\pi_{def,t}(s^t) := \pi_{def}(s_t)$ , and  $r_{def,t}(s^t) := r_{def}(s_t)$  and *the deflationary value sequence* is defined and denoted as  $\{w_{def,t}(s^t)\}_{t=1}^{\infty}$  such that  $w_{def,t}(s^t) := w_{def}(s_t)$ .

*The deflationary plan*,  $(\sigma_g^{def}, \sigma_p^{def})$ , consists of the following government strategy

- $\sigma_{g,1}^{def}(s_1) = r_{def}(s_1)$  for any  $s_1 \in \mathbb{S}$
- $\sigma_{g,t}^{def}(r^{t-1}, s^t) = r_{def}(s_t)$  for any  $s^t \in \mathbb{S}^t$  and any  $r^{t-1} \in \mathbb{R}^{t-1}$

and the following private-sector strategy

- $\sigma_{p,t}^{def}(r^t, s^t) = (c_{def}(s_t), \pi_{def}(s_t))$  if  $r_t = r_{def}(s_t)$
- $\sigma_{p,t}^{def}(r^t, s^t) = (c_{br,def}(s_t, r_t), \pi_{br,def}(s_t, r_t))$  otherwise<sup>29</sup>

where

$$c_{br,def}(s_t, r_t) = E_t c_{def,t+1}(s^{t+1}) - \frac{1}{\chi_c} \left[ [r_t - E_t \pi_{def,t+1}(s^{t+1})] - s_t \right]$$

$$\pi_{br,def}(s_t, r_t) = \kappa c_{br,def}(s_t, r_t) + \beta E_t \pi_{def,t+1}(s^{t+1})$$

The government strategy instructs the government to choose the nominal interest rate consistent with the deflationary outcome, regardless of the history of past nominal interest rates. The private sector strategy instructs the household and firms to choose consumption and inflation consistent with the deflationary outcome, as long as today's nominal interest rate chosen by the government is consistent with the deflationary outcome. If the government chooses an interest rate that is not consistent with the deflationary outcome, then the private sector strategy instructs the household and firms to optimally choose today's consumption under the belief that the government in the future will not deviate again.

By construction, the deflationary plan induces the deflationary outcome, and the value sequence implied by the deflationary plan is identical to the deflationary value sequence.

**Proposition C.1:** The deflationary plan is credible.

The proof for this proposition proceeds in the same as the proof for proposition 1.

## C.2 The revert-to-deflation plan

The revert-to-deflation plan,  $(\sigma_g^{rtdef}, \sigma_p^{rtdef})$ , consists of the following government strategy

- $\sigma_{g,1}^{rtdef}(s_1) = r_{ram,1}(s_1)$  for any  $s_1 \in \mathbb{S}$
- $\sigma_{g,t}^{rtdef}(r^{t-1}, s^t) = r_{ram,t}(s^t)$  if  $r_j = r_{ram,j}(s^j)$  for all  $j \leq t-1$
- $\sigma_{g,t}^{rtdef}(r^{t-1}, s^t) = \sigma_{g,t}^{def}(r^{t-1}, s^t)$  otherwise.

and the following private-sector strategy

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<sup>29</sup>Subscript *br* stands for *best response*.

- $\sigma_{p,t}^{rtdef}(r^t, s^t) = (c_{ram,t}(s^t), \pi_{ram,t}(s^t))$  if  $r_j = r_{ram,j}(s^j)$  for all  $j \leq t$
- $\sigma_{p,t}^{rtdef}(r^t, s^t) = \sigma_{p,t}^{def}(r^t, s^t)$  otherwise.

The government strategy instructs the government to choose the nominal interest rate consistent with the deflationary outcome, but chooses the interest rate consistent with the deflationary outcome if it has deviated from the Ramsey outcome at some point in the past. The private sector strategy instructs the household and firms to choose consumption and inflation consistent with the Ramsey outcome as long as the government has never deviated from the Ramsey outcome. If the government has ever deviated from the nominal interest rate consistent with the Ramsey outcome, the private sector strategy instructs the household and firms to choose consumption and inflation today based on the belief that the government in the future will choose the nominal interest rate consistent with the deflationary outcome.

By construction, the revert-to-deflation plan induces the Ramsey outcome, and the implied value sequence is identical to the Ramsey value sequence.

**Proposition C.2:** The revert-to-deflation plan is credible if and only if  $w_{ram,t}(s^t) \geq w_{def,t}(s^t)$  for all  $t \geq 1$  and all  $s^t \in \mathbb{S}^t$ .

The proof for this proposition proceeds in the same way as the proof for proposition 2.

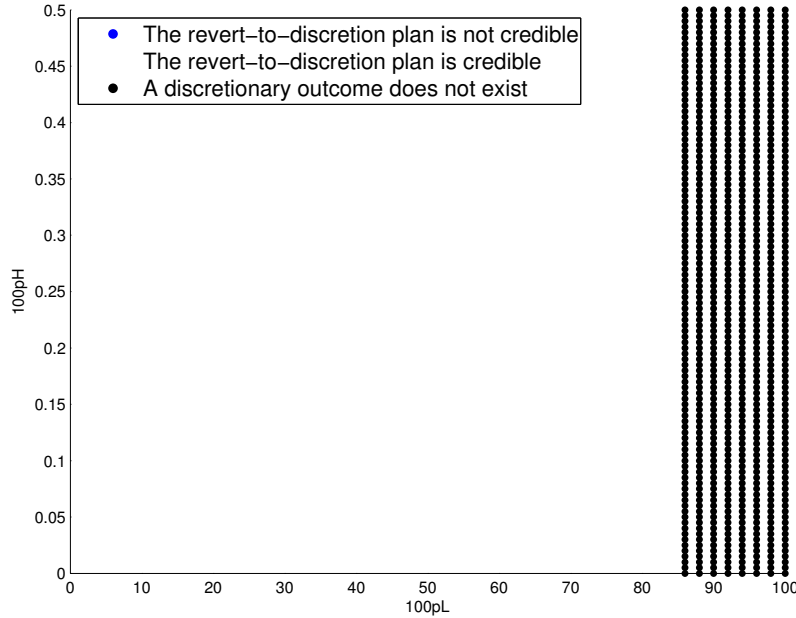
### C.3 Credibility of the revert-to-deflation plan

Figure C.1 shows how the credibility of the revert-to-deflation plan depends on the frequency and severity of the shocks. Blank areas indicate the combinations of  $(p_H, p_L)$  for which the revert-to-discretionary plan is credible. Blue dots indicate the combinations of  $(p_H, p_L)$  for which the revert-to-discretionary plan is not credible. Black dots indicate the combinations of  $(p_H, p_L)$  for which the revert-to-discretionary plan is not defined because the discretionary outcome does not exist.

According to the figure, the revert-to-deflation plan is credible for any pairs of frequency and persistence. To understand why, notice that the deflationary outcome is associated with deflation and consumption declines even in the high-state. After the shock disappears, if the government were to renege the promise of zero nominal interest rate and raises the nominal interest rate, the private sector agents adjust their expectations and believe the economy will be followed by deflationary outcome in the future. By renegeing on the Ramsey promise after the shock disappears, the government would see deflation and below-trend consumption in the period of renegeing, instead of above-trend consumption and high inflation. Inflation and consumption in the high state of the deflationary outcome are so low that there is no short-run gain from renegeing on the promise. Thus, the revert-to-deflation plan is credible for any combinations of frequency and persistence.



Figure C.1: Credibility of the revert-to-deflation plan



## D The revert-to-discretion(N) plan

One may feel that the private sector’s punishment strategy of reverting to the discretionary outcome forever after the government’s deviation may be too harsh. In reality, the household and firms do not live forever. The head of the central bank changes at some frequencies. Even if the same central banker is in charge for an extended period of time, central bank doctrines can change over the course of his/her tenure.<sup>30</sup> Based on these considerations, I explore plans in which the punishment regime lasts for a finite period of time and the economy reverts back to the Ramsey outcome afterwards. After introducing a few concepts, I formally define the revert-to-discretion(N) plan in which the punishment lasts for a finite N periods and discuss a proposition that is useful in analyzing its credibility. The results are discussed in the paper.

### D.1 Setup

**Definition of the resurrected Ramsey outcome:** For any positive integer  $j$ , the resurrected Ramsey outcome starting at time  $j$  is defined as, and denoted by,  $\{c_{ram,t}^j(s^t), \pi_{ram,t}^j(s^t), r_{ram,t}^j(s^t)\}_{t=j}^{\infty}$  such that  $c_{ram,t}^j(s^t) := c_{ram,t-j+1}(s_j^t)$ ,  $\pi_{ram,t}^j(s^t) := r_{ram,t-j+1}(s_j^t)$ , and  $r_{ram,t}^j(s^t) := r_{ram,t-j+1}(s_j^t)$  where  $t \geq j$  and  $s_j^t$  denotes a recent history of  $s^t$  starting at time  $j$  (i.e.,  $s_j^t := \{s_j, s_{j+1}, \dots, s_t\}$ ).

This outcome specifies the outcome the Ramsey planner would choose if the economy hypothetically starts at time  $j$ . This will be the outcome the economy reverts to after the punishment

<sup>30</sup>For example, consider the gradual move toward transparency during the tenure of Alan Greenspan.

period ends at time  $j-1$ . *The resurrected Ramsey value starting at time  $j$*  is defined as the value sequence associated with the resurrected Ramsey outcome starting at time  $j$ .

### Government's problem in the temporary punishment periods

Consider the following problem of the discretionary government in the final period of the temporary punishment regime. Taking as given the fact that the resurrected Ramsey outcome prevails starting from tomorrow, say  $T$ , , the government chooses today's consumption, inflation, and the nominal interest rate in order to maximize the value at  $T-1$ .

$$w_{T-1}(s_{T-1}) = \max_{\{c_{T-1} \in \mathbb{C}, \pi_{T-1} \in \Pi, r_{T-1} \in \mathbb{R}\}} u(c_{T-1}, \pi_{T-1}) + \beta E_{T-1} w_{ram,T}^T(s_T^T)$$

subject to

$$\chi_c c_{T-1} = \chi_c E_{T-1} c_{ram,T}^T(s_T^T) - [r_{T-1} - E_{T-1} \pi_{ram,T}^T(s_T^T)] + s_{T-1} \quad (22)$$

$$\pi_{T-1} = \kappa c_{T-1} + \beta E_{T-1} \pi_{ram,T}^T(s_T^T) \quad (23)$$

Let us denote the solution to this problem by  $\{c_{d,1}(\cdot), \pi_{d,1}(\cdot), r_{d,1}(\cdot), w_{d,1}(\cdot)\}$ . The discretionary government in the prior period takes them as given and maximizes the value at time  $T-2$ . As such, the allocations and value during the temporary punishment regime are recursively defined as the follows. For any  $j \geq 2$ , the discretionary government's problem is given by

$$w_{T-j}(s_{T-j}) = \max_{\{c_{T-j} \in \mathbb{C}, \pi_{T-j} \in \Pi, r_{T-j} \in \mathbb{R}\}} u(c_{T-j}, \pi_{T-j}) + \beta E_{T-j} w_{d,j+1}(s_{T-j+1}^{T-j+1})$$

subject to

$$\chi_c c_{T-j} = \chi_c E_{T-j} c_{d,T-j+1}(s_{T-j+1}) - [r_{T-j} - E_{T-j+1} \pi_{d,T-j+1}(s_{T-j+1})] + s_{T-j} \quad (24)$$

$$\pi_{T-j} = \kappa c_{T-j} + \beta E_{T-j} \pi_{d,T-j+1}(s_{T-j+1}) \quad (25)$$

Let  $\{c_{d,j}(\cdot), \pi_{d,j}(\cdot), r_{d,j}(\cdot), w_{d,j}(\cdot)\}$  be the solution to these problems at time  $T-j$ .  $\{c_{d,j}(\cdot), \pi_{d,j}(\cdot), r_{d,j}(\cdot), w_{d,j}(\cdot)\}_{j=1}^T$  will be the allocations that would prevail during the temporary punishment regime of length  $N$ .

## D.2 Definition of the revert-to-discretion(N) plan

For any positive integer  $N$ , *the revert-to-discretion(N) plan*,  $(\sigma_g^{rtd(N)}, \sigma_p^{rtd(N)})$ , consists of the following government and private sector strategies.

### D.2.1 The government strategy

For  $t = 1$ , set  $\sigma_{g,1}^{rtd(N)}(s_1) = r_{ram,1}(s_1)$  for any  $s_1 \in \mathbb{S}$ .

For  $t \geq 2$ , determine  $\sigma_{g,t}^{rtd(N)}(s^t)$  as follows.

**Case 1:** If  $r_k = r_{ram,k}(s^k)$  for all  $1 \leq k \leq t - 1$ , then

- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_{ram,t}(s^t)$

**Case 2:** If  $r_k = r_{ram,k}(s^k)$  for some  $1 \leq k \leq t - 1$ , let  $k^{(1)}$  be the period of the first deviation from the Ramsey policy. In other words,

$$k^{(1)} = \begin{cases} 1 & \text{if } r_1 \neq r_{ram,1}(s_1) \\ \min\{k > 1 | r_k \neq r_{ram,k}(s^k) \text{ and } r_{k-1} \neq r_{ram,k-1}(s^{k-1})\} & \text{otherwise} \end{cases}$$

**Case 2.A** [when  $k^{(1)} = t - 1$ ]: Set  $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_{d,N}(s^t)$

**Case 2.B** [when  $t - N \leq k^{(1)} \leq t - 2$ ]: Let  $j := t - k^{(1)}$ .

- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_{d,N+1-j}(s_t)$  if  $r_{t-h+1} = r_{d,N+1-h}(s_{t-h+1})$  for all  $1 \leq h \leq j - 1$
- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_d(s_t)$  otherwise.

**Case 2.C** [when  $k^{(1)} \leq t - N - 1$ ]: If  $r_{k^{(1)}+j} \neq r_{d,N-j+1}(s_{k^{(1)}+j})$  for some  $1 \leq j \leq N$ ,

- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_d(s_t)$

If  $r_{k^{(1)}+j} = r_{d,N-j+1}(s_{k^{(1)}+j})$  for all  $1 \leq j \leq N$ , then set  $m = 1$  and follow the steps below.

#### Recursive steps to follow if you are following the resurrected Ramsey outcome

**Case 1:** If  $r_k = r_{ram,k}^{k^{(m)}+N+1}(s^k)$  for all  $k^{(m)} + N + 1 \leq k \leq t - 1$ , then

- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_{ram,k}^{k^{(m)}+N+1}(s^t)$

**Case 2:** If  $r_k = r_{ram,k}^{k^{(m)}+N+1}(s^k)$  for some  $k^{(m)} + N + 1 \leq k \leq t - 1$ , let  $k^{(m+1)}$  be the period of the first deviation from the resurrected Ramsey policy starting at  $k^{(m)} + N + 1$ . In other words,

$$k^{(m+1)} = \begin{cases} k^{(m)} + N + 1 & \text{if } r_{k^{(m)}+N+1} \neq r_{ram,k^{(m)}+N+1}^{k^{(m)}+N+1}(s^{k^{(m)}+N+1}) \\ \min\{k > k^{(m)} + N + 1 | r_k \neq r_{ram,k}^{k^{(m)}+N+1}(s^k) \text{ and } r_{k-1} \neq r_{ram,k-1}^{k^{(m)}+N+1}(s^{k-1})\} & \text{otherwise} \end{cases}$$

**Case 2.A** [when  $k^{(m+1)} = t - 1$ ]: Set  $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_{d,N}(s^t)$

**Case 2.B** [when  $t - N \leq k^{(m+1)} \leq t - 2$ ]: Let  $j := t - k^{(m+1)}$ .

- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_{d,N+1-j}(s_t)$  if  $r_{t-h+1} = r_{d,N+1-h}(s_{t-h+1})$  for all  $k^{(m+1)}+1 \leq h \leq k^{(m+1)}+j-1$
- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_d(s_t)$  otherwise.

**Case 2.C** [when  $k^{(m+1)} \leq t - N - 1$ ]: If  $r_{k^{(m+1)}+j} \neq r_{d,N-j+1}(s^{k^{(m+1)}+j})$  for some  $k^{(m+1)} + 1 \leq j \leq k^{(m+1)} + N$ ,

- $\sigma_{g,t}^{rtd(N)}(r^{t-1}, s^t) = r_d(s_t)$

If  $r_{k^{(m+1)}+j} = r_{d,N-j+1}(s_{k^{(m+1)}+j})$  for all  $1 \leq j \leq N$ , then set  $m = m + 1$  and go back to the beginning of the recursive step.

At each time  $t$ , this government strategy instructs the government to choose the nominal interest rate consistent with the Ramsey outcome as long as the past government has not deviated from the Ramsey prescription. If the past government deviated from either the Ramsey prescription or the resurrected Ramsey outcome at some  $t \leq t - N - 1$ , but chosen the nominal interest rate consistent with temporary punishment regime afterward, then the strategy instructs the government to choose the nominal interest rate consistent with the resurrected Ramsey outcome. If the government deviated from either the Ramsey or resurrected Ramsey policies in the recent past  $t - N \leq k \leq t - 1$ , then the strategy instructs the government to choose the nominal interest rate consistent with the temporary punishment regime. If the government has ever deviated from the temporary punishment regime during some punishment periods, then the strategy prescribes the government to choose the nominal interest rate consistent with the discretionary outcome.

## D.2.2 The private sector strategy

For any positive integer  $t$ , determine  $\sigma_{p,t}^{rtd(N)}(r^t, s^t)$  as follows.

**Case 1:** If  $r_k = r_{ram,k}(s^k)$  for all  $1 \leq k \leq t$ , then

- $\sigma_{p,t}^{rtd(N)}(r^t, s^t) = (c_{ram,t}(s^t), \pi_{ram,t}(s^t))$

**Case 2:** If  $r_k = r_{ram,k}(s^k)$  for some  $1 \leq k \leq t$ , let  $k^{(1)}$  be the period of the first deviation from the Ramsey policy. In other words,

$$k^{(1)} = \begin{cases} 1 & \text{if } r_1 \neq r_{ram,1}(s_1) \\ \min\{k > 1 \mid r_k \neq r_{ram,k}(s^k) \text{ and } r_{k-1} \neq r_{ram,k-1}(s^{k-1})\} & \text{otherwise} \end{cases}$$

**Case 2.A** [when  $k^{(1)} = t$ ]: Set  $\sigma_{p,t}^{rtd(N)}(r^t, s^t) = (c_{d,N}(s_t), \pi_{d,N}(s_t))$

**Case 2.B** [when  $t - N - 1 \leq k^{(1)} \leq t - 1$ ]: Let  $j := t - k^{(1)}$ .

- $\sigma_{p,t}^{rtd(N)}(r^t, s^t) = (c_{d,N+1-j}(s_t), \pi_{d,N+1-j}(s_t))$  if  $r_{t-h+1} = r_{d,N+1-h}(s_{t-h+1})$  for all  $1 \leq h \leq j - 1$

- $\sigma_{p,t}^{rtd(N)}(r^t, s^t) = (c_d(s_t), \pi_d(s_t))$  otherwise.

**Case 2.C** [when  $k^{(1)} \leq t - N - 2$ ]: If  $r_{k^{(1)+j} \neq r_{d,N-j+1}(s_{k^{(1)+j})}$  for some  $1 \leq j \leq N$ ,

- $\sigma_{p,t}^{rtd(N)}(r^t, s^t) = (c_d(s_t), \pi_d(s_t))$

If  $r_{k^{(1)+j} = r_{d,N-j+1}(s_{k^{(1)+j})}$  for all  $1 \leq j \leq N$ , then set  $m = 1$  and follow the steps below.

### Recursive steps to follow if you are following the resurrected Ramsey outcome

**Case 1:** If  $r_k = r_{ram,k}^{k^{(m)}+N+1}(s^k)$  for all  $k^{(m)} + N + 1 \leq k \leq t$ , then

- $\sigma_{p,t}^{rtd(N)}(r^t, s^t) = (c_{ram,k}^{k^{(m)}+N+1}(s^t), \pi_{ram,k}^{k^{(m)}+N+1}(s^t))$

**Case 2:** If  $r_k = r_{ram,k}^{k^{(m)}+N+1}(s^k)$  for some  $k^{(m)} + N + 1 \leq k \leq t$ , let  $k^{(m+1)}$  be the period of the first deviation from the resurrected Ramsey policy starting at  $k^{(m)} + N + 1$ . In other words,

$$k^{(m+1)} = \begin{cases} k^{(m)} + N + 1 & \text{if } r_{k^{(m)}+N+1} \neq r_{ram,k^{(m)}+N+1}^{k^{(m)}+N+1}(s^{k^{(m)}+N+1}) \\ \min\{k > k^{(m)} + N + 1 \mid r_k \neq r_{ram,k}^{k^{(m)}+N+1}(s^k) \text{ and } r_{k-1} \neq r_{ram,k-1}^{k^{(m)}+N+1}(s^{k-1})\} & \text{otherwise} \end{cases}$$

**Case 2.A** [when  $k^{(m+1)} = t - 1$ ]: Set  $\sigma_{p,t}^{rtd(N)}(r^t, s^t) = r_{d,N}(s_t)$

**Case 2.B** [when  $t - N \leq k^{(m+1)} \leq t - 2$ ]: Let  $j := t - k^{(m+1)}$ .

- $\sigma_{p,t}^{rtd(N)}(r^t, s^t) = (c_{d,N+1-j}(s_t), \pi_{d,N+1-j}(s_t))$  if  $r_{t-h+1} = r_{d,N+1-h}(s_{t-h+1})$  for all  $k^{(m+1)} + 1 \leq h \leq k^{(m+1)} + j - 1$
- $\sigma_{p,t}^{rtd(N)}(r^t, s^t) = (c_d(s_t), \pi_d(s_t))$  otherwise.

**Case 2.C** [When  $k^{(m+1)} \leq t - N - 1$ ]: If  $r_{k^{(m+1)+j} \neq r_{d,N-j+1}(s_{k^{(m+1)+j})}$  for some  $k^{(m+1)} + 1 \leq j \leq k^{(m+1)} + N$ ,

- $\sigma_{p,t}^{rtd(N)}(r^t, s^t) = (c_d(s_t), \pi_d(s_t))$

If  $r_{k^{(g+1)+j} = r_{d,N-j+1}(s_{k^{(m+1)+j})}$  for all  $1 \leq j \leq N$ , then set  $m = m + 1$  and go back to the beginning of the recursive step.

At each time  $t$ , this private sector strategy instructs the household and firms to choose consumption and inflation consistent with the Ramsey outcome as long as the past government has not deviated from the Ramsey prescription. If the government deviated from either the Ramsey prescription or the resurrected Ramsey prescription at some  $t \leq t - N$ , but chose the nominal interest rate consistent with temporary punishment regime afterward, then the strategy instructs the household and firms to choose consumption and inflation consistent with the resurrected Ramsey outcome. If the government deviated from either the Ramsey or resurrected Ramsey policies in

the recent past  $t - N - 1 \leq k \leq t$ , then the strategy instructs the household and firms to choose consumption and inflation consistent with the temporary punishment regime. If the government has ever deviated from the temporary punishment regime during some punishment periods, then the strategy prescribes the household and firms to choose consumption and inflation consistent with the discretionary outcome.

### D.3 Credibility of the revert-to-discretion(N) plan

**Proposition D.1:** The revert-to-discretion(N) plan is credible if and only if (i)  $w_{ram,t}(s^t) \geq w_{d,N}(s_t)$  for all  $t$  and  $s^t \in \mathbb{S}^t$  and (ii)  $w_{d,j}(s_t) \geq w_d(s_t)$  for all  $1 \leq j \leq N$ .

The proof proceeds similarly to the proof of proposition 2. The first condition makes sure that the government does not have incentives to deviate from either the Ramsey or resurrected Ramsey outcomes. The second condition guarantees that the government does not have incentives to deviate from the strategy's prescription in the temporary punishment regime.

## E Detailed Sensitivity Analyses

This section studies how variations in  $(\beta, \chi_c, \lambda, L, \kappa)$  affect the credibility of the revert-to-discretion plan. Throughout the section,  $(p_H, p_L) = (0.001, 0.5)$  and other parameters are set to their baseline values, except for the parameter being investigated.

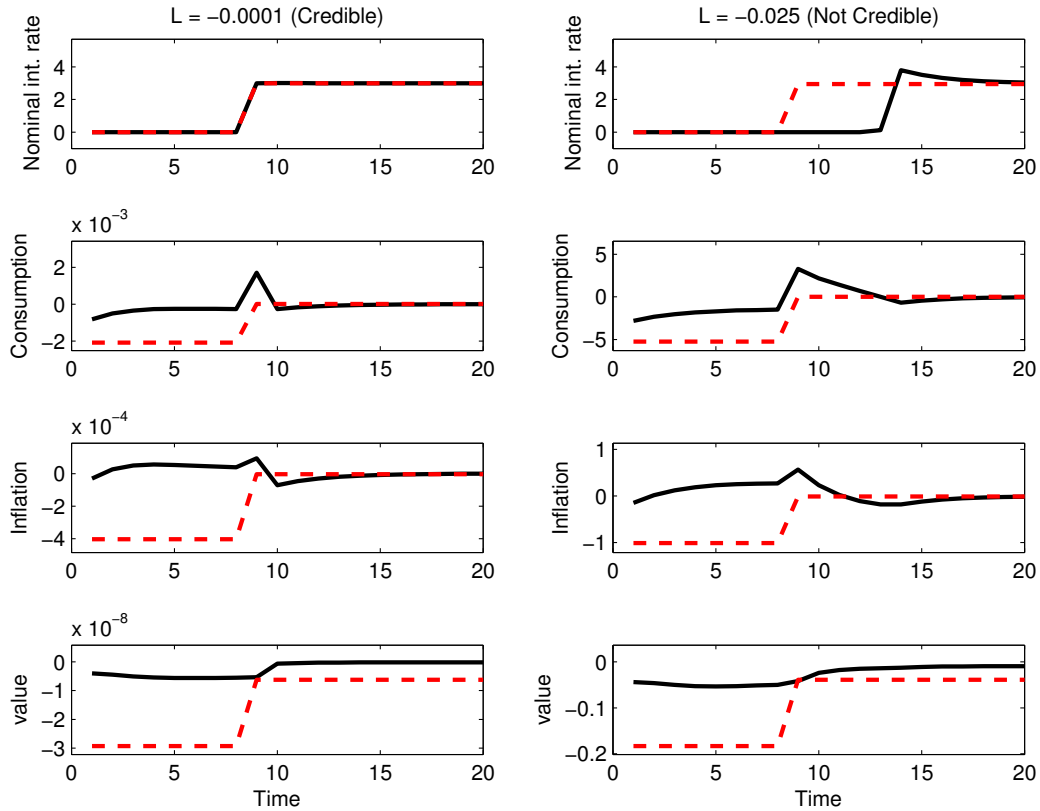
### Severity of the shock (L)

The left and right panels in Figure E.1 show the impulse response functions for the discretionary/Ramsey outcomes and values in the economies with small and large shocks (small  $|L|$  and large  $|L|$ ), respectively. The contractionary shock is assumed to take a low value for the first eight periods and to return to the high value afterward.

Comparing the IRFs for consumption and inflation across two economies shows that a more severe shock leads to larger declines in consumption and inflation under both discretionary and Ramsey outcomes. However, the Ramsey planner can promise a higher inflation, a larger consumption boom, and a longer period of zero nominal interest rates to mitigate the declines in consumption and inflation during the period of contractionary shocks. Thus, a marginal increase in the shock severity leads to larger marginal declines in consumption and inflation in the low state in the discretionary outcome than in the Ramsey outcome, implying large marginal declines in both the high-state and low-state values. This means that the long-run loss from renegeing on the Ramsey promise and reverting back to the discretionary outcome is larger in the economy with more severe shocks, as depicted by the solid black line in Figure E.2.

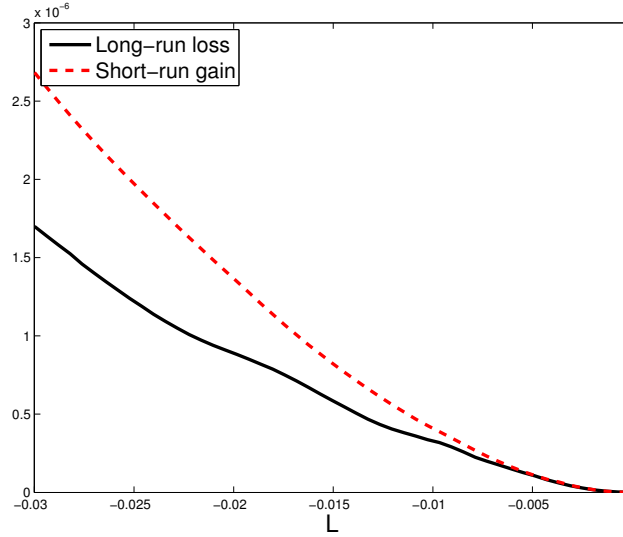
On the other hand, as the Ramsey promise entails a higher inflation and larger consumption boom, the short-run gain from renegeing on the promise is also larger with a larger shock, as depicted in the dashed red lines in Figure E.2. As such, the overall effects are mixed. The panel in the main

Figure E.1: The discretionary/Ramsey outcomes and values with alternative sizes of the shock



\*\*The long-run loss shows the loss in the continuation value if the government deviates from the Ramsey policy at  $t = 9$ , given by  $\beta E_9 [w_{ram,10}(s^{10}) - w_{d,10}(s^{10})]$ , and the short-run gain shows the gain in today's utility flow if the government deviates from the Ramsey policy at  $t = 9$ , given by  $[u(c_{d,9}(s^9), \pi_{d,9}(s^9)) - u(c_{ram,9}(s^9), \pi_{ram,9}(s^9))]$ , where  $s_t = L$  for  $1 \leq t \leq 8$  and  $s_9 = H$ .

Figure E.2: Long-run loss and short-run gain with alternative sizes of the shock



\*\*The continuation values of following and deviating from the Ramsey plan at  $t = 9$  are respectively given by  $\beta E_9 w_{ram,10}(s^{10})$  and  $\beta E_9 w_{d,10}(s^{10})$  where  $s_t = L$  for  $1 \leq t \leq 8$  and  $s_9 = H$ .

text shows that, while the threshold frequency is higher when the shock is larger in the economy with highly persistent shocks, the threshold frequency is lower when the shock is larger in the economy in which the shock persistence is low. For the specific choice of parameter values used in Figure E.1, the Ramsey outcome is credible if the shock size is sufficiently small.

### Discount rate ( $\beta$ )

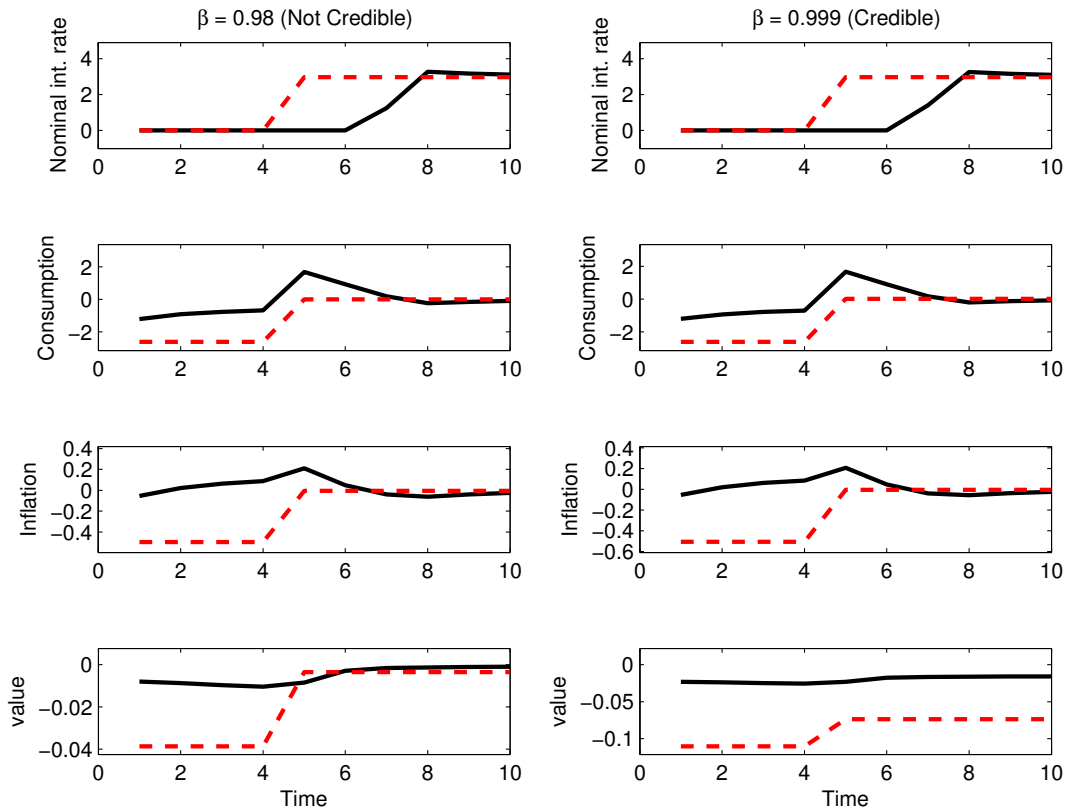
The left panels in Figure E.1 show the impulse response functions for the discretionary/Ramsey outcomes and values in the economy with a small discount factor. The right panels show the same objects in the economy with a large discount factor. The contractionary shock is assumed to take a low value for the first four periods and return to the high value afterward.

When  $\beta$  is large, inflation today depends more on future marginal costs. As a result, a promise of future inflation is more effective in mitigating the declines in inflation in the low state and the Ramsey planner promises higher inflation in the economy with higher  $\beta$ . However, this effect is quantitatively negligible, as the comparison of solid black lines in the right and left column in Figure E.3 shows. As a result, the short-run gain from reneging on the Ramsey promise and thus stabilizing consumption and inflation are essentially insensitive to the discount rate, as captured by the flat dashed red line in Figure E.4.

On the other hand, with a higher  $\beta$ , the same difference between the discretionary and Ramsey continuation values translates into a larger difference between discounted continuation values. As a result, a high discount factor implies a larger long-run loss of reneging on the promise, which is captured in the solid black line in Figure E.4. Accordingly, the threshold  $p_H$  above which the

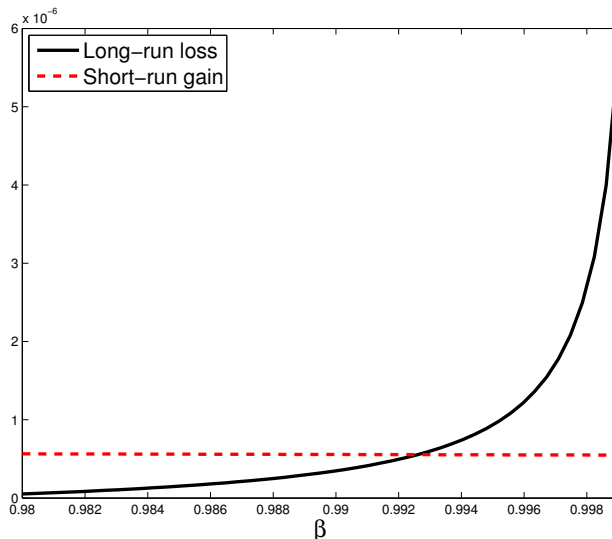


Figure E.3: The discretionary/Ramsey outcomes and values with alternative discount rates



\*\*The long-run loss shows the loss in the continuation value if the government deviates from the Ramsey policy at  $t = 5$ , given by  $\beta E_5[w_{ram,6}(s^6) - w_{d,6}(s^6)]$ , and the short-run gain shows the gain in today's utility flow if the government deviates from the Ramsey policy at  $t = 5$ , given by  $[u(c_{d,5}(s^5), \pi_{d,5}(s^5)) - u(c_{ram,5}(s^5), \pi_{ram,5}(s^5))]$ , where  $s_t = L$  for  $1 \leq t \leq 4$  and  $s_5 = H$ .

Figure E.4: Long-run loss and short-run gain with alternative discount rates



\*\*The continuation values of following and deviating from the Ramsey plan at  $t=5$  are respectively given by  $\beta E_5 w_{ram,6}(s^6)$  and  $\beta E_5 w_{d,6}(s^6)$  where  $s_t = L$  for  $1 \leq t \leq 4$  and  $s_5 = H$ .

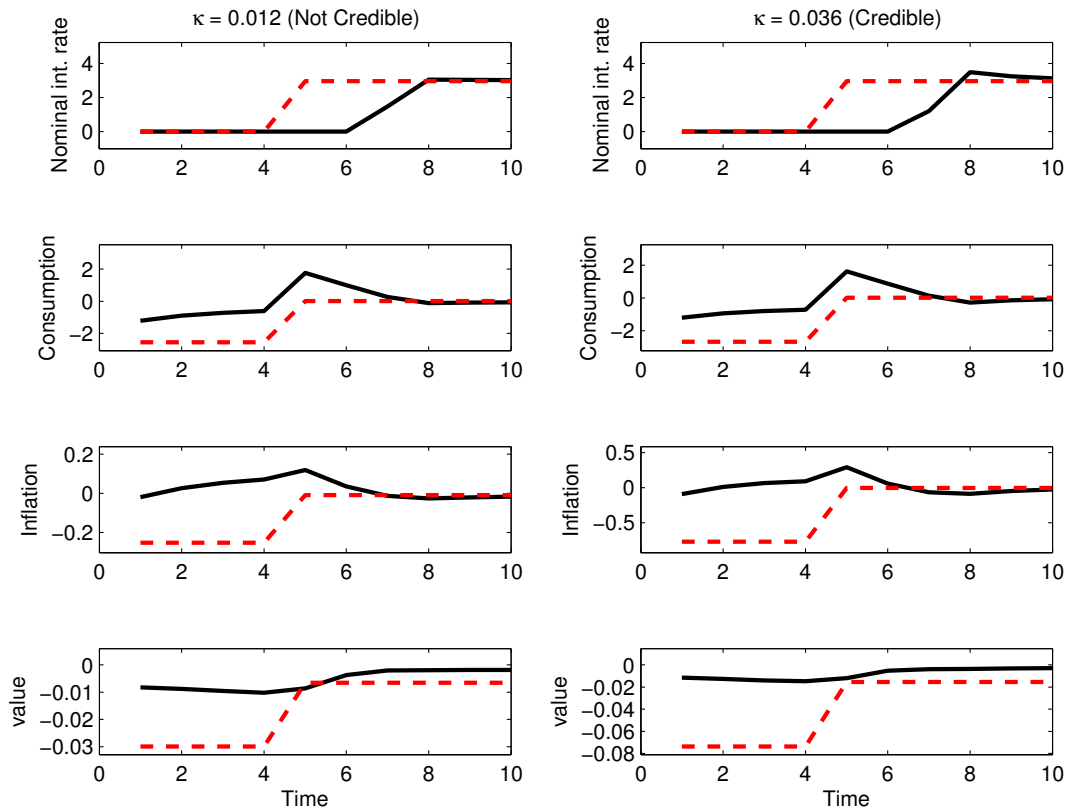
revert-to-discretion plan is credible is lower in the economy with a larger  $\beta$ . This result is consistent with previous literature on credible plans which has shown that a sufficiently large  $\beta$  can make the Ramsey policy credible in various contexts (see, for example, Chari and Kehoe (1990), Phelan and Stacchetti (2001), and Kurozumi (2008).)

### Slope of the Phillips Curve ( $\kappa$ )

The left panels in Figure E.5 show the impulse response functions for the discretionary/Ramsey outcomes and values in the economy with a small  $\kappa$ . The right panels show the same objects in the economy a large  $\kappa$ . The contractionary shock is assumed to take a low value for the first four periods and return to the high value afterward.

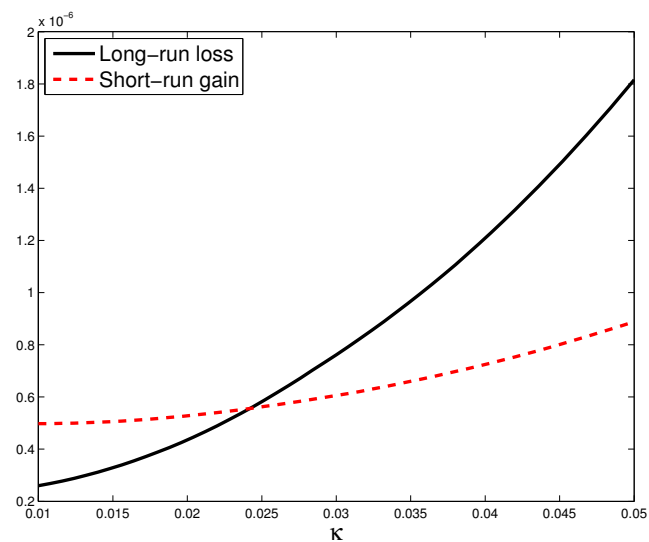
Comparison of left and right panels tells us that declines in consumption and inflation in the low state are exacerbated under both discretionary and Ramsey outcomes under more flexible prices (i.e. higher slope of the Phillips Curve). When the prices are more flexible, the same shock leads to larger deflation in the low state, which in turn amplifies the decline in low-state consumption by increasing the expected real interest rate. However, the Ramsey planner mitigates those declines by promising a higher inflation, consumption booms, and a longer period of the zero lower bound. Thus, a marginal increase in the slope parameter leads to larger marginal declines in consumption and inflation, and thus values, in the discretionary outcome than in the Ramsey outcome. Accordingly, the long-run loss from reverting back to the discretionary plan is larger in the economy with more flexible prices, as captured in the solid black line in Figure E.6.

Figure E.5: The discretionary/Ramsey outcomes and values with alternative slopes of the Phillips curve



\*See the footnote in Figure E.3

Figure E.6: Long-run loss and short-run gain with alternative slopes of the Phillips curve



\*See the footnote in Figure E.4

On the other hand, the Ramsey promise of higher inflation and larger consumption booms means that short-run gain from renegeing on the promise once the shock disappears is higher under a more flexible price environment, which is captured in the dashed red line in Figure E.6. Quantitatively, for the calibration considered in this paper, the second effect dominates the first effect. The threshold value of  $p_H$  above which the revert-to-discretion plan is credible is lower for any given  $p_L$ .

### Inverse IES ( $\chi_c$ )

The left panels in Figure E.7 show the impulse response functions for the discretionary/Ramsey outcomes and values in the economy with a small shock. The right panels show the same objects in the economy with a large shock. The contractionary shock is assumed to take a low value for the first eight periods and return to the high value afterward.

Comparison of the IRFs across left and right panels shows that a higher  $\chi_c$  implies larger declines in consumption and inflation in the low state under both discretionary and Ramsey outcomes. When the inverse IES is high, the household's consumption decision is more sensitive to the fluctuations in  $s_t$ . Since firms' pricing today depends on consumption today, inflation today is more sensitive to the fluctuations in  $s_t$  with a higher  $\chi_c$ . While the Ramsey planner can mitigate declines in low-state consumption and inflation by future promises, the discretionary government has no tool to mitigate them. As a result, a marginal increase in the inverse IES leads to larger marginal declines in low-state consumption and inflation under the discretionary outcome than in the Ramsey outcome. Since lower low-state consumption and inflation reduce values in both states, the long-run loss from renegeing on the promise is large with a smaller  $\chi_c$ , as shown in the dashed red lines in Figure E.8.

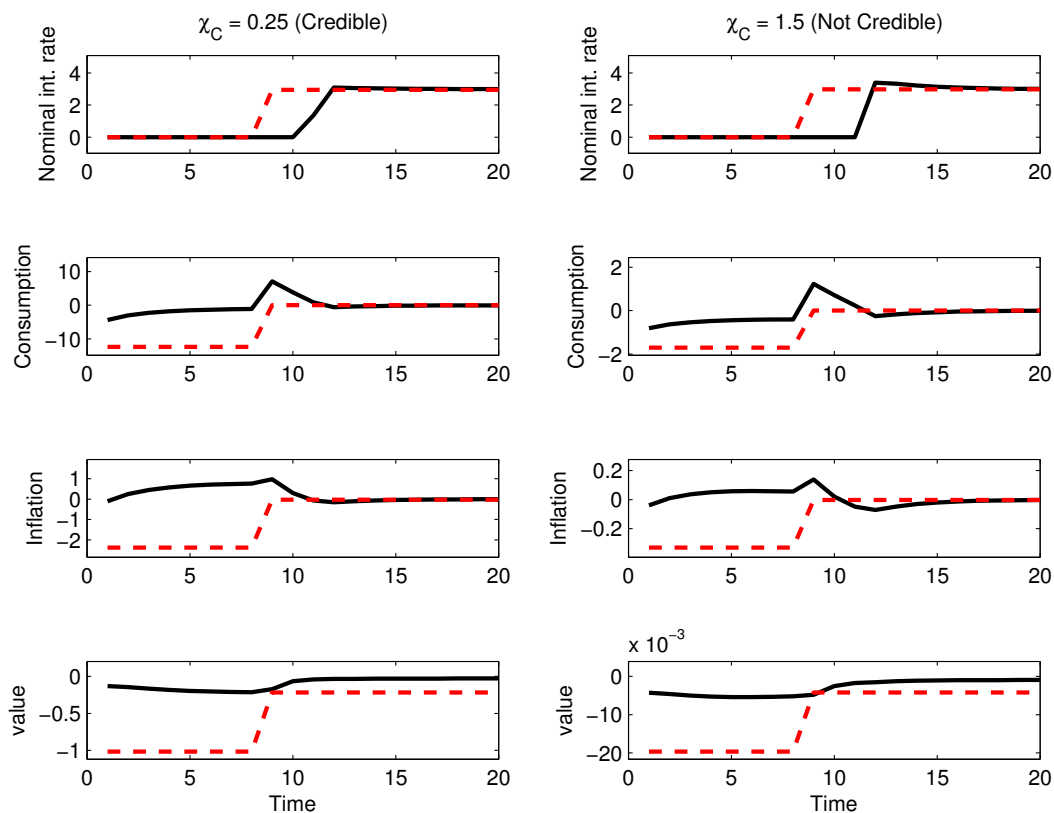
On the other hand, higher promised consumption and inflation with a larger  $\chi_c$  mean that short-run gain from renegeing on the promise is larger when  $\chi_c$  is larger, as shown in the dashed red lines in Figure E.8. Thus, the overall effect of  $\chi_c$  on credibility of the revert-to-discretion plan is mixed. Similarly with the severity of shocks, while the threshold frequency is higher when the inverse IES is larger in the economy with highly persistent shocks, the threshold frequency is lower when the inverse IES is larger in the economy in which the shock persistence is low.

### Weight on consumption volatility ( $\lambda$ )

The left panels in Figure E.9 show the impulse response functions for the discretionary/Ramsey outcomes and values in the economy with a small weight on consumption volatility in the government's objective function. The right panels show the same objects in the economy with a large weight. The contractionary shock is assumed to take a low value for the first four periods and return to the high value afterward.

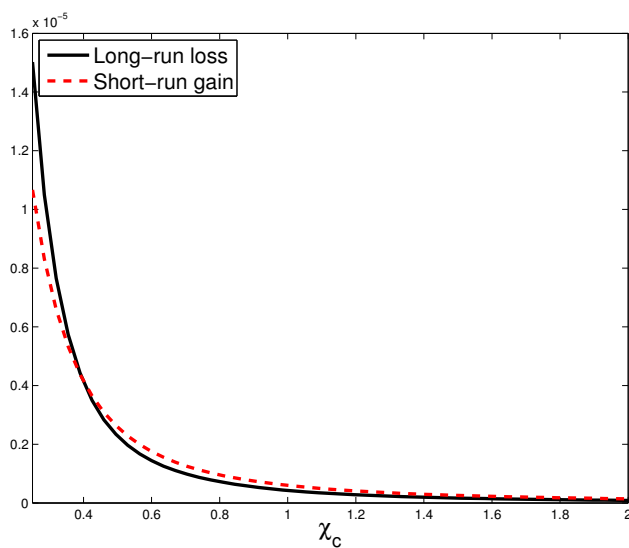
A larger  $\lambda$  means that the government cares more about consumption volatility relative to inflation volatility. Under the discretionary outcome, a greater concern for consumption volatility exacerbates the deflation bias in the high state, in turn amplifying deflation and consumption drops in the low state. The Ramsey planner can mitigate this effect by promising a higher, and more prolonged, inflation and consumption booms in the future, and marginal increases in the weight

Figure E.7: The discretionary/Ramsey outcomes and values with alternative risk aversion



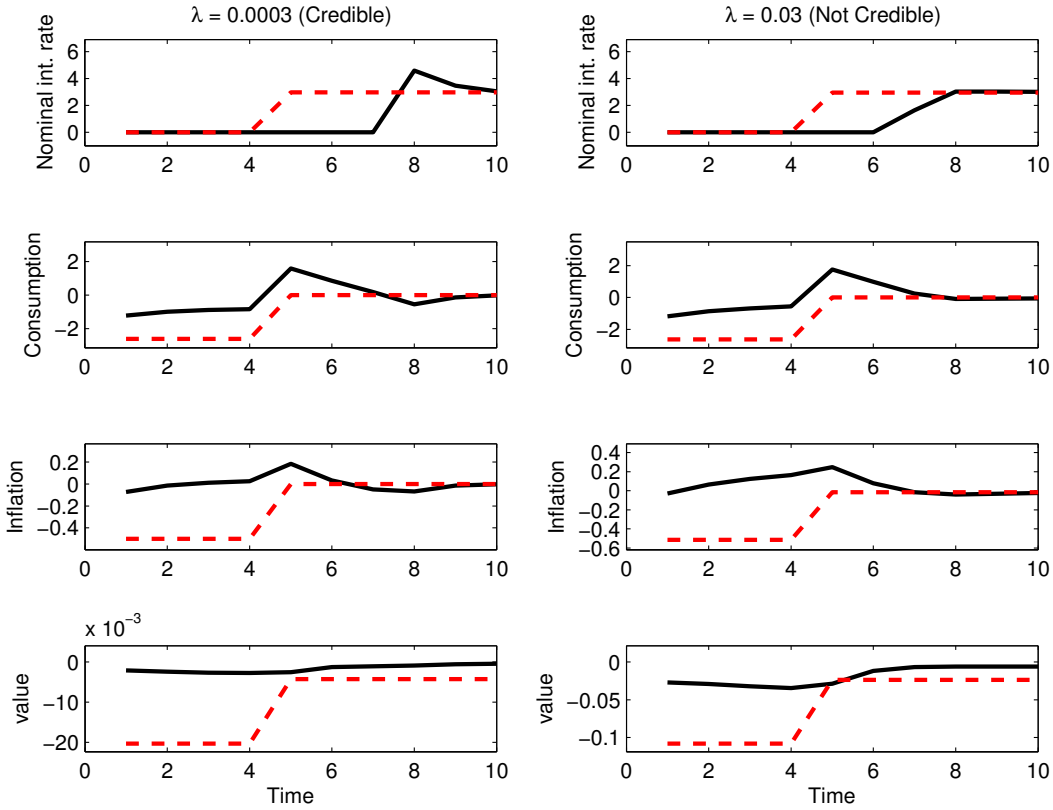
\*See the footnote in Figure E.1

Figure E.8: Long-run loss and short-run gain with alternative risk aversion



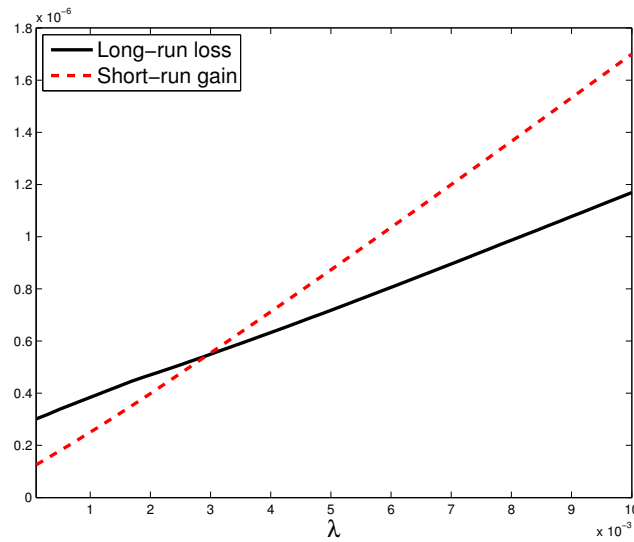
\*See the footnote in Figure E.2

Figure E.9: The discretionary/Ramsey outcomes and values with alternative weights on consumption stabilization



\*See the footnote in Figure E.3

Figure E.10: Long-run loss and short-run gain with alternative weights on consumption stabilization



\*See the footnote in Figure E.4

on consumption volatility reduces the low-state consumption and inflation, and also the values in both states, by more under the discretionary outcome than under the Ramsey outcome. Thus, the long-run loss of renegeing on the promise, and therefore accepting the continuation value associated with the discretionary outcome, is higher as captured by the solid black line in Figure E.10.

On the other hand, promises of higher inflation and consumption hikes mean that the short-run gain of deviating from the promise is larger, as shown by the dashed red line in Figure E.10. Quantitatively, the second effect dominates the first effect unless the persistence of the shock is very high. For most values of  $p_L$ , the threshold frequency above which the revert-to-discretionary plan is credible is higher when the central bank places a greater weight on consumption volatility in its objective function.