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Investment Horizon and Repo in the Over-the-Counter Market

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Hajime Tomura*

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Abstract

This paper presents a three-period model featuring a short-term investor in the over-the-counter bond market. A short-term investor stores cash because of a need to pay cash at some future date. If a short-term investor buys bonds, then a deadline for retrieving cash lowers the resale price of bonds for the investor through bilateral bargaining in the bond market. Ex-ante, this hold-up problem explains the use of a repo by a short-term investor, the existence of a haircut, and the vulnerability of a repo market to counterparty risk. This result holds without any uncertainty about bond returns or asymmetric information.

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Keywords: Repo; Over-the-counter market; Securities broker-dealer; Short-term investor; Haircut.

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1 Introduction

A repo is one of the primary instruments in the money market. In this transaction, a short-term investor buys long-term bonds with a repurchase agreement in which the seller of the bonds promises to buy back the bonds at a later date. From the seller's point of view, this transaction is akin to a secured loan with the underlying bonds as collateral. A question remains, however, regarding why a short-term investor needs a repurchase agreement when the investor can simply resell bonds to a third party in a spot market. The answer to this question is not immediately clear, as the bonds traded in the repo market include Treasury securities and agency mortgage-backed securities, for which a secondary market is available.

This paper presents a simple model to show that a short-term investor's need for a repo arises from the investor's short investment horizon and the fact that the bond market is an over-the-counter (OTC) market.[1] This paper first illustrates a hold-up problem for a short-term investor in an OTC bond market. If a short-term investor buys a long-term bond, then the investor needs to resell the bond by the time to pay out cash. This time constraint weakens the bargaining position of the investor against the buyer of the investor's bond; thus the buyer can negotiate down the bond price through bilateral bargaining in an OTC bond market. Ex-ante, this hold-up problem caps the highest spot bond price that a short-term investor can pay to a bond seller, resulting in no spot transaction between them.

A repo solves this hold-up problem. Given an ex-post discount on the resale price of a short-term investor's bond, a bond seller can lower the initial selling price of a bond—or pay a haircut—ex-ante to ensure a sufficiently high yield on a repo for a short-term investor. Lowering the initial selling price is viable for a bond seller, because the bond seller can repurchase the bond at a low price at the maturity of the repo. This arrangement is robust to limited commitment that prevents a bond seller from financing through an unsecured loan.

[1]See Harris (2003) and Biais and Green (2007) for the institutional details of the bond market.

A corollary of this result is that a long-term investor who does not have to resell a bond before the maturity can buy a bond in a spot transaction. Thus, the result explains the co-existence of a repo and a spot transaction by a difference in investment horizons among investors.

Given this result, the model explains why a repo market is vulnerable to counterparty risk. A short-term investor in a repo must resell the underlying bond to a third party whenever the counterparty goes bankrupt. If there exists a possibility of such an event, then the same friction arises as in a spot transaction: with some probability, the initial bond seller cannot gain from the future resale of a short-term investor's bond. As a result, the counterparty risk reduces the trading surplus between a bond seller and a short-term investor in a repo ex-ante. This effect makes a repo market collapse if the counterparty risk is too high.

This result has an implication for the recent discussion of automatic stay on a repo. If a repo is not exempted from automatic stay, then a short-term investor with a defaulted repo cannot resell the underlying bond in a timely manner to retrieve cash within the investor's investment horizon. Therefore, committing to reselling a bond to a bond seller ex-ante is now costly. This effect makes a repo market collapse in a wider range of counterparty risk with automatic stay.

1.1 Related literature

This paper adds to the literature on the theoretical analysis of a repo market. Dang, Gorton, and Holmström (2011) characterize a repo as a secured loan, and explain the existence of a haircut by an adverse selection problem regarding the quality of collateral. Also, Martin, Skeie and von Thadden (2010) analyze the fragility of the U.S. tri-party repo market, which was about to collapse in 2008. Given an exogenous bond liquidation cost, they show that a repo market can collapse due to coordination failure among investors if a clearing bank unwinds a repo every day as in the U.S. tri-party repo market. Adding to these papers, this paper shows the fragility of a repo market due to an endogenous bond liquidation cost without any uncertainty about bond returns or asymmetric information. Thus, this paper confirms that the safety of underlying bonds does not preclude the fragility of a repo market.

There also exist papers analyzing a repo in an OTC market. Monnet and Narajabad (2011) characterize a repo as an asset rental, and show that investors both buy and rent assets at the same time if they have idiosyncratic shocks to the utility from holding assets. Mills and Reed (2012) consider limited commitment by both parties in a repo, including a failure to return collateral by a lender, and derive a repo endogenously in an optimal contracting problem. Fujiki (2014) extends their framework to cross-border trade.

Another related work is the paper by Antinolfi et al. (2012), which characterizes a repo as a secured loan and analyzes the general-equilibrium effects of automatic stay on a repo. This paper adds to their work by analyzing the effect of automatic stay on a repo for a short-term investor.[2]

Finally, this paper is related to the literature on OTC spot trading.[3] In this literature, the motive for an asset sale is typically assumed to be some idiosyncratic shock that raises the seller's asset holding cost. Given a search friction that delays the seller's contact with another buyer, such a shock lowers the threat point for the seller in a bilateral transaction with a buyer, resulting in a low bid price. In the context of this result in the literature, the novel finding of this paper is a linkage between the investment horizon and a hold-up problem: even without any shock, a short-term investor endogenously falls into a hold-up problem ex-post if buying a long-term bond. This linkage is crucial to explain why a short-term investor is a main user of a repo in practice.

The remainder of this paper is organized as follows. Section 2 briefly summarizes the

[2]There exist other strands in the literature on a repo. Duffie (1996) presents a model to analyze a special repo rate on a reverse repo between a security lender and a short seller. Vayanos and Weill (2008) analyze the on-the-run premium on Treasury securities by considering reverse repos in a dynamic search model. In contrast to these papers, this paper focuses on a repo for a short-term investor.

[3]For example, see Spulber (1996), Rust and Hall (2003), Duffie, Gârleanu, and Pedersen (2005), Miao (2006), Vayanos and Wang (2007), Lagos and Rocheteau (2010), Lagos, Rocheteau and Weill (2011), and Chiu and Koeppl (2011).

stylized features of the repo market in practice. Section 3 presents the baseline model with limited commitment. Section 4 introduces counterparty risk into the baseline model. Section 5 analyzes the effect of automatic stay on a repo. Section 6 concludes.

2 Features of the repo market in practice

This section briefly summarizes the empirical features of the repo market that motivate this paper's analysis. Copeland, Martin, and Walker (2010) analyze clearing banks' data on the U.S. tri-party repo market, and report that typically a short-term investor, such as a money market mutual fund (MMF) or a security lender, buys long-term bonds from a securities broker-dealer in the market.[4] They also find that most of the bonds traded in the market are government-guaranteed bonds, such as Treasury securities, agency mortgage-backed securities, and agency debt. Similarly, Krishnamurthy, Nagel, and Orlov (2011) construct data on repos held by MMFs and securities lenders, and confirm that government-guaranteed bonds occupy a large share of assets that underlie these investors' repos.

The two papers also report a haircut on repos in their samples. A haircut is defined as $1 - p/v$, where p denotes the initial selling price of securities in a repo and v denotes the quoted market value of the securities. They report that the haircut on repos with government-guaranteed bonds is consistently around 2% throughout their sample periods.

In addition, a recent experience shows the fragility of the repo market. Despite the government guarantees on most of the underlying bonds in the U.S. tri-party repo market, this market was about to collapse in the run-up to the failure of Bear Stearns in March 2008. Even though no public data are available for this incident, there exists anecdotal evidence of the turmoil. For example, the then Federal Reserve Chairman Ben Bernanke provides his account of the turmoil in the Financial Crisis Inquiry Report:

> The $2.8 trillion tri-party repo market had "really [begun] to break down,"

[4] This market is called "tri-party" because a clearing bank participates into a repo as a third party.

> Bernanke said. "As the fear increased," short-term lenders began demanding more collateral, "which was making it more and more difficult for the financial firms to finance themselves and creating more and more liquidity pressure on them. And, it was heading sort of to a black hole." He saw the collapse of Bear Stearns as threatening to freeze the tri-party repo market, leaving the short-term lenders with collateral they would try to "dump on the market. You would have a big crunch in asset prices." (Financial Crisis Inquiry Commission 2010, pp. 290-291.)

Also, Adrian, Burke, and McAndrews (2009) report that haircuts rose significantly across the tri-party repo market during the two-week period before the failure of Bear Stearns. This observation is puzzling given the fact that the investors in the market were protected by safe underlying bonds, such as Treasury securities.

In summary, this paper analyzes the following features of the repo market:

1. A short-term investor needs a repo to buy safe bonds;

2. A haircut is positive in a repo held by a short-term investor;

3. An increase in counterparty risk raises a haircut, or even causes a market collapse, in a repo market with safe underlying bonds.

In the following, this paper explains these features of the repo market by the investment horizon of a short-term investor and the fact that the bond market is an OTC market.

To clarify, even though the description of the repo market here is mostly about the tri-party repo market, this is just due to data availability. This paper's analysis is general because the model is based only on the basic feature of the repo market, that is, a short-term investor buys bonds through a repo in an OTC market. Thus, the result is applicable to a bilateral repo, if an investor puts cash on it for short-term investment.[5]

[5] See Gorton and Metrick (2012) for more details about the bilateral repo market.

3 Baseline model

Time is discrete and indexed by 0, 1, 2. There are three risk-neutral agents, A, B, C. Agent A is endowed with an amount e of cash in period 0 and can consume cash in periods 1 and 2. Agent A, however, discounts the utility of consumption in period 2 by β ($\in [0, 1)$). Agent B can invest cash into a project in period 0, which returns an amount Z (> 1) of cash in period 1 per invested cash. Agent C is endowed with an amount e of cash in period 1. Both agents B and C gain utility from consuming cash in period 2. Storage technology is available so that each agent can store cash without appreciation or depreciation between consecutive periods. Each agent maximizes its expected utility of consumption.

Given $Z > 1$, the first-best allocation is for agent B to invest all of agent A's cash into the project in period 0, promising agent A a rate of return higher than, or equal to, that on storage in period 1. The return on the project, however, is not pledgeable; thus agent B cannot take an unsecured loan from agent A.

Instead, agent B is endowed with a bond in period 0. The bond returns an amount R (> 0) of cash in period 2. Any agent can receive the return on the bond. The bond market is an over-the-counter market. Agents A and B can meet to bargain over the terms of a bond sale from agent B to A in period 0. If agent A buys the bond, then it can meet with agents B and C sequentially to resell the bond in period 1. Agent A can commit in period 0 to which agent to meet first in period 1. The outcome of each bilateral meeting is determined by Nash bargaining with agent A's bargaining power equal to α ($\in (0, 1)$).

Assume that

$$e > R. \tag{1}$$

Also assume that agent B is endowed with an amount e of cash in period 1. These assumptions simplify the analysis of the model by ensuring that each agent has enough cash to buy a bond in any event.[6] An equilibrium is a subgame perfect Nash equilibrium. See Table 1

[6] If agent A buys a bond at a sufficiently low price in period 0, then agent B cannot pay the period-1 bond

for a summary of the baseline model.

3.1 Interpretation of the baseline model

Agent A represents a short-term investor storing cash until the time for a cash payment in the near future. The discount on agent A's utility of consumption in period 2, β, represents a cost of missing a timely cash payment in period 1, such as a penalty for violating a rigid due date for a payment obligation in practice. This cost makes agent A impatient, and the agent's investment horizon shorter than the maturity of the bond. Consumption maximization is equivalent to maximizing the return on cash while storing cash.[7]

The gross rate of return on Agent B's project, Z, can be interpreted literally as the gross return on investment for a borrower using a repo, or as the opportunity cost for a securities broker-dealer to finance its bond holdings with its own capital. In the latter interpretation, agent B holds a bond in period 0 as a result of a transaction with a client outside the model.

There are two ways to finance agent B's funding need. One is a chain of spot bond sales from agent A to B and then to C (Figure 1(a)). The other is a spot bond purchase by agent A with a repurchase by B, which is a repo (Figure 1(b)). In the social planner's allocation, they are indifferent: given limited commitment, the social planner can provide agent B with as much cash as an amount R by requiring agent B to submit a bond as collateral in period 0; the social planner can obtain that amount of cash from agent A by passing on the bond; and the social planner can repay an amount R of cash to agent A in period 1 by retrieving the bond from agent A and passing it on to agent B or C in exchange for an amount R of cash. Note that all of these exchanges are incentive-compatible for each agent at each point

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price shown below without a cash endowment in the period. Incorporating such an off-equilibrium path only complicates the presentation of the equilibrium without a merit.

[7]It is not crucial to assume that agent A literally exits from the economy after consumption. The implication of the model would be robust if an infinite-lived investor received a sufficiently severe punishment in terms of utility or wealth for missing a scheduled payment. This paper adopts a three-period model to keep the analysis as simple as possible. This approach is similar to the literature on liquidity insurance, such as Diamond and Dybvig (1983) and Allen and Gale (1998). While the timing of consumption is stochastic in this literature, it is not necessary for this paper's result.

in time. This paper shows that this indifference result is overturned if agents bargain over the terms of each bond trade bilaterally, as in an OTC market in practice.

Here, a repo is constrained by limited commitment by agent B, as the repurchase bond price must be consistent with the expected result of ex-post bargaining. This assumption is in line with the assumption that an unsecured loan is unavailable for agent B due to limited commitment.

On the other hand, agent A can commit to which agent to meet first in period 1. As shown below, this assumption is not crucial in the current set-up, as agent A is indifferent to whether to meet first with agent B or C ex-post, given the same bargaining power of the two agents against agent A. This is not the case, however, if agent C has lower bargaining power than agent B. In such a case, agents A and B cannot arrange a repo without agent A's commitment to meeting again with agent B, because agent C can offer a higher ask price for agent A's bond than agent B in period 1. By assuming agent A's commitment ability, the model categorically excludes an unrealistic case that a repo market collapses because of lack of a short-term investor's commitment to a repo.[8]

3.2 Bargaining over a bond resale in period 1

Now solve the model backward from period 1. Suppose agent A buys a bond in period 0. As shown below, agent A resells its bond in the first meeting in period 1 in equilibrium. Because agent A has the same bargaining power, α, against agents B and C, the bargaining outcome in period 1 is identical whether agent A meets first with agent B or C in that period.

Let us start from the second meeting in period 1. If agent A does not resell its bond in the first meeting in the period, then the bargaining problem for the second meeting is

$$\max_{p_1'}(p_1' - \beta R)^{\alpha}(R - p_1')^{1-\alpha}, \tag{2}$$

[8]This feature of the model is consistent with the observation that the possibility of a settlement fail does not make the repo market collapse in reality.

where p_1' is the resale bond price that agent A receives from its counterparty in the second meeting, and the left and the right parenthesis are the trade surpluses for agent A and the counterparty, respectively. In the left parenthesis, βR appears as the threat point for agent A, because agent A discounts the utility of consumption by β if it retains the bond until period 2. In the right parenthesis is the profit for the counterparty from buying the bond in period 1.

The solution for this bargaining problem is[9]

$$p_1' = [\alpha + (1-\alpha)\beta]R. \tag{3}$$

Given $\alpha \in (0,1)$ and $\beta \in [0,1)$, $p_1' < R$; thus, the resale bond price that agent A can receive in the second meeting in period 1 is lower than the return on the bond in period 2. This result is due to a hold-up problem. Agent A wants to retrieve cash in period 1 because it must incur a cost if it misses the timely consumption of cash in period 1. The counterparty for agent A can take advantage of this short investment horizon of agent A, negotiating down the bond price that it pays to the agent.

Given (3), the bargaining problem in the first meeting is

$$\max_{p_1} (p_1 - p_1')^{\alpha}(R - p_1)^{1-\alpha}, \tag{4}$$

where p_1 is the resale bond price that agent A receives from its counterparty in the first meeting, and the left and the right parenthesis are the trade surpluses for agent A and the counterparty, respectively. In the left parenthesis, p_1' is the threat point for agent A, because agent A can move to the second meeting if the first meeting fails.

The solution for the bargaining problem in the first meeting is

$$p_1 = \alpha R + (1-\alpha)p_1' = [1 - (1-\alpha)^2(1-\beta)]R. \tag{5}$$

Given $\alpha \in (0,1)$ and $\beta \in [0,1)$, $p_1 < R$. Thus, agent A suffers a discount on the resale price

[9] This solution is feasible as the counterparty has enough cash to pay p_1' under assumption (1).

of its bond in period 1, because agent A's short investment horizon affects p_1 through p_1'. Nonetheless, agent A resells its bond in the first meeting in period 1, as p_1 is higher than p_1'.

3.3 Bargaining over a bond sale in period 0

Move back to period 0. In this period, agent B meets with agent A to sell a bond. The bargaining problem for this initial bond sale in period 0 is

$$\max_{d\in\{0,1\},\, p_0} (p_1 - p_0)^\alpha [Zp_0 - R + d(R - p_1)]^{1-\alpha}, \tag{6}$$

where: p_0 is the initial selling price of the bond that agent A pays to agent B; and d equals one if agent A commits to meeting first with agent B in period 1, and zero otherwise.[10] In the left parenthesis is the trade surplus for agent A, which is the profit from a bond purchase for the agent, given the resale price of the bond, p_1, in period 1. In the right square bracket is the trade surplus for agent B. In this term, the gross rate of return on agent B's project, Z, is multiplied to p_0, as agent B can invest the revenue from a bond sale into the project in period 0. The opportunity cost of a bond sale is R. If $d = 1$, then agent B can expect an additional profit, $R - p_1$, in a bond repurchase from agent A in period 1.

First, consider a chain of spot bond sales from agent A to B and then to C. In this case, agent A meets first with agent C in period 1. Thus, $d = 0$. The total trade surplus for agents A and B with $d = 0$ can be non-negative if and only if

$$p_1 - \frac{R}{Z} \geq 0, \tag{7}$$

which is equivalent to

$$Z \geq \frac{1}{1-(1-\alpha)^2(1-\beta)}, \tag{8}$$

as implied by (5).

[10]The result does not change even if agents A and B can choose a mixed strategy (i.e., $d \in [0,1]$), because $d = 1$ is strictly dominant as shown below.

If this condition is violated, then agents A and B cannot find an agreeable bond price with $d = 0$. To see why a spot bond trade can break down, note that agent A can pay only a low bond price in period 0 because of a discount on the resale price of its bond in period 1. If (7) is violated, then agent A cannot pay a sufficiently high bond price to compensate for the present discounted value of the bond for agent B (i.e., R/Z). Thus, a low resale bond price for agent A due to its short investment horizon prevents a spot bond trade in period 0.

Next, consider the case that agent A commits to meeting first with agent B in period 1, that is, $d = 1$. In this case, the transaction between agents A and B can be interpreted as a repo, because agent B repurchases a bond in period 1 after selling it to agent A in period 0.

The total trade surplus for agents A and B is always higher with $d = 1$ than with $d = 0$:

$$p_1 - \frac{p_1}{Z} > p_1 - \frac{R}{Z}, \tag{9}$$

as implied by (5), given $\alpha \in (0, 1)$ and $\beta \in [0, 1)$. The left- and the right-hand side are the total trade surpluses with $d = 1$ and with $d = 0$, respectively. Thus, agents A and B prefer a repo (i.e., $d = 1$) to a chain of spot bond sales (i.e., $d = 0$). Moreover, arranging a repo is feasible even if (7) is violated, because the total trade surplus with $d = 1$ on the left-hand side of (9) is always positive, given $Z > 1$ and $p_1 > 0$ as implied by (5).

A repo works because agent B can ultimately take back its bond at the maturity of a repo. In a repo, only the ratio between the initial selling price, p_0, and the repurchase price, p_1, of the bond—that is, the yield—matters. Thus, agent B can lower p_0 to offer a sufficiently high yield for agent A to buy the bond, given p_1. Both agents A and B can gain from this trade, as agent B can offer agent A a yield higher than that on storage, 1, but lower than that on agent B's project, Z, given $Z > 1$.

With $d = 1$, the solution for the bargaining problem, (6), yields

$$p_0 = \left(\frac{\alpha}{Z} + 1 - \alpha\right) p_1 = \left(\frac{\alpha}{Z} + 1 - \alpha\right) [1 - (1 - \alpha)^2(1 - \beta)]R. \tag{10}$$

Given this result, the interest rate on a repo can be computed as

$$r \equiv \frac{p_1}{p_0} - 1 = \frac{\alpha(Z-1)}{\alpha + (1-\alpha)Z}, \tag{11}$$

where r denotes the interest rate. These terms of a repo are determined through bilateral bargaining endogenously.

3.4 Introduction of a long-term investor and an implied repo haircut

It has been shown that the need for a repo by agent A results from the agent's short investment horizon that makes the agent resell a bond before the maturity. A corollary of this result is that a long-term investor can buy a bond without a repo, because a long-term investor does not have to resell a bond before the maturity. Thus, the co-existence of a repo and a spot transaction can be explained by different investment horizons among investors.

To illustrate this result, replace agent A by another agent with $\beta = 1$; thus, this agent does not discount the utility of consumption in period 2. Call this agent agent D. The other characteristics of agent D are as same as those of agent A. Because agent D can hold the bond at no cost until period 2, the sequential bargaining game among agents is simplified to: agent D meets with agent B to buy a bond in period 0; if agent D buys a bond in period 0, then it holds the bond until period 2; otherwise agent D meets with agent B in period 1 to bargain over a bond purchase again.[11]

The bargaining problems between agents B and D in periods 1 and 0 are respectively:

$$\max_{v_1} (R - v_1)^{\alpha} (v_1 - R)^{1-\alpha}, \tag{12}$$

$$\max_{v_0} [R - v_0 - (R - v_1)]^{\alpha} [Z v_0 - R - (v_1 - R)]^{1-\alpha}, \tag{13}$$

[11]In fact, the equilibrium bond price paid by agent D in period 0 is the same even if the sequential bargaining game is as same as that for agent A. In this case, $p_1' = p_1 = R$ in period 1 under $\beta = 1$, as implied by (3) and (5). In period 0, agent D is indifferent between a repo (i.e., $d = 1$) and a chain of spot bond sales (i.e., $d = 0$); thus it always has a positive trade surplus in the bond trade with agent B in period 0. The equilibrium value of p_0 is the same as the value of v_0 shown in (15).

where v_t denotes the bond price that agent D pays to agent B if agent D buys the bond in period t for $t = 0, 1$. In each bargaining problem, the left parenthesis is the trade surplus for agent D and the right one is the trade surplus for agent B. In the right parenthesis of (12), the gross rate of return on agent B's project, Z, is not multiplied to v_1 because agent B can invest cash into the project only in period 0. If agent B sells its bond in period 1, then it just stores the revenue from the bond sale, v_1, until period 2.

Agents B and D can have non-negative trade surpluses in both (12) and (13), given $Z > 1$. Thus, agent D can buy a bond without a repo in period 0.

The solutions for the bargaining problems yield

$$v_1 = R, \tag{14}$$

$$v_0 = \left(\frac{\alpha}{Z} + 1 - \alpha\right) R. \tag{15}$$

With (15), it is possible to calculate a haircut on a repo. A haircut is defined as $1 - p_0/v_0$, that is, the difference between the quoted spot price of the bond, v_0, and the initial selling price of the bond in a repo, p_0, divided by v_0. It is straightforward to show that

$$1 - \frac{p_0}{v_0} = (1 - \alpha)^2 (1 - \beta), \tag{16}$$

which is positive given $\alpha \in (0, 1)$ and $\beta \in [0, 1)$.

A haircut is necessary to make a repo robust to limited commitment. Because agent B can commit only to a discounted repurchase price, p_1, due to agent A's short investment horizon, agent A requires a reduction in the initial selling price of a bond, p_0, in period 0. As a result, p_0 is set lower than the quoted market value of the bond, v_0. This result implies the presence of a haircut specific to a short-term investor. It adds to the standard explanation for a haircut based on value-at-risk of underlying securities.[12]

[12] See Geanakoplos (2009) and Brunnermeier and Pedersen (2009) for a haircut based on value-at-risk.

3.5 Testable implications for the terms of a repo

The baseline model has two testable implications for the terms of a repo. First, (16) shows that a haircut is decreasing in α and β. This result holds because the resale price of a short-term investor's bond declines as the investor has less bargaining power in the OTC market (less α) and is more impatient to retrieve cash within an investment horizon (less β). Thus, a haircut on a repo is increasing in the illiquidity of underlying bonds for a short-term investor. The illiquidity of bonds may depend on each short-term investor, as different investors can have different values of α and β.

Second, (11) shows that the impatience of a short-term investor, β, does not affect the interest rate on a repo. This result holds because a haircut fully takes care of the illiquidity of underlying bonds for a short-term investor. A short-term investor's bargaining power in the OTC market, α, still affects the interest rate, as the interest rate splits the surplus between the yield on cash investment by a bond seller, Z, and that on alternative storage available for a short-term investor, 1. These features of the model can be tested in future research.

3.6 Underlying friction and market set-up behind the existence of a repo

Before moving to the next section, let us briefly discuss the underlying friction and market set-up behind the existence of a repo. First, if $\alpha = 1$, then the resale price of a bond for agent A in period 1, p_1, equals the face value of the bond, R, as implied by (5). In this case, the total trade surplus for agents A and B is the same between a chain of spot bond sales from agent A to B and then to C (i.e., $d = 0$) and a repo between agents A and B (i.e., $d = 1$). This result can be confirmed by substituting $p_1 = R$ into (9).[13] Thus, the partial bargaining power of agent A is necessary for a hold-up problem in an OTC market and the need for a repo.

[13]The total trade surplus is $p_1 - p_1/Z$ with $d = 1$ and $p_1 - R/Z$ with $d = 0$.

Second, if agent C does not exist, then agents A and B automatically enter into a repo if agent A buys a bond in period 0, because agent A can resell the bond only to agent B in period 1. Thus, the distinction between a repo and a chain of spot bond sales becomes meaningful if there exists another counterparty, agent C, which can purchase a bond from agent A in period 1.

Third, if agent B can choose between meetings with a short-term investor (agent A) and a long-term investor (agent D) in period 0, then it prefers a long-term investor because it can obtain a higher bond price, and thus more cash for its project.[14] It is implicitly assumed that agent B cannot choose its counterparty in period 0 due to some random matching process. This feature of the model is consistent with the fact that a securities broker-dealer in practice finances its inventory of a security through a repo until it finds a buyer. This observation implies that a securities broker-dealer prefers to meet with a long-term investor, but cannot always do so. This paper abstracts from modeling a search and matching process formally to illustrate the underlying reason for a repo in a simple set-up.

4 Counterparty risk and the fragility of a repo market

Now investigate the sensitivity of a repo to counterparty risk. Suppose that agent B goes bankrupt exogenously with probability μ ($\in (0,1)$) at the beginning of period 1. In case of bankruptcy, agent B simply exits from the model. In this case, agent A can meet only with agent C in period 1. The other part of the model remains as same as in the baseline model. Thus, agent A is allowed to resell its bond to agent C immediately in case of agent B's bankruptcy. This assumption reflects the exemption of a repo from automatic stay in the current U.S. bankruptcy code. The case without the exemption from automatic stay will be analyzed in the next section.

[14] The trade surplus for agent B in period 0 equals $(Z-1)(1-\alpha)p_1$ in the meeting with agent A, and $(Z-1)(1-\alpha)R$ in the meeting with agent D. Given $p_1 < R$ as implied by (5), the latter is greater than the former.

With the possibility of agent B's bankruptcy in period 1, the bargaining problem for the initial bond sale from agent B to A in period 0, (6), is modified to:

$$\max_{d\in\{0,1\},\, p_0} [\mu p_1' + (1-\mu)p_1 - p_0]^{\alpha}\{\mu \cdot 0 + (1-\mu)[Zp_0 - R + d(R - p_1)]\}^{1-\alpha}, \tag{17}$$

where: the left square and the right curly bracket are the trade surpluses for agents A and B, respectively. In the left square bracket, $\mu p_1' + (1-\mu)p_1$ is the expected resale price of a bond for agent A in period 1. With probability μ, agent A can resell its bond only to agent C because of agent B's bankruptcy. In this case, the bargaining problem becomes as same as (2) due to lack of an alternative counterparty for agent A other than agent C. As a result, the resale bond price in period 1 becomes p_1' as shown in (3). In the right curly bracket, $\mu \cdot 0$ appears because agent B's consumption is zero in case of bankruptcy, whatever agent B does in period 0.

Given $p_1 < R$ as implied by (5), the total trade surplus for agents A and B is larger with $d = 1$ than with $d = 0$:

$$\mu p_1' + (1-\mu)p_1 - \frac{p_1}{Z} > \mu p_1' + (1-\mu)p_1 - \frac{R}{Z}, \tag{18}$$

where the left- and the right-hand side are the total trade surpluses with $d = 1$ and $d = 0$, respectively. Thus, the bargaining problem, (17), implies that

$$(d,\ p_0) = \left(1,\ \frac{\alpha p_1}{Z} + (1-\alpha)[\mu p_1' + (1-\mu)p_1]\right), \tag{19}$$

provided that the total trade surplus with $d = 1$ on the left-hand side of (18) is non-negative. This condition is equivalent to

$$\mu \leq \frac{(Z-1)[1-(1-\alpha)^2(1-\beta)]}{Z\alpha(1-\alpha)(1-\beta)}, \tag{20}$$

given (3) and (5).

If (20) is satisfied, then the probability of agent B's default, μ, is small enough that agents A and B can arrange a repo in period 0. In this case, both the haircut, $1 - p_0/v_0$, and the interest rate, $p_1/p_0 - 1$, on a repo are increasing in μ, because p_0 is decreasing in μ given $p_1' < p_1$. Note that both the repurchase bond price paid by agent B in case of no bankruptcy, p_1, and the quoted spot bond price in period 0, v_0, are independent of μ.[15] Thus, higher counterparty risk makes a short-term investor in a repo require more protection from default as well as more interest, because the investor must liquidate a bond at a discounted price, p_1', if its counterparty defaults.

If (20) is not satisfied, agents A and B cannot find an agreeable initial selling price of a bond in a repo because counterparty risk, μ, is too high. In this case, the bond price that agent A can pay to agent B in period 0 is significantly capped, as agent A must resell its bond to agent C at a discounted price, p_1', with a high probability. Agent B cannot accept the bond price that agent A can pay in period 0, because agent B cannot repurchase the bond at p_1' in period 1 when it goes bankrupt.[16]

In summary, a repo is vulnerable to counterparty risk, because a short-term investor in a repo must resell the underlying bond to a third party whenever the counterparty goes bankrupt. If there exists a possibility of such an event, then the initial bond seller cannot gain from the future resale of a short-term investor's bond with some probability. As a result, the counterparty risk reduces the trading surplus between a bond seller and a short-term investor in a repo ex-ante. This effect makes a repo market collapse if the counterparty risk is too high.[17]

[15]It is obvious that μ does not affect the bond price paid by agent B in period 1 once agent B turns out to be solvent. Note that there are no gains from trade between a long-term investor (i.e., agent D) and agent B in period 1, as implied by (12); thus, the bargaining problem in period 0, (13), and hence v_0, is unchanged even if agent D loses an opportunity to meet with agent B again in period 1.

[16]Note that the result is not driven by the absolute level of the number of counterparties for agent A. If there were not agent C from the beginning, then agent B would be the only counterparty for agent A. In this case, agent B's repurchase price of the bond in period 1 would be p_1'. Without counterparty risk, however, agent B would be able to lower the initial selling price of its bond sufficiently that agent A buys the bond in period 0, because agent B would be able to repurchase the bond at p_1'. Thus, the key to the result is a change in the number of counterparties due to agent B's bankruptcy, and a resulting change in the resale price of agent A's bond from p_1 to p_1'.

[17]Given the risk neutrality of agents, the volatility of R does not affect the results. If risk management is introduced into the model, however, then there can be a linkage between the volatility of bond returns and endogenous counterparty risk. Suppose that R is stochastic and realizes in period 2. The volatility of R is

An implicit assumption behind this result is that agent B is not immediately replaced by a new entrant into the market in case of bankruptcy. To interpret this assumption, consider agents B and C as major counterparties in the repo market, such as large investment banks, which can act as a counterparty for a short-term investor with a large volume of cash. Agent A represents such a large short-term investor. The model assumes that if one of the major counterparties exits from the repo market, then it is difficult to find a replacement right away. This oligopolistic structure of the repo market makes counterparty risk a concern for a short-term investor ex-ante.

5 Effect of automatic stay on a repo

So far, it has been assumed that a repo is exempted from automatic stay, so that agent A can resell its bond to agent C in case of agent B's bankruptcy. This assumption is consistent with the current U.S. bankruptcy code. Currently, there is discussion whether the exemption should be removed or not. This section describes the effect of automatic stay in the model.

When a repo is subject to automatic stay, agent A cannot resell its bond to agent C in period 1 if it arranges a repo with agent B in period 0 and then agent B goes bankrupt in period 1.[18] With this assumption, the bargaining problem between agents A and B in period

updated in period 1; it increases to a certain value with probability p and remains the same as in period 0 otherwise. Assume that agents B and C have some threshold for value-at-risk of their asset holdings, and if an asset violates this threshold, then they do not hold the asset. Now assume that if the volatility of R increases in period 1, then it violates the value-at-risk restriction on agents B and C. In this case, agent B defaults on a repo endogenously, and agent A cannot find any buyer for its bond in period 1. As implied by the result shown in section 5, the possibility of such an event raises the haircut and the interest rate on a repo, or even makes a repo market collapse, in period 0. The formal analysis of this issue, including why investors set a cut-off threshold for value-at-risk rather than pricing the risk, is beyond the scope of this paper and left for future research.

[18] Assume that the court can identify a repo from the terms of a transaction between agents A and B, even if a repo takes a form of an implicit contract. Thus, agents A and B cannot avoid the application of automatic stay on a repo just by writing no explicit contract. This assumption is consistent with the practice that a repo is regarded as a secured loan contract even if it does not take an explicit form of such a contract.

0, (17), is modified to:

$$\max_{d\in\{0,1\},\ p_0} \{\mu[d\beta R + (1-d)p_1'] + (1-\mu)p_1 - p_0\}^{\alpha}\{\mu\cdot 0 + (1-\mu)[Zp_0 - R + d(R-p_1)]\}^{1-\alpha}. \tag{21}$$

In the left curly bracket, p_1' in (17) is replaced with $d\beta R+(1-d)p_1'$. This term is the value of the bond for agent A in period 1 in case of agent B's bankruptcy. If a repo is arranged in period 0 (i.e., $d=1$), this term becomes βR because agent A must hold its bond until period 2 in case of agent B's bankruptcy. If agent A buys a bond without a repo in period 0 (i.e., $d=0$), then it can resell the bond to agent C at p_1' when agent B goes bankrupt in period 1.

The choice of d depends on the total trade surplus from each type of trade in period 0. The conditions for non-negative total trade surpluses from a repo ($d=1$) and from a spot bond trade ($d=0$) are respectively:

$$\mu\beta R + (1-\mu)p_1 - \frac{p_1}{Z} \geq 0 \quad \left(\Leftrightarrow \quad \mu \leq f(Z) \equiv \frac{(Z-1)[1-(1-\alpha)^2(1-\beta)]}{Z\alpha(2-\alpha)(1-\beta)}\right), \tag{22}$$

$$\mu p_1' + (1-\mu)p_1 - \frac{R}{Z} \geq 0 \quad \left(\Leftrightarrow \quad \mu \leq g(Z) \equiv \frac{Z[1-(1-\alpha)^2(1-\beta)]-1}{Z\alpha(1-\alpha)(1-\beta)}\right), \tag{23}$$

where the equivalent condition in each parenthesis is calculated from (3) and (5). The total trade surplus from a repo dominates that from a spot bond trade if and only if

$$\mu\beta R + (1-\mu)p_1 - \frac{p_1}{Z} \geq \mu p_1' + (1-\mu)p_1 - \frac{R}{Z} \quad \left(\Leftrightarrow \quad \mu \leq h(Z) \equiv \frac{(1-\alpha)^2}{Z\alpha}\right). \tag{24}$$

A repo and a spot bond trade are indifferent if (24) holds in equality. Thus, the solution for the bargaining problem, (21), can be written as[19]

$$(d,\ p_0) = \begin{cases} \left(1,\ \dfrac{\alpha p_1}{Z} + (1-\alpha)[\mu\beta R + (1-\mu)p_1]\right), & \text{if } \mu \leq f(Z) \text{ and } \mu \leq h(Z), \\ \left(0,\ \dfrac{\alpha R}{Z} + (1-\alpha)[\mu p_1' + (1-\mu)p_1]\right), & \text{if } \mu \leq g(Z) \text{ and } \mu > h(Z). \end{cases} \tag{25}$$

If $\mu > f(Z)$ and $\mu > g(Z)$, agents A and B do not agree on any deal in period 0.

[19] Here, without a loss of generality, it is assumed that agents A and B choose a repo if they are indifferent between a repo and a spot bond trade.

Figure 2 draws the functions f, g, and h to illustrate the behavior of agents A and B over the parameter space spanned by counterparty risk, μ, and the gross rate of return on agent B's project, Z. For comparison purpose, the figure also contains a panel for the case without automatic stay. In the figure, agent A's bargaining power, α, is fixed to 0.5 and the discount on agent A's utility of consumption in period 2, β, is set to zero. The results described below, however, hold for any value of $\alpha \in (0, 1)$ and $\beta \in [0, 1)$.[20]

Figure 2 indicates two results. First, automatic stay expands the parameter region for no trade. It reduces the trade surplus from a repo, because it prevents agent A from reselling a bond within the agent's investment horizon if agent B goes bankrupt. As a result, agents A and B find no gains from trade in a repo in a wider range of counterparty risk.[21] Also, the comparison between (19) and the first line in (25) imply that even if agents A and B arrange a repo, the cost of automatic stay makes agent A pay a lower initial bond price, p_0, than in the case without automatic stay.[22] Thus, automatic stay hampers agent B's funding through a repo by raising the haircut, $1 - p_0/v_0$, if a repo market remains open.

Second, for a sufficiently high value of Z, there is a range of μ in which agents A and B choose a spot bond trade in period 0. This result contrasts with the case without automatic stay, in which a spot bond trade is never chosen. In this parameter region, counterparty risk, μ, is so high that the expected cost of automatic stay makes a repo less attractive than a spot bond trade. Agents A and B still find gains from trade in a spot bond trade, as a high return on agent B's project, Z, makes agent B willing to give up its bond to finance its funding need.

In the second case, the spot bond price that agent A pays to agent B in period 0 is higher

[20]For all $Z > 1$, $\alpha \in (0, 1)$ and $\beta \in [0, 1)$, $g' > f' > 0$ and $h' < 0$. Also, $\lim_{Z\to 1} f(Z) > \lim_{Z\to 1} g(Z)$. Thus, f, g, and h intersect only once at the same value of Z in their domain, $Z > 1$. Denoting this value of Z by Z^*, it is immediate to show that $f(Z) > 0$ and $f(Z) > g(Z)$ for $Z \in (1, Z^*)$ and that $g(Z) > f(Z) > 0$ for $Z > Z^*$. These properties of f, g, and h are sufficient for the two results described below as an implication of Figure 2.

[21]To confirm this result analytically, compare (20) and (22).

[22]Note that $p_1' > \beta R$ as implied by (3), given $\alpha \in (0, 1)$ and $\beta \in [0, 1)$.

than the initial selling price of a bond in a repo without automatic stay. Compare p_0 in (19) and the second line in (25) to confirm this result. In a spot bond trade, agent B requires a higher bond price for its bond to compensate for the opportunity cost of giving up its bond for good. Thus, agent B's investment into its project in period 0, and hence aggregate income, increases if automatic stay on a repo induces agents to switch from a repo to a spot bond trade.[23]

In summary, if a repo is not exempted from automatic stay, then a short-term investor in a repo cannot retrieve cash within the investor's investment horizon in case of the counterparty's default. Therefore, committing to reselling a bond to a bond seller ex-ante is now costly. This effect makes a repo market collapse in a wider range of counterparty risk with automatic stay. If a bond seller's funding need is sufficiently strong in such a case, then a short-term investor and a bond seller choose a spot bond trade to keep financing the bond seller's funding need while avoiding automatic stay on a repo.

6 Conclusions

This paper presents a simple model featuring a short-term investor in the OTC bond market. The model illustrates that the investment horizon of a short-term investor and bilateral bargaining in the OTC bond market result in a hold-up problem for a short-term investor. The hold-up problem explains the use of a repo by a short-term investor as well as the vulnerability of a repo market to counterparty risk. Also, the co-existence of a repo and a spot transaction is explained by a difference in investment horizons among investors. These results hold without any uncertainty about bond returns or asymmetric information.

Just considering a repo as a secured loan is insufficient for the results in this paper. Using a risk-free bond as collateral should protect a borrower from counterparty risk, if the bond

[23] Despite the expansion of aggregate income, the total trade surplus for agents A and B is higher in a repo without automatic stay than in a spot bond trade. The increased surplus due to higher aggregate income is absorbed by agent C, which can buy a bond from agent A at a discounted price in period 1 if agents A and B arrange a spot bond trade in period 0.

market is competitive. Such a result, however, does not hold because the liquidation price of a bond in an OTC market differs according to the characteristics of each seller. This structure of the bond market makes a short-term investor vulnerable to counterparty risk in a repo. It also explains why such a vulnerable investor is a main user of a repo in the first place.

In this paper, it is taken as given that the bond market is an OTC market. A question remains regarding the optimal market design, such as whether to introduce a centralized bond market. Also, it remains an issue to introduce a repo into a richer model of a bond market. This paper keeps the model as simple as possible to illustrate the underlying reason for the need for a repo and its fragility. The remaining issues are left for future research.

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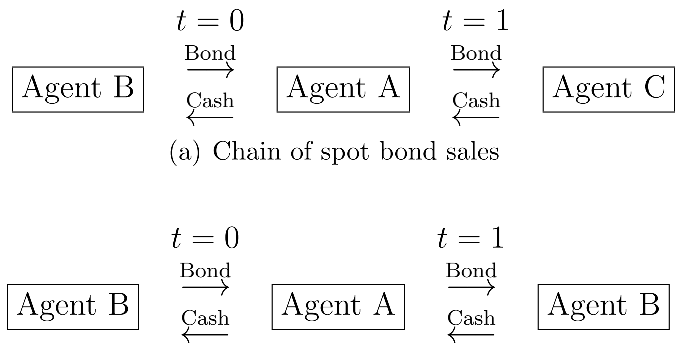
Table 1: Summary of the baseline model

	Period 0	Period 1	Period 2
Agent A	Endowed with an amount e of cash.	Can consume cash.	Can consume cash, but the utility of consumption is discounted by β.
Agent B	Can invest cash into a project.	The project returns an amount Z of cash per invested cash. The return is non-pledgeable. Endowed with an amount e of cash.	Consume cash.
Agent C		Endowed with an amount e of cash.	Consume cash.
Bond	Agent B is endowed with a bond, and meets with agent A to sell the bond.	If holding a bond, agent A can meet agent B and then agent C, or agent C and then agent B, to resell the bond.	The holder of the bond receives an amount R of cash.
	Nash bargaining determines the outcome of each bilateral bond trade. Agent A's bargaining power is α.		
Storage	Each agent can store cash between consecutive periods.		

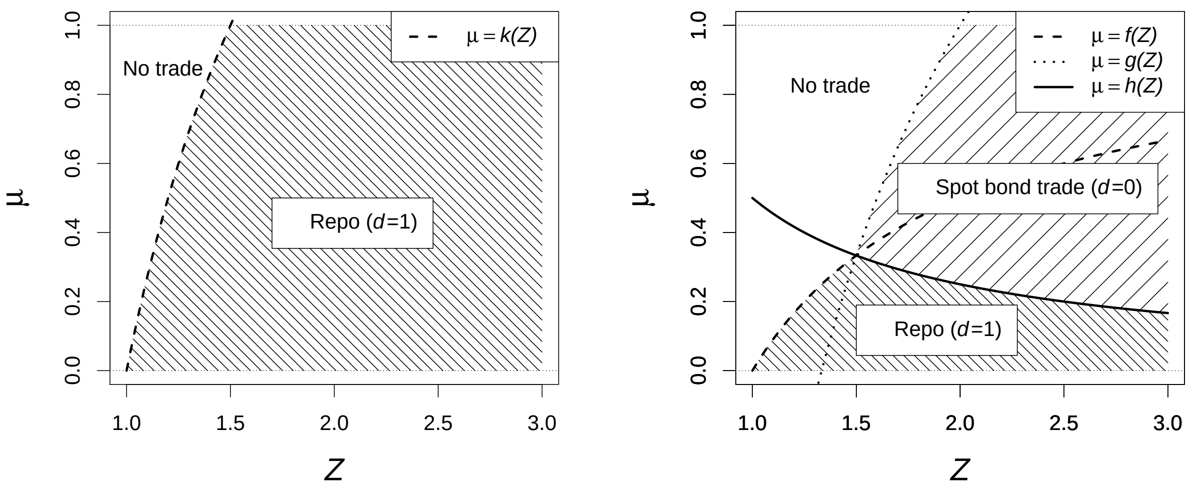
Figure 1: Two ways to finance agent B's funding need



(a) Chain of spot bond sales

(b) Repo

Figure 2: Type of trade in period 0



(a) Without automatic stay on a repo

(b) With automatic stay on a repo

Note: $\alpha = 0.5$. $\beta = 0$. The domain of μ in each panel does not include $\mu = 1$, as it is defined over $\mu \in [0, 1)$. Without automatic stay (panel a), a repo ($d = 1$) is chosen in period 0 if and only if $\mu \leq k(Z) \equiv (Z-1)[1-(1-\alpha)^2(1-\beta)]/[Z\alpha(1-\alpha)(1-\beta)]$, as shown in (20). Otherwise, there is no agreeable trade in period 0 as a spot bond trade ($d = 0$) never dominates a repo without automatic stay. In the presence of automatic stay (panel b), the definition of the functions f, g, and h is provided by (22)-(24), given α and β. These functions have the following properties: the total trade surpluses from a repo and from a spot bond trade are non-negative if and only if $\mu \leq f(Z)$ and $\mu \leq g(Z)$, respectively; a repo dominates a spot bond trade if and only if $\mu < h(Z)$; and they are indifferent if and only if $\mu = h(Z)$.