# The Relation between Inventory Investment and Price Dynamics in a Distributive Firm

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# The Relation between Inventory Investment and Price Dynamics in a Distributive Firm[*]

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**Abstract**

In this paper, we examine the role of inventory in the price-setting behavior of a distributive firm. Empirically, we show the 5 empirical facts relating to pricing behavior and selling quantity of a certain consumer goods based on daily scanner data to examine the relation between store properties and pricing behavior. These results denote that price stickiness varies by the retailers' characteristics. We consider that the hidden mechanism of price stickiness comes from the retailer's policy for inventory investment. A partial equilibrium model of the retailer's optimization behavior with inventory is constructed so as to replicate the five empirical facts. The results of the numerical experiments in the constructed model suggest that price change frequency depends on the retailer's order cost, storage cost, and menu cost, not on the price elasticity of demand.

**Keywords:** Inventory; Price Stickiness; Numerical Experiment; (S, s) Policy
**JEL Classification:** D22, E27, E31

# 1. Introduction

## 1.1. Background

Price stickiness is one of the most important and controversial concepts in macroeconomics. Many macroeconomists consider it as a key concept of the real effect of monetary policy in a macroeconomic model. So far, they have turned to the theory of price dynamics and investigated data to establish empirical facts. This paper studies the mechanism of price stickiness by examining the role of inventory in the price-setting behavior of a distributive firm empirically using micro-data scanned in retail stores, and through numerical experiments of a quantitative model of a distributive firm.

Figure 1 shows the sales prices and quantities sold of a brand of cup noodles in three supermarkets in Japan. We find retailers take different pricing strategies, and their price stickiness, frequency of discount sales, and sales concentration vary. Figure 1(A) shows that one retailer does not change the price in the window period. However, the quantity sold changes due to demand shocks, and consequently the retailer adopts the strategy of setting the item's price constant. Figure 1(B) shows that another retailer implements discount sales periodically, during which consumers purchase a significant amount of the bargain item. Figure 1(C) shows that a third retailer very rarely holds discount sales with large discounts, and most of the items are sold during the discount sales. From the concentration of quantities sold, we infer the accumulation of the item's inventory not shown in the figure.

In this paper, we attempt to answer three questions: First, what causes the difference in the price-setting strategies among retailers? Second, what is the role of inventory investment in the price-setting behavior of a distributive firm? Finally, how does the business environment affects a retailer's price change probability and discount sales frequency? To this end, we explore the concept of price dynamics.

## 1.2. Previous Studies

There are abundant studies in price dynamics using micro price data. The Eurosystem Inflation Persistence Network is a pioneer in this research area; one of its studies was conducted by Fabiani, Loupias, Martins, and Sabbatini (2007). In the US, Cecchetti (1986), Kashyap (1995), Bils and Klenow (2004), and Nakamura and Steinsson (2007) conducted empirical studies on price rigidity. In Japan, Saito and Watanabe (2007), and Matsuoka (2012) conducted empirical research on price dynamics using daily scanner data of retail stores in Japan. Abe and Tonogi (2010) found that there is a variety of pricing strategies among retailers, and that retailers' discount sale behavior is important in price dynamics. Matsuoka (2012) examined empirically the relationship between mo-

nopolistic power and frequency of price change and found a negative relationship between them. In response to these studies, we examines the mechanism that generates price rigidity through inventory holdings with numerical experiments of a quantitative (S, s) model.

There have been several early theoretical studies of inventory investment, which is related to (S, s) policy. In an (S, s) type model, a firm optimally picks some level of inventory, s, below which the firm orders inventory stocks in bunching manner, and increases the stocks to an optimally chsen level, S. Thus S minus s is the optimally lot size of the order. Arrow, Harris, and Marschak (1951) were the first to study (S, s)-type inventory behavior. Arrow, Karlin, and Scarf (1958) also performed seminal work on this type of model. Blinder (1982) examined price stickiness and inventory investment in (S, s)-type models and concluded that "the model helps provide an explanation for sluggish relative prices."[1] This paper adopts the concept of retailer's monopolistic power in retail market and so the firm is able to set his selling price from Blinder's model. Aguirregabiria (1999) also conducted important work in this research field. In his study, he constructed a model of the interaction between price and inventory decisions in retail firms and estimated the model parameters using retail data. He concluded inventory and order costs play important roles in sales promotion behavior.

Recently, several studies on the relation between price stickiness and inventory holding have been performed. That by Kryvtsov and Midrigan (2012) is representative of these studies. Our study is founded on all of these previous studies, and we contribute to the literature with the use of scanner data and numerical experiments.

### 1.3.    Contributions of the Paper

Through this paper, we contribute to the study of price dynamics related to inventory holding in three ways. First, we investigate the empirical relations among a firm's probability of price change, business scale, frequency of discount sales, and price elasticity of demand using daily scanner data. Second, we focus on the relation between price rigidity and periodic discount sale behavior, which is often ignored in the macroeconomic context. Third, we construct a numerical model with an (S, s) inventory policy and examine the dynamic nature of price and quantity behavior.

### 1.4.    Organization of the Paper

The rest of this paper is organized as follows. In the next section, we determine the empirical properties for price and quantity of an item (i.e., a popular brand of cup noo-

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[1]  In page 347, Blinder (1982).

dles) in retailers in Japan. In section 3, we describe the development of the model of a distributive firm with an inventory level that optimizes the sum of the current and discounted future profit in a monopolistic environment. In section 4, we describe the numerical experiments we conducted using the model. We also show the changes in price and discount sales moments depending on the cost parameters. In section 5, we present the conclusions.

## 2. Empirical Facts

In this section, we examine daily scanner data collected from retail stores throughout Japan to clarify empirically the relation between pricing behavior and retailers' characteristics.

### 2.1. Daily Scanner Data

We use Nikkei POS Data from Nikkei Digital Media.[2] Our investigation focuses on a representative item, instant cup noodles, which is a very popular processed food in Japan and whose quality has not changed for a long time. The Nikkei POS Data we examine in this paper covers the period January 1, 2008 to December 31, 2008. We chose this period because it has the most number of stores (235) that sell the item. The data includes quantity sold, price, and number of store visitors. Table 1 shows the pooled summary of statistics for the panel data.

### 2.2. Moments and Implications

To investigate empirically the relationship between pricing behavior and retailers' characteristics, we compare the moments of the individual stores. Table 2 shows the summary of the moments across stores. Although all the retail stores deal with the same brand of instant cup noodles, their average prices varied from 104 yen to 159 yen. Thus, the standard deviations of the prices varied from 0.2 to 27.9. Similarly, there are significant differences in the probability of price changes and frequency of discount sales among stores. The price elasticity is estimated by the regression in equation (1):

$$log(p_t^i) = \alpha^i + \beta_1^i \, log(z_t^i) + \beta_2^i \, log(v_t^i) + \epsilon_t^i. \tag{1}$$

$p_t^i$ denotes the price of the item at store $i$ in period $t$. $z_t^i$ denotes the quantity sold for the item at store $i$ in period $t$. $v_t^i$ denotes the number of visitors to store $i$ in period $t$. $\epsilon_t^i$

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[2] Nikkei-POS Data are compiled by Nikkei Digital Media Inc. The data set contains daily transaction for a large number of products in various retail shops throughout Japan from March 1, 1988 to thepresent. A more detailed description are   in Abe and Tonogi (2010) and Matsuoka (2012).

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denotes the residual of the estimation at store $i$ in the period $t$. $\beta_1^i$ represents the estimated price elasticity of demand for store $i$. The price elasticity for demand varies widely, from 5.1 to 471.7. This is because each store faces a different shaped demand function, which indicates each store has a different level of monopolistic power in the market. These facts denote that various pricing strategies are adopted by the retail stores. These facts also raise the question: What causes the difference in pricing strategies?

Figure 3 shows scatter plots of the moments of the scanner data. Figure 3(A) correlation between frequencies of price changes and frequencies of discount sales among stores. There is a significant linear relationship, which means that price movement in discount sales is an important cause of price change. Figure 3(B) shows the correlation between frequencies of price changes and coefficients of variation for sales quantities. We can see a significant positive linear relationship between these variables. It means frequency of price changes has a positive relationship to capacity of goods storage in a retail store. Figure 3(C) shows the correlation between frequencies of price changes and average price level. We can see a significant negative linear relationship between these variables. It means that the store which changes price more frequently set the price lower in average. Figure 3(D) shows the correlation between average markdown ratio on discount sales days and estimated price elasticity of demand. We can see a significant negative linear relationship between these variables. It means that the store which has more monopolistic power cuts prices more on discount sales day.

Figure 3(E) shows the correlation between frequency of discount sales and average markdown ratio on discount sales days. We cannot see a significant linear relationship between these variables. There might be no relation between monopolistic power and discount sales' frequency.

The empirical facts regarding the relation between pricing behavior and retailers' characteristics are summarized as follows:

- Fact 1: Price movements in discount sales are important causes of price changes.
- Fact 2: Probability of price changes has a positive correlation to capacities of goods storage in a retail store.
- Fact 3: Probability of price changes has a negative correlation to the average price level of a retail store.
- Fact 4: Average markdown rate has a negative correlation with price elasticity of demand.
- Fact 5: Frequency of discount sales has no correlation to average markdown rate.

In the following section, we investigate these relations in a model of a retail store with an (S, s)-type inventory investment so as to replicate these five empirical facts.

## 3. The Model

We construct a model of a distributive firm that purchases goods in the wholesale market, stocks inventory, and sells this inventory in the retail market. Blinder (1982) investigated the same model in a monopolistic environment. He pointed out that the model explains the sluggish price reaction to cost. Although the model used in this paper is similar to that of Blinder (1982), it also differs from the latter on a few points. First, in our model, we assume a linear cost function because of the assumption that the retailer is a price taker in the wholesale market. Second, we introduce a fixed order cost in order to create an (S, s) policy for inventory investment.[3] Third, we incorporate a stock-out penalty cost, which prompts a store to avoid stock outs, into the model.[4] We construct the model to analyze numerically the relationship between a retailer's price-setting behavior and inventory investment.

### 3.1. Environment

The empirical facts established in previous section come from an analysis of retail stores' scanner data. In this section, we construct a mathematical model of a retail store's optimal behavior, and then test the model in numerical experiments in the next section.

The model has several differences from the model of a perfect competitive market. First, the distributive firm addresses the demand function of consumers instead of a given price. This assumption is similar to that for Blinder's (1981) model; however, in our model, we do not suppose a linear demand function but a power function, as follows:

$$p_t = a_t x_t^{-\frac{1}{\rho}}, \qquad \rho > 0, \tag{2}$$

where $p_t$ is the price of the item sold in the selling phase in the retail market, $x_t$ is the quantity of the item sold, $a_t$ is the exogenous demand state in the retail market, and $\rho$ is the parameter of price elasticity of demand. Meanwhile, the distributive firm is supposed to be a price taker in the purchasing phase in wholesale market. The firm obtains its operating fund from the profit of its buying and selling operation wherein it purchases the optimal quantity of inventory at an optimal timing In addition, we assume a penalty for stock outs, which would motivate the firm to ensure it has sufficient stock.

When the distributive firm orders inventory in the wholesale market, it incurs a fixed order cost regardless of the quantity it purchased. This order cost gives the firm an in-

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[3] Blinder (1981) analyzed the (S, s) policy of inventory investment in detail.
[4] Kahn (1987) constructed the stock-out avoidance model of inventory investment.

centive to order in bulk instead of purchasing each period. Consequently, an (S, s)-type inventory policy is generated.

We also suppose that the firm faces two other costs: inventory holding cost and menu cost. The inventory holding cost is assumed as a quadratic power function imposed on the inventory stock level held from the previous to the current period. The menu cost is a small cost imposed on changing the selling price. These costs are expected to affect the pricing behavior of the firm. Summing up the distributive firm's environment, the firm's current profit function is expressed as follows:

$$\pi_t = p_t x_t - q_t y_t - c^s s_t^\phi - c^x x_t - c^o i_t^o - c^p i_t^p - c^n i_t^n, \tag{3}$$

where $q_t$ is the price of the item purchased in the purchasing phase in the wholesale market, $y_t$ is the quantity of the item purchased, and $s_t$ is the inventory holding stock. Meanwhile, $c^s$ and $\phi$ are the cost parameters in the inventory holding cost function; $c^x$ is the operating cost, $c^o$ is the order cost, $c^p$ is the menu cost, and $c^n$ is the stock-out penalty cost. $i_t^o, i_t^p,$ and $i_t^n$ are indicator functions that take the following values:

$$i_t^o \begin{cases} = 1, & \text{if } y_t > 0 \\ = 0, & \text{if } y_t = 0 \end{cases}, \quad i_t^p \begin{cases} = 1, & \text{if } p_t \neq p_{t-1} \\ = 0, & \text{if } p_t = p_{t-1} \end{cases}, \quad i_t^n \begin{cases} = 1, & \text{if } s_t = 0 \\ = 0, & \text{if } s_t > 0 \end{cases}. \tag{4}$$

In addition, the firm is subject to two constraints: cash-in-advance constraint and inventory-in-advance constraint. The cash-in-advance constraint means the firm's purchase amount should not exceed the cash on hand:

$$q_t y_t \leq f_t, \tag{5}$$

The inventory-in-advance constraint means the firm's selling quantity should not exceed the inventory in the storage:

$$x_t \leq s_t, \tag{6}$$

The state variables $s_t$ and $f_t$ are subject to the following transition equations:

$$s_t = (1 - \delta)(s_t - x_t) + y_t, \tag{7}$$
$$f_t = f_t + \pi_t. \tag{8}$$

where $\delta$ is the depreciation rate of inventory stock. Both state variables are also subject to non-negativity constraints.

The timeline of the distributive firm activities in period $t$ is summarized in Table 3. After the state variables $s_t, f_t,$ and $p_{t-1}$, which are determined based on the firm's behavior in the previous period, $a_t$ and $q_t$ are derived from the data-generating process exogenously,

enabling the distributive firm to determine the control variables $(x_t, y_t, p_t)$ optimally.

## 3.2.  Set Up

The problem of the distributive firm is represented as follows:

$$\max_{x_t, y_t} E\left[\sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \pi_t | s_t, f_t, p_{t-1}, q_t, a_t\right],$$

$$\text{s.t.} \quad \pi_t = p_t x_t - q_t y_t - c^s s_t^{\phi} - c^x x_t - c^o i_t^o - c^p i_t^p - c^n i_t^n,$$

$$\phi > 0, \ c^s \geq 0, \ c^o \geq 0, c^p \geq 0, c^n \geq 0,$$

$$i_t^o \begin{cases} = 1, \ \text{if } y_t > 0 \\ = 0, \ \text{if } y_t = 0 \end{cases}, \quad i_t^p \begin{cases} = 1, \ \text{if } p_t \neq p_{t-1} \\ = 0, \ \text{if } p_t = p_{t-1} \end{cases}, \quad i_t^n \begin{cases} = 1, \ \text{if } s_t = 0 \\ = 0, \ \text{if } s_t > 0 \end{cases},$$

$$s_{t+1} = (1-\delta)(s_t - x_t) + y_t, \ s_0 \text{ given},$$

$$f_{t+1} = f_t + \pi_t, \ f_0 \text{ given},$$

$$q_t y_t \leq f_t, \ x_t \leq s_t, \ x_t \geq 0, \ y_t \geq 0, \ s_{t+1} \geq 0, \ f_t \geq 0,$$

$$\text{where} \quad p_t = a_t x_t^{-\frac{1}{\rho}}, \ \rho > 0,$$

$$a_t = 1, \forall t,$$

$$q_t = 1, \forall t.$$

(9)

The definitions of the variables and parameters are provided in Table 4. Since the model is non-linear and has many possible binding constraints and indicator functions that are not differentiable, we cannot solve the policy function of the problem by a closed-form analysis, even if we formalize the data generating processes of $a_t$ and $q_t$. Thus, we use a numerical method to solve the problem computationally. In concrete terms, we use the method of discrete space dynamic programming with an interpolation method for the value evaluation of the state variables. Our interpolation method is a multi-dimensional linear interpolation method.

## 3.3.  Parameterizations

When we solve the model numerically, it is necessary to specify the concrete functional forms and give values to the parameters. Since the functional forms are already specified in the previous subsection, we calibrate the values of the parameters in this subsection.

First, we parameterize the model of a benchmark case. In the benchmark case, the value of the price elasticity of price $\rho$ is set to 18, which is the median of the estimated values for the elasticity in the previous subsection. The value of the curvature parameter of inventory stock cost, $\phi$, is set to 2, indicating the cost function is quadratic. Inventory cost

technology, $c^s$, is set to 0.01. Since we have no a priori information on the inventory holding cost, we have to determine the values of the parameters in the function arbitrarily. The depreciation rate, $\delta$, is set to 0.00. In this paper, the deterioration of products is not considered for simplicity. The discount rate, $r$, is set to 0.01. This value, however, is too high for making daily decisions, and thus, we adopt the value from the convenience of a value function convergence computation. Although, from the perspective of the manager of a retail store, who moves to another store as part of the personnel changes once every few years, this discount rate is not so high. The values of the order cost, operating cost, menu cost, and stock-out cost are 2.00, 0.05, 0.03, and 5.00 respectively. At present, we have no a priori information about these parameters. To address this issue, we investigate empirically the cost structure of the distributive firm. We summarize the parameterization in Table 5.

## 4.   Numerical Experiments

We solve the parameterized model numerically using an algorithm of the value function iteration on the discrete state space. The state spaces are divided into 20 grids. Therefore, the state variables $s_t, f_t,$ and $p_{t-1}$ each have 20 grids. Likewise, the control variables $x_t$ and $y_t$ each have 20 grids. The values are evaluated in $20 \times 20 \times 20 \times 20 \times 20$ points, and we seek optimal policies from control candidates. We suppose $a_t$ and $q_t$ are constant throughout all periods here, that is, $a_t = 1, \forall t,$ and $q_t = 0.6, \forall t.$

Figure 3 shows the value functions on the state grids of inventory stock and operation fund in the case of $p_{t-1} = 1$. Figures 4 and 5 plot the policy functions for optimal selling quantity and purchasing quantity on their respective grid spaces. The optimal selling quantity in the retail market basically depends on the inventory stock and also somewhat on the operating fund. Likewise, the optimal purchasing quantity in the wholesale market depends mostly on inventory stock. If the retailer experiences a stock out, it has to purchase a significant amount of inventory to replenish its stock. The selling and buying policy function generates the (S, s) behavior of inventory stock.

### 4.1.   Benchmark Case

We demonstrate the deterministic simulation of the distributive firm that is parameterized in the benchmark case in order to understand the behavior of the numerical model. Figures 6 and 7 plot the simulated paths of the variables in the window, which focuses on the last 50 periods of the 20,000-period simulation. Figure 6 shows the periodic discount sales in an environment without selling price fluctuations and demand state fluctuations.

Figure 6 resembles the actual retail store's behavior presented in Figure 1(C). The firm purchases inventory stock in the wholesale market once every seven periods in order to save on order cost. The firm also holds discount sales in the retail market in the period after purchasing inventory in order to save on inventory stock cost. After the discount sales, the firm hikes up the price in the retail market and then fixes the price for six periods in order to save on menu cost. Figure 7 shows the firm's (S, s) behavior of inventory and similar reversal movements of the operating fund.

After purchasing inventory in bulk, the firm accumulates the operating fund required to purchase inventory next time. We perform deterministic simulations of the model, assigning various values to the cost and demand function parameters. Table 6 summarizes the moments of price dynamics and discount sale behavior. The discount sale frequency is calculated as follows: we indicate 1 when the difference between the mode price and sold price is more than 2 and indicate 0 otherwise. The average of these scores is the frequency of discount sales. The price change frequency is calculated as follows: we indicate 1 when $p_{t-1} \neq p_t$ and indicate 0 otherwise. The average of these scores is the frequency of price change.

## 4.2. Menu Cost

The menu cost affects the price-setting and discount sale behaviors. A higher menu cost leads to a lower probability of price change and a lower frequency of discount sales. On the other hand, a higher menu cost leads to a higher average change rate of price. Figure 8 shows that a higher menu cost prompts the distributive firm to prolong the interval between discount sales.

## 4.3. Storage Cost

Storage cost plays an important role in the cyclical discount sale behavior. Figure 9 shows the selling price movements in the various storage cost parameters. If the storage cost is set to zero, the firm sets a low price every day without holding discount sales (Figure 9 (A)). If the cost of storage increases, the firm has an incentive to implement discount sales immediately after it purchases inventory in order to save on storage cost (Figures 9(B), (C)). Without the storage cost, the firm purchases inventory in a large quantity at once, and then sets a low price and sells greater quantities than in the benchmark case. If the storage cost increases even further, the firm is prompted to sell more of the inventory than usual not only for the period after purchasing the inventory but for several periods after that (Figure 9(D)).

### 4.4. Order Cost

The interval between discount sales is directly affected by the order cost. Figure 10(A) shows that the firm has no incentive to hold discount sales in the case of a low order cost ($c^o = 1$). Because of the low order cost, the firm purchases inventory not in bulk but in a constant quantity every period. Thus, the firm's inventory stock level and selling quantity are always constant. When the firm implements discount sales frequently, it incurs a higher order cost. To save on order cost, the firm thus holds discount sales less frequently (Figures 10(B), (C), and (D)). The probability of price changes is zero in the case that the order cost is set to 1, but the higher order cost leads to a lower probability of price changes in the case that the order cost is set to above 1.5 (Table 6).

### 4.5. Price Elasticity of Demand

The price elasticity of demand affects the price dispersion, as indicated in measures such as the standard deviation, max-min difference, average change rate, and average discount rate (Figure 11). The higher elasticity leads to a lower average discount rate and average change rate of price (Table 6). Meanwhile, the price elasticity of demand does not affect the frequency of discount sales and price change.

### 4.6. Discussion

In this subsection, we compare and examine the results of the empirical analysis in section 2 and the numerical experiments on the model of a distributive firm.

First, the empirical results indicate that price movements in discount sales are important causes of price changes. In our model, movements of preices are accompanied with the enodogenous discount sales. So the model behaviors close to the empirical situations.

Second, the empirical results show that the probability of price changes has a positive correlation to capacities of goods storage in a retail store. The retailer, which has a flexible capacity of goods but a low marginal cost for storing inventory, purchases inventory frequently and sells much of it every period without much account for the menu cost. Therefore, the positive relation between the coefficient of variation for the quantities sold and probability of price change may come from the inventory holding cost structure.

Third, the empirical results also suggest that the probability of price change has negative correlation with the average price level. Likewise, the experimental results indicate that a rise in the menu cost leads to an increase in the average price level and a decrease in the frequency of price changes. We find the same relationship when the order cost arises over 0.01. Thus, the negative correlation between probability of price changes and average price level may be explained by the retailer's menu cost and order cost.

Fourth, the empirical results show that the average markdown rate has a negative correlation with the price elasticity of demand. In other words, the retailer who has larger monopolistic power in its sales market intends to cut prices more on the discount sales days. It is consistet with the implication of the numerical experiments for the price elastisity of demand varyig.

Finally, the empirical results show that the frequency of discount sales has no correlation to average markdown rate. It is also consistet with the implication of the numerical experiments for the price elastisity of demand varyig. The price elasticity of demand has no effect on discount sale's frequency in our model. It only affects the markdown ratio in the discount sale.

## 5. Conclusion

This paper examines the role of inventory in the price-setting behavior of a distributive firm. This paper shows the five empirical facts using daily POS data of a storable processed food. 1) Price movements in discount sales are important causes of price changes. 2) Probability of price changes has a positive correlation to capacities of goods storage in a retail store. 3) Probability of price changes has a negative correlation to the average price level of a retail store. 4) Average markdown rate has a negative correlation with price elasticity of demand. 5) Frequency of discount sales has no correlation to average markdown rate. These findings denote that price setting behavior varies depending on the characteristics of a retailer.

This paper considers that the hidden mechanism of pricesetting comes from retailer's policy for inventory investment. We develop a partial equilibrium model of a retailer's optimization behavior with inventory investment. The distributive firm's model which generates an endogenous discount sales behavior without demand state or purchasing price movements is constructed. The model is incorporated with monopolistic power in selling market, fixed order cost, power cost function of inventory holding, menu cost, and the stock out penalty cost.

We implement several numerical experiments with various parameter values, and show that the model succeeds to replicate the features of the five empirical facts for daily behaviors of retail shops in the numerical experiments.

# References

- Abe, N. and A. Tonogi, 2010. "Micro and Macro Price Dynamics in Daily Data," *Journal of Monetary Economics*, 57(6), 716-728.

- Aguirregabiria, V., 1999. "The Dynamics of Markups and Inventories in Retailing Firms," *The Review of Economic Studies*, 66(2), 275-308.

- Arrow, K. J., T. Harris, and J. Marschak, 1951. "Optimal Inventory Policy," *Econometrica*, 19(3), 250-272.

- Arrow, K. J., S. Karlin, and H. Scarf, 1958. *Studies in Mathematical Theory of Inventory and Production*, Stanford University Press.

- Bils, M. and P. J. Klenow, 2004. "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112(5), 947-985.

- Blinder, A. S., 1981. "Retail Inventory Behavior and Business Fluctuations," *Brookings Papers on Economic Activity*, 12(2), 443-520.

- Blinder, A. S., 1982. "Inventories and Sticky Prices: More on the Microfoundations of Macroeconomics," *American Economic Review*, 72(3), 334-348.

- Cecchetti, S. G., 1986. "The Frequency of Price Adjustment: A Study of the Newsstand Prices of Magazines," *Journal of Econometrics* 31(3), 255-274.

- Fabiani, S., C. Loupias, F. Martins, and R. Sabbatini, 2007. *Pricing Decisions in the Euro Area: How Firms Set Prices and Why*, Oxford University Press.

- Kahn, J. A. 1987. "Inventories and the Volatility of Production," *The American Economic Review*, 77(4), 667-679.

- Kashyap, A. K., 1995. "Sticky Prices: New Evidence from Retail Catalogs," *Quarterly Journal of Economics*, 110(1), 245-274.

- Kryvtsov, O. and V. Midrigan, 2012. "Inventories, Markups, and Real Rigidities in Menu Cost Models," *The Review of Economic Studies*, 80(1), 249-276.

- Matsuoka, T., "Price Rigidity and Market Structure: Evidence from the Japanese Scanner Data," Research Center for Price Dynamics Working Paper Series, 4.

- Nakamura, E. and J. Steinsson, 2007. "Five Facts about Prices: A Reevaluation of Menu Cost Model," *Quarterly Journal of Economics*, 123(4), 1415-1464.

- Saito, Y. and T. Watanabe, 2007. "Menu Costs and Price Change Distributions: Evidence from Japanese Scanner Data," *Research Center for Price Dynamics Working Paper Series*, 17.

# Tables and Figures

## Figure 1: Price Setting and Quantity Sold in Retailers

(A)



(B)



(C)
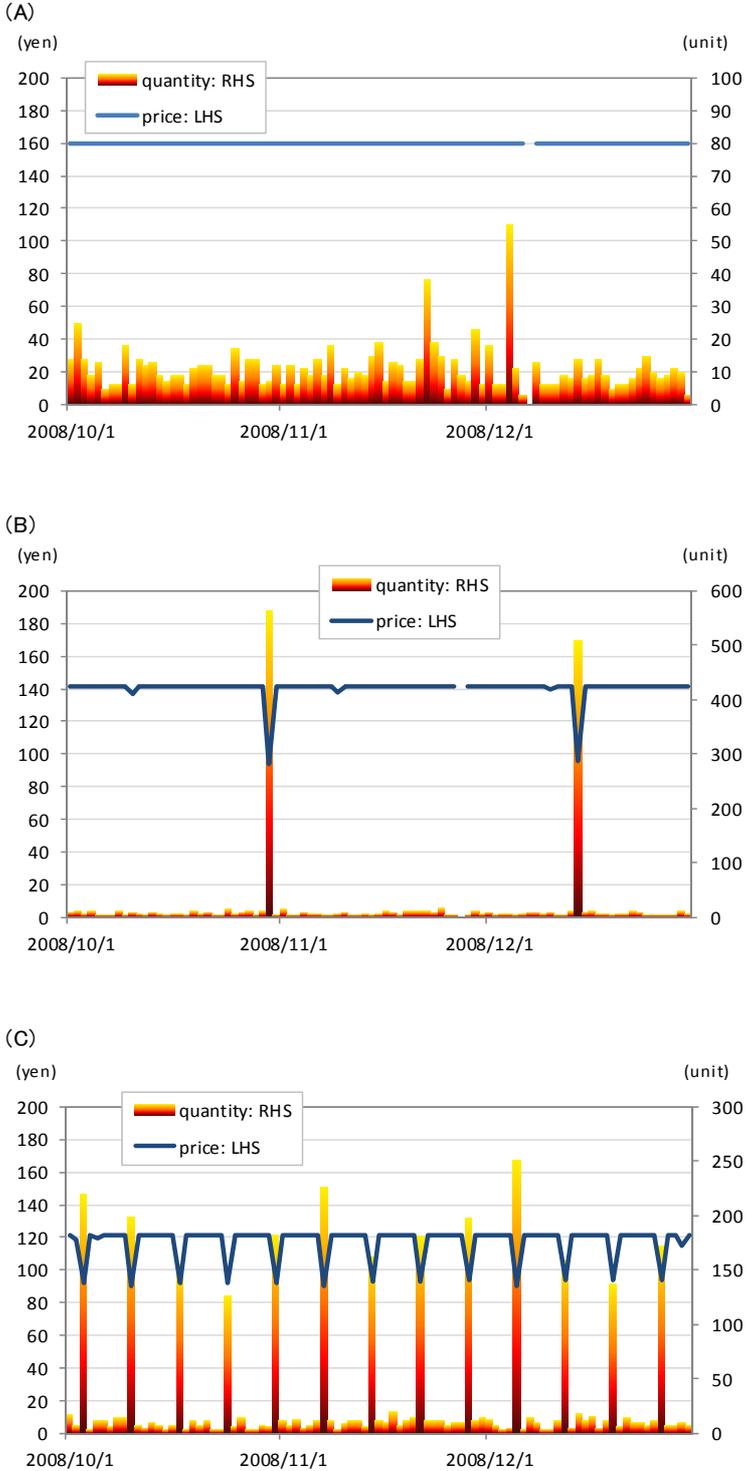
## Table 1: Summary of Statistics for Pooled Data

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| storecode | 92,598 | 609.0 | 379.9 | 1 | 1152 |
| date | 92,598 | - | 105.7 | 2008/1/1 | 2008/12/31 |
| quantity | 85,207 | 20.8 | 49.0 | 1 | 3,529 |
| price | 85,207 | 128.7 | 16.3 | 16 | 178 |
| visitor | 92,163 | 4,064.8 | 2,748.7 | 169 | 29,149 |

## Table 2: Summary of Moments for Retail Stores

| Within Moment | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| # of Days | 253 | 364 | 2 | 344 | 366 |
| Visitors/ day | 253 | 4,062 | 2,499 | 300 | 11,608 |
| Coefficient of Variance (Visitors/Day) | 253 | 0.196 | 0.074 | 0.077 | 0.426 |
| Average Price | 253 | 129.421 | 11.436 | 103.922 | 159.266 |
| Mode Price | 253 | 133.269 | 13.800 | 110.000 | 160.000 |
| Median Price | 253 | 133.269 | 13.800 | 110.000 | 160.000 |
| Standard Deviation of Price Level | 253 | 11.475 | 4.130 | 0.194 | 27.874 |
| Skewness of Price | 253 | -1.108 | 1.406 | -7.858 | 2.541 |
| Kurtosis of Price | 253 | 6.957 | 10.402 | 1.250 | 104.189 |
| Probability of Price Changes | 253 | 0.299 | 0.164 | 0.003 | 0.726 |
| Average Rate of Price Changes | 253 | 0.273 | 0.669 | -0.358 | 7.522 |
| Standard Deviation of Price Change Rate | 253 | 5.857 | 5.569 | 0.076 | 64.085 |
| Probability of Bargain Sales | 253 | 0.283 | 0.150 | 0.000 | 0.662 |
| Average Discount Rate | 252 | -14.455 | 5.584 | -31.685 | -3.770 |
| Standard Deviation of Discount Rate | 252 | 6.439 | 2.999 | 0.000 | 20.006 |
| Sales Quantity/Day | 253 | 20 | 16 | 2 | 89 |
| Sales Amount/Day | 253 | 2,270 | 1,749 | 202 | 9,853 |
| Coefficient of Variance (Sales Quantity/Day) | 253 | 1.72 | 0.95 | 0.53 | 5.72 |
| Coefficient of Variance (Sales Amount/Day) | 253 | 1.48 | 0.71 | 0.53 | 4.24 |
| Price Elastisity of Demand | 252 | 27.00 | 40.75 | 5.10 | 471.70 |

# Figure 2: Relations among Moments in Scanner Data

(A) Price Change Probability and Frequency of Discount sales

(B) Price Change Probability and Coefficient of Variation for Quantities Sold
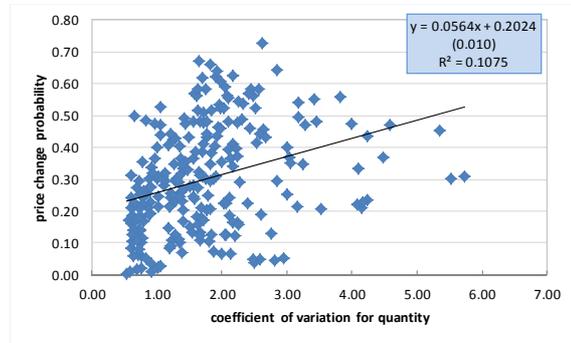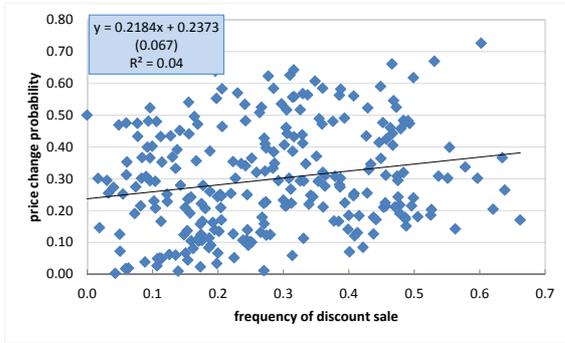


(C) Price Change Probability and Average Sales Price

(D) Average Markdown ratio in Discount Sales and Monopolistic Power of Retailers
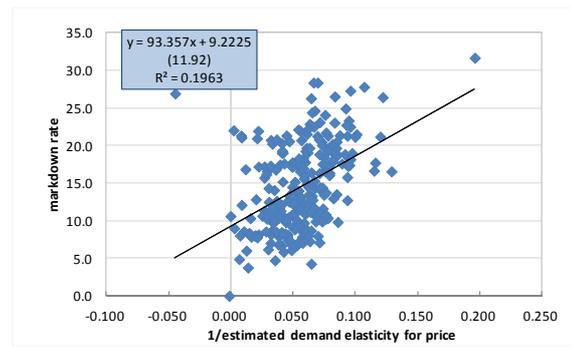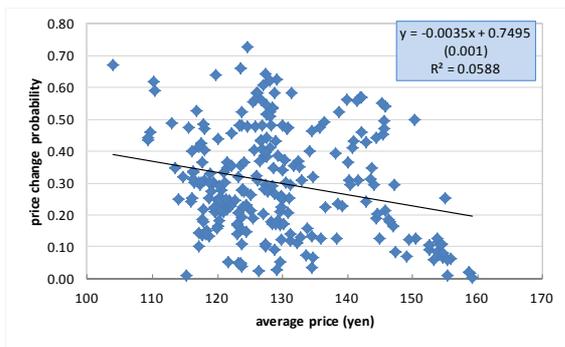


(E) Frequency of Discount Sales and Average Markdown Ratio in Discount Sales
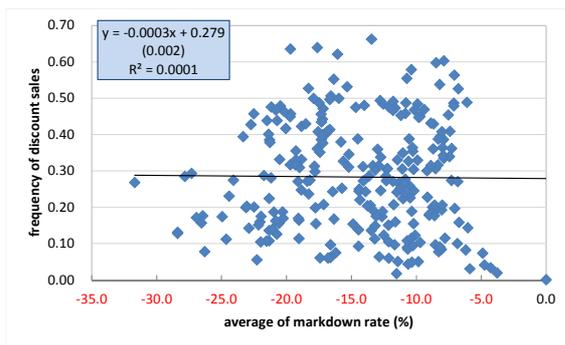


Note: Figures in parentheses are standard deviations of the coefficients.

**Table 3: Time Line of a Distributive Firm's Decision**

| period | session | activity | | indicator function |
|---|---|---|---|---|
| t-1 | | Endogenous | Exogenous | |
| | state variables (t) | $s(t)$ $f(t)$ $p(t-1)$ | $a(t)$ $q(t)$ | |
| t | selling time | selling goods in retatil market: $p(t)x(t)$<br>s.t. $x(t) \leqq s(t)$ (selling ceiling)<br>$p(t)=D[x(t)]$ (demand function)<br>$x(t) \geqq 0$ | | if $p(t) \neq p(t-1)$ then $i\_p(t)=1$<br>otherwise $i\_p(t)=0$ |
| | purchasing time | purchasing goods in wholesale market: $q(t)y(t)$<br>s.t. $q(t)y(t) \leqq f(t)$<br>if $f(t)<0$ then $y(t)=0$<br>$y(t) \geqq 0$ | | if $y(t)>0$ then $i\_o(t)=1$<br>otherwise $i\_o(t)=0$ |
| | settlement time | profit function<br>$\pi(t)=p(t)x(t)-q(t)y(t)-c\_x*x(t)-c\_s(s(t))^\phi$<br>$-c\_o*i\_o(t)-c\_p*i\_p(t)-c\_n(t)*i\_n(t)$<br>calculating state variables of next period<br>$s(t+1)=s(t)-x(t)+y(t)$<br>$f(t+1)=f(t)+\pi(t)$ | | if $s(t)=0$ then $i\_s(t)=1$<br>otherwise $i\_s(t)=0$ |
| | | Endogenous | Exogenous | |
| t+1 | state variables (t+1) | $s(t+1)$ $f(t+1)$ $p(t)$ | $a(t+1)$ $q(t+1)$ | |

**Table 4: Definitions of Variables and Parameters**

| Variables | | Parameters | |
|---|---|---|---|
| $p_t$ | Selling price | $\rho$ | Price elasticity of demand |
| $x_t$ | Selling quantity | $\phi$ | Curvature of inventory cost |
| $a_t$ | Demand state | $\delta$ | Depreciation rate of inventory |
| $q_t$ | Purchasing price | $r$ | Discount rate |
| $y_t$ | Purchasing quantity | $c^s$ | Inventory cost technology |
| $s_t$ | Inventory stock | $c^o$ | Order cost |
| $f_t$ | Operating fund | $c^x$ | Operating cost |
| $i_t^o$ | Indicator of order | $c^n$ | Stock-out cost |
| $i_t^p$ | Indicator of price change | | |
| $i_t^n$ | Indicator of stock-out | | |
| $\pi_t$ | Current profit | | |

**Table 5: Parameterization in a Basic Case**

| | Parameters | Value |
|---|---|---|
| $\rho$ | Price elasticity of demand | 18.0 |
| $\phi$ | Curvature of inventory cost | 2.00 |
| $\delta$ | Depreciation rate of inventory | 0.00 |
| $r$ | Discount rate | 0.01 |
| $c^s$ | Inventory cost technology | 0.01 |
| $c^o$ | Order cost | 2.00 |
| $c^x$ | Operating cost | 0.05 |
| $c^n$ | Stock-out cost | 5.00 |
| $c^p$ | Menu cost | 0.03 |

**Figure 3: Value Function on the Grid of $p_{t-1} = 1$**
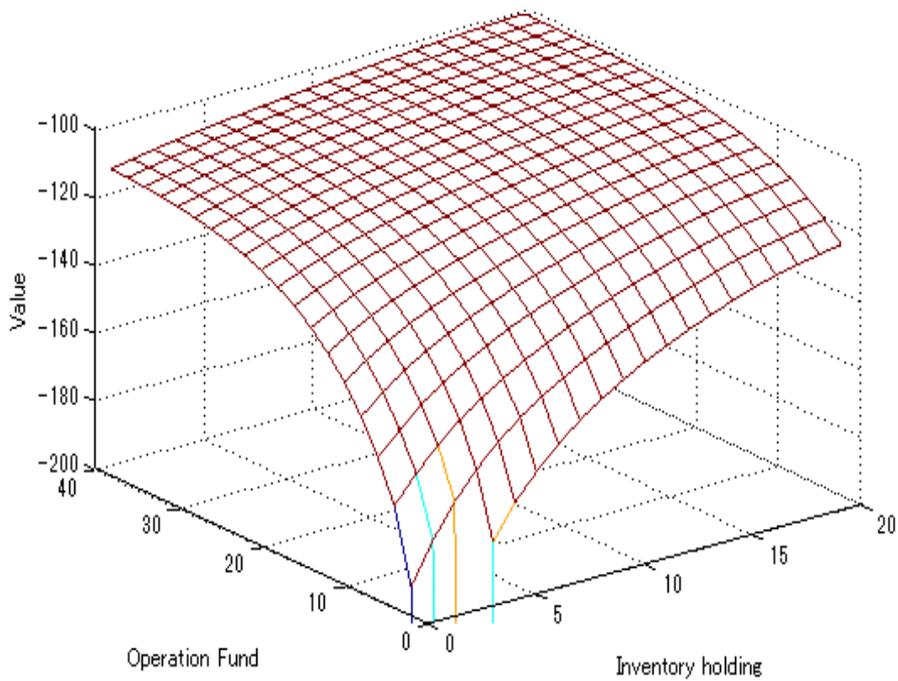
**Figure 4: Policy Function of Selling Quantity on the Grid of $p_{t-1} = 1$**
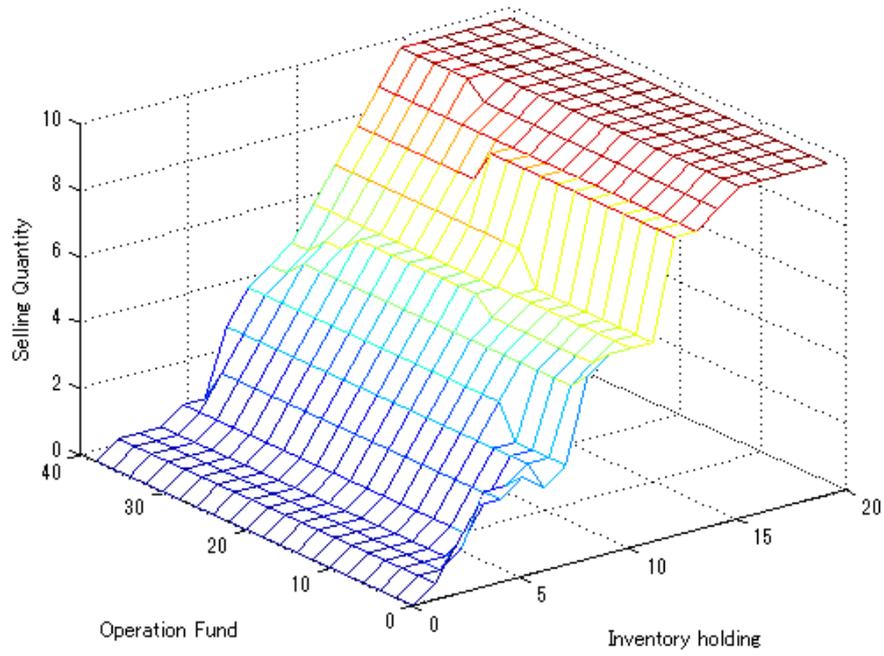


**Figure 5: Policy Function of Purchasing Quantity on the Grid of $p_{t-1} = 1$**
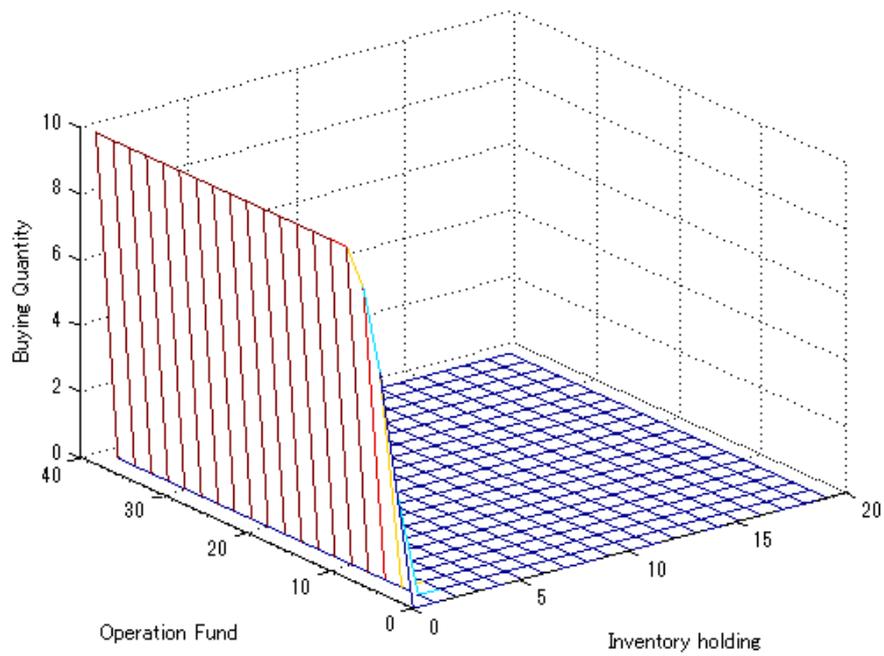
**Figure 6: Deterministic Simulation in a Benchmark Case: Price and Quantity**
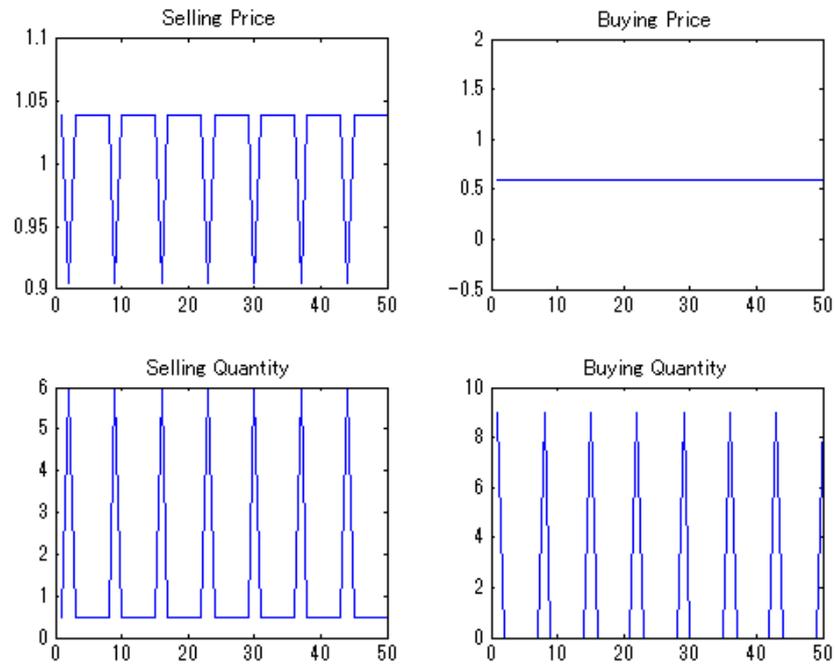


**Figure 7: Deterministic Simulation in a Benchmark Case: State and Profit**
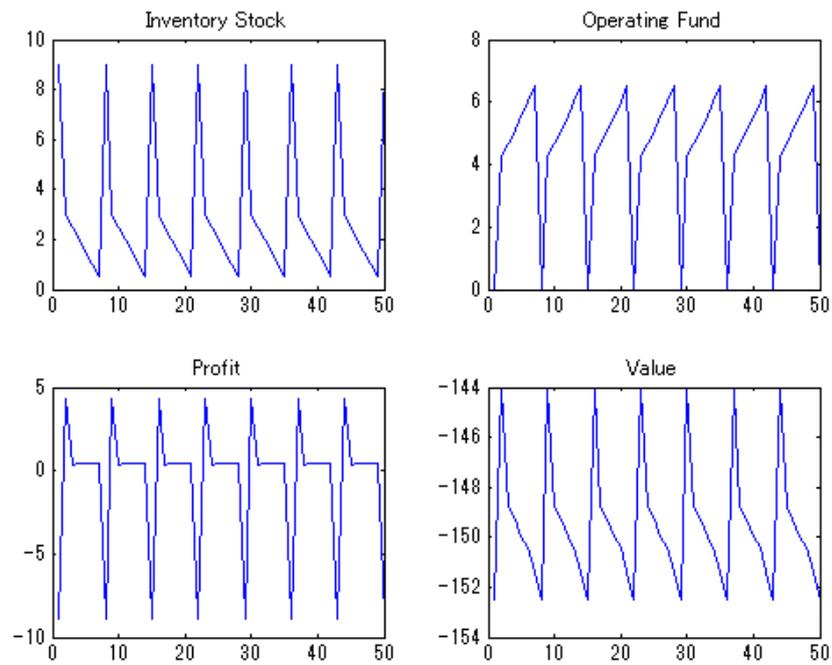
**Table 6: Moments for Price Dynamics and Distributive Firm Characteristics**

| elasticity of demand | order cost | storing cost | menu cost | selling price | | | | | selling quantity | | | | | price change | | bargain sale | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho$ | $c^o$ | $c^s$ | $c^p$ | average | SD | max | min | mode | average | SD | max | min | mode | frequency | average rate | frequency | average rate of discount |
| | | | 0.00 | 1.00 | 0.05 | 1.04 | 0.91 | 1.04 | 1.67 | 2.01 | 6.00 | 0.50 | 0.50 | 0.50 | 9.20 | 0.33 | 10.75 |
| | | | 0.01 | 1.01 | 0.05 | 1.04 | 0.91 | 1.04 | 1.43 | 1.88 | 6.00 | 0.50 | 0.50 | 0.43 | 9.21 | 0.43 | 7.17 |
| 18 | 2.00 | 0.010 | 0.03 | 1.02 | 0.05 | 1.04 | 0.91 | 1.04 | 1.28 | 1.92 | 6.00 | 0.50 | 0.50 | 0.29 | 13.81 | 0.14 | 13.81 |
| | | | 0.06 | 1.02 | 0.05 | 1.04 | 0.91 | 1.04 | 1.28 | 1.92 | 6.00 | 0.50 | 0.50 | 0.29 | 13.81 | 0.14 | 13.81 |
| | | | 0.10 | 1.02 | 0.04 | 1.04 | 0.91 | 1.04 | 1.11 | 1.73 | 6.00 | 0.50 | 0.50 | 0.22 | 13.81 | 0.11 | 13.81 |
| | | 0.000 | | 0.88 | 0.00 | 0.89 | 0.88 | 0.89 | 9.50 | 0.50 | 10.00 | 9.00 | 10.00 | 0.50 | 0.59 | 0.00 | NaN |
| | | 0.005 | | 0.92 | 0.03 | 0.94 | 0.88 | 0.94 | 5.33 | 3.30 | 10.00 | 3.00 | 3.00 | 0.67 | 6.69 | 0.33 | 6.69 |
| 18 | 2.00 | 0.010 | 0.03 | 1.02 | 0.05 | 1.04 | 0.91 | 1.04 | 1.28 | 1.92 | 6.00 | 0.50 | 0.50 | 0.29 | 13.81 | 0.14 | 13.81 |
| | | 0.015 | | 1.02 | 0.05 | 1.04 | 0.91 | 1.04 | 1.28 | 1.92 | 6.00 | 0.50 | 0.50 | 0.29 | 13.81 | 0.14 | 13.81 |
| | | 0.020 | | 1.01 | 0.05 | 1.04 | 0.91 | 1.04 | 1.33 | 1.65 | 5.00 | 0.50 | 0.50 | 0.50 | 8.53 | 0.33 | 8.33 |
| | 1.00 | | | 0.89 | 0.00 | 0.89 | 0.89 | 0.89 | 9.00 | 0.00 | 9.00 | 9.00 | 9.00 | 0.00 | NaN | 0.00 | NaN |
| | 1.50 | | | 1.00 | 0.05 | 1.04 | 0.90 | 1.04 | 2.00 | 2.51 | 7.00 | 0.50 | 1.00 | 0.60 | 9.77 | 0.60 | 7.45 |
| 18 | 2.00 | 0.010 | 0.03 | 1.02 | 0.05 | 1.04 | 0.91 | 1.04 | 1.28 | 1.92 | 6.00 | 0.50 | 0.50 | 0.29 | 13.81 | 0.14 | 13.81 |
| | 2.50 | | | 1.02 | 0.04 | 1.04 | 0.91 | 1.04 | 1.11 | 1.73 | 6.00 | 0.50 | 0.50 | 0.22 | 13.81 | 0.11 | 13.81 |
| | 3.00 | | | 1.02 | 0.04 | 1.04 | 0.91 | 1.04 | 1.11 | 1.73 | 6.00 | 0.50 | 0.50 | 0.22 | 13.81 | 0.11 | 13.81 |
| 14 | | | | 1.03 | 0.06 | 1.05 | 0.88 | 1.05 | 1.29 | 1.93 | 6.00 | 0.50 | 0.50 | 0.29 | 17.75 | 0.14 | 17.75 |
| 18 | 2.00 | 0.010 | 0.03 | 1.02 | 0.05 | 1.04 | 0.91 | 1.04 | 1.28 | 1.92 | 6.00 | 0.50 | 0.50 | 0.29 | 13.81 | 0.14 | 13.81 |
| 26 | | | | 1.01 | 0.04 | 1.03 | 0.92 | 1.03 | 1.57 | 2.62 | 8.00 | 0.50 | 0.50 | 0.29 | 10.66 | 0.14 | 10.66 |

Note: The discount sale frequency is calculated as follows: we indicate 1 when the difference between mode price and sold price is more than 2 and indicate 0 otherwise. The average of these scores is the frequency of discount sales. The price change frequency is calculated as follows: we indicate 1 when $p_{(t-1)} \neq p_t$ and indicate 0 otherwise. The average of these scores is the frequency of price changes.

## Figure 8: Numerical Experiments for the Menu Costs

(A) $c^p = 0.00$



(B) $c^p = 0.01$



(C) $c^p = 0.03$ (Benchmark Case)



(D) $c^p = 0.10$
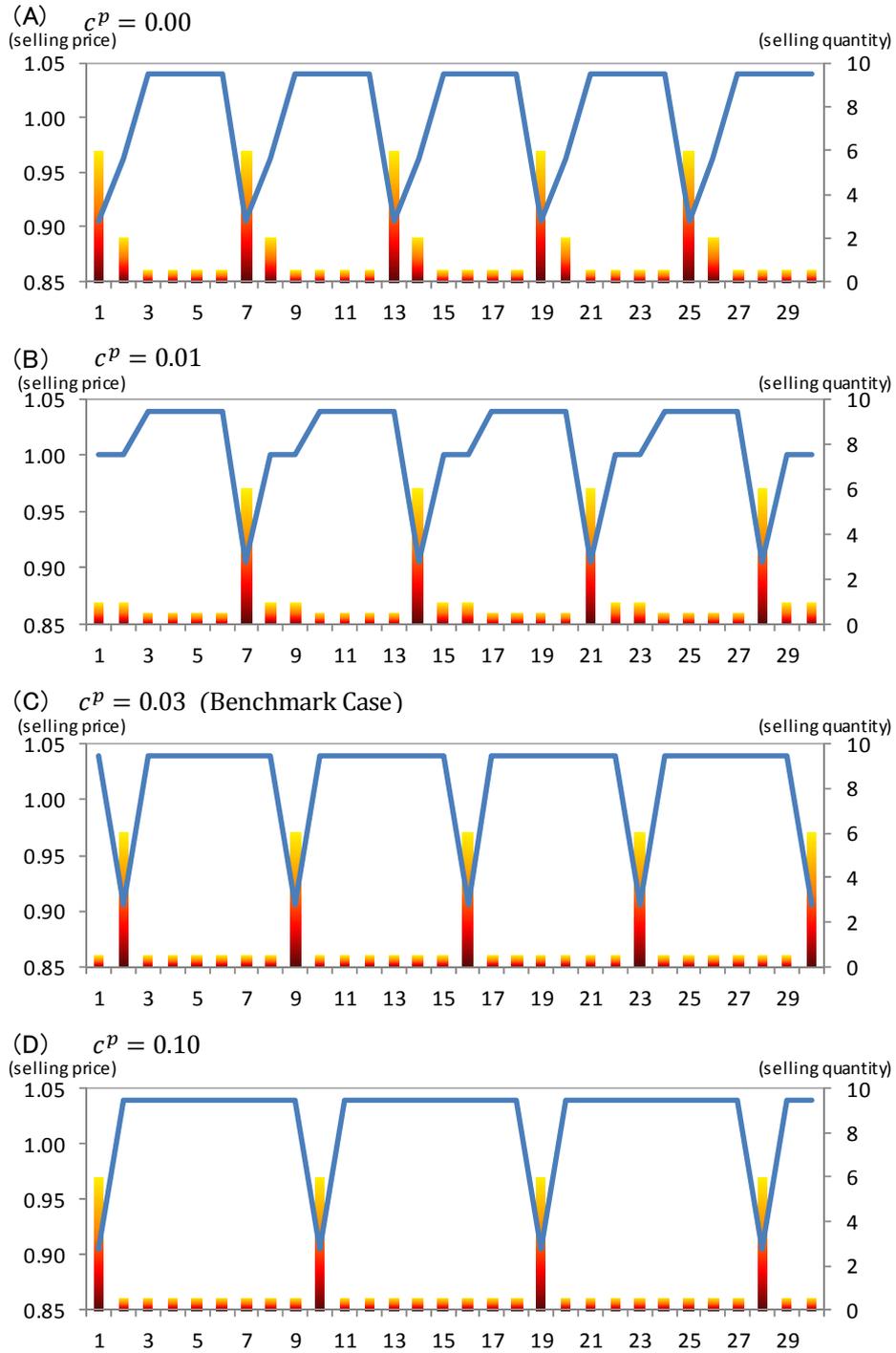


Note: line denotes selling price (LHS) and bar denotes selling quantity (RHS)

# Figure 9: Numerical Experiments for the Storage Costs

（A）　$c^s = 0.00$



（B）　$c^s = 0.005$



（C）　$c^s = 0.010$　(Benchmark Case)



（D）　$c^s = 0.020$
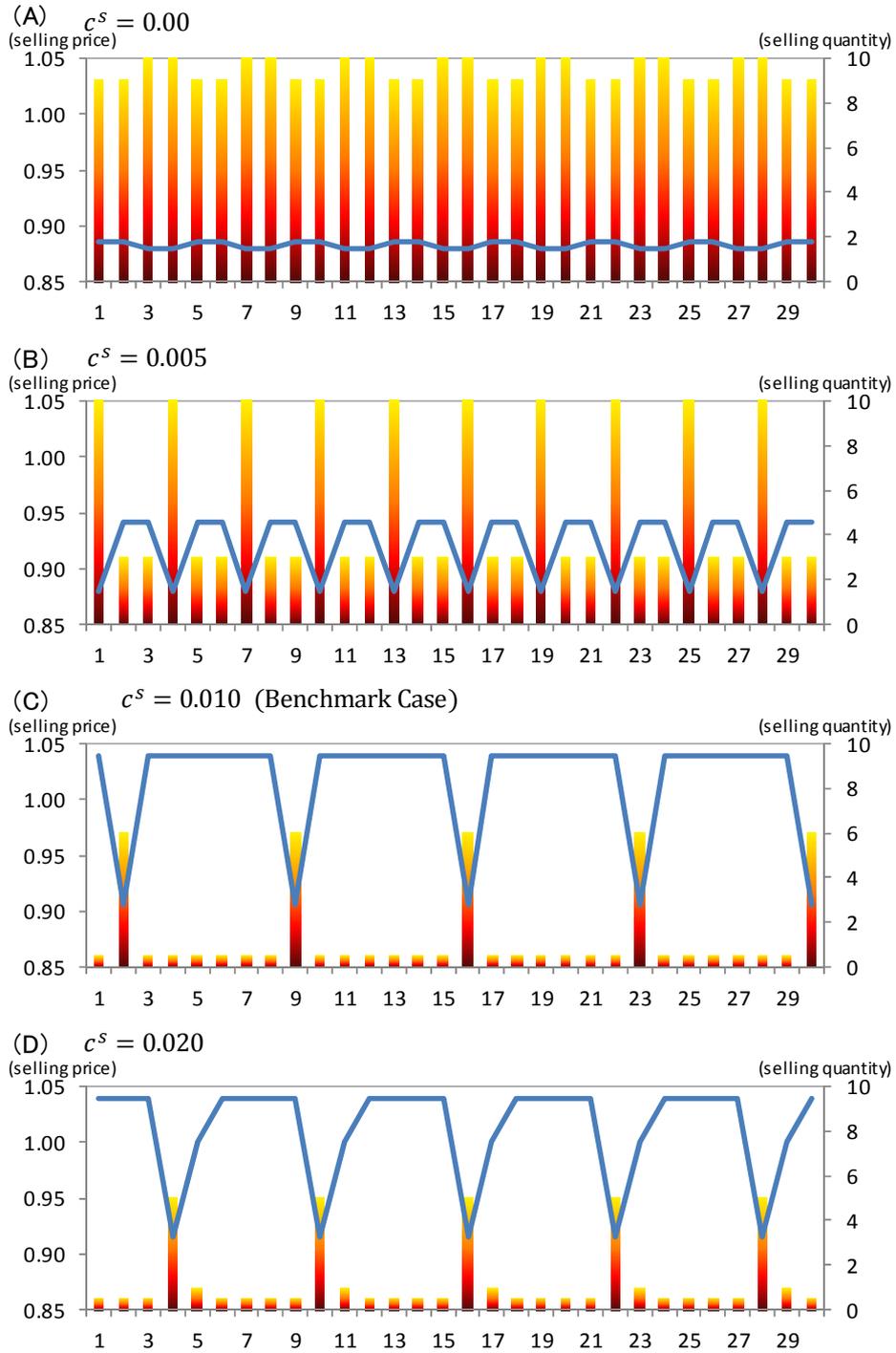


Note: line denotes selling price（LHS）and bar denotes selling quantity（RHS）

# Figure 10: Numerical Experiments for the Order Costs

（A）　$c^o = 1.00$

(selling price)　　　　　　　　　　　　　　　　　　　　　　　　(selling quantity)



（B）　$c^o = 1.50$

(selling price)　　　　　　　　　　　　　　　　　　　　　　　　(selling quantity)



（C）　　　$c^o = 2.00$　(Benchmark Case)

(selling price)　　　　　　　　　　　　　　　　　　　　　　　　(selling quantity)



（D）　$c^o = 3.00$
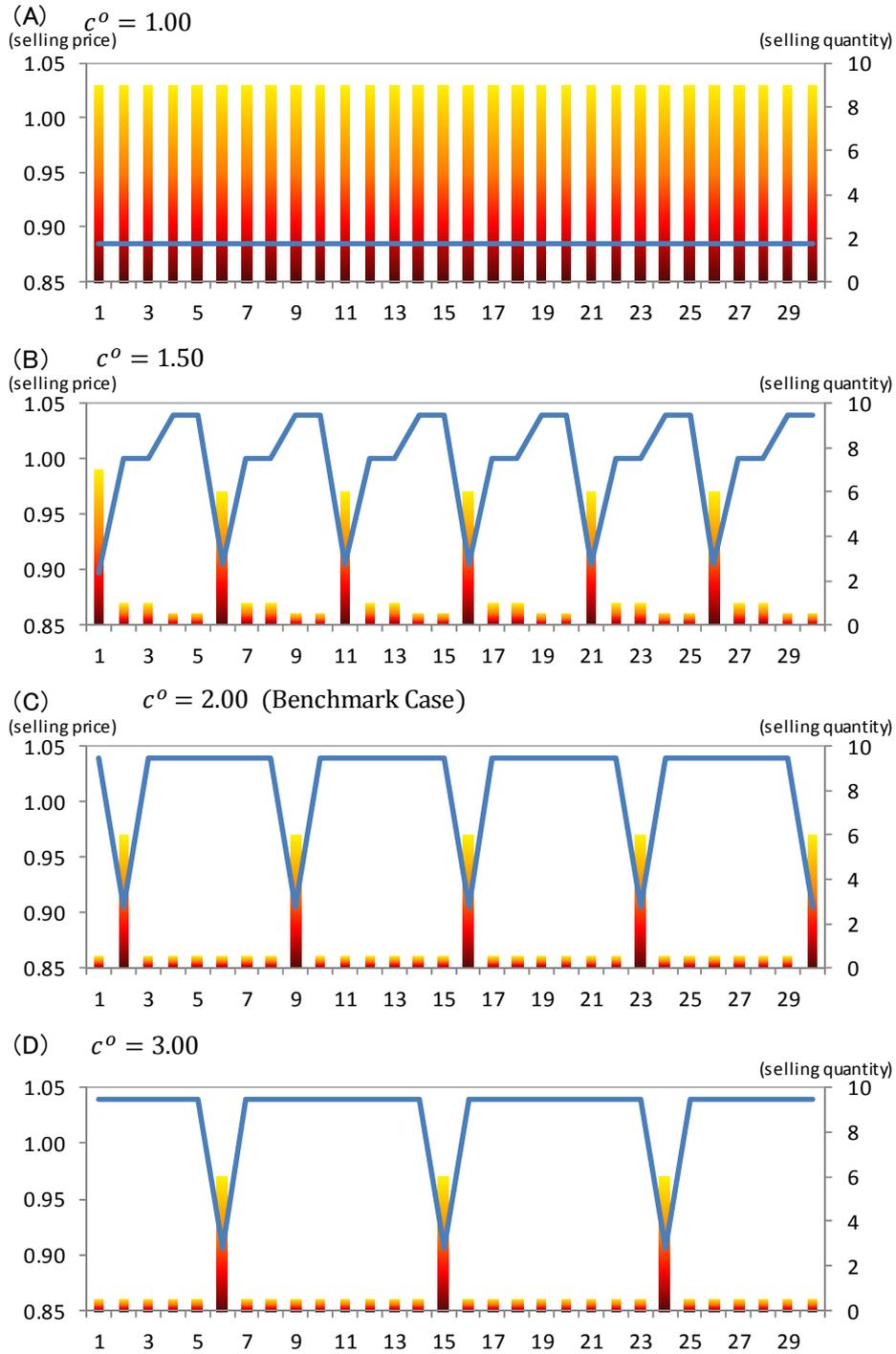
　　　　　　　　　　　　　　　　　　　　　　　　　　　　　(selling quantity)



Note: line denotes selling price（LHS）and bar denotes selling quantity（RHS）

# Figure 11: Numerical Experiments for Price Elasticity of Demand

（A）　$\rho = 14$



（B）　$\rho = 18$　（Benchmark Case）



（C）　$\rho = 26$



Note: line denotes selling price（LHS）and bar denotes selling quantity（RHS）