Term Structure Models with Negative Interest Rates

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NOTE: Views expressed in this paper are those of author and do not necessarily reflect those of the Bank of Japan or the Institute for Monetary and Economic Studies.

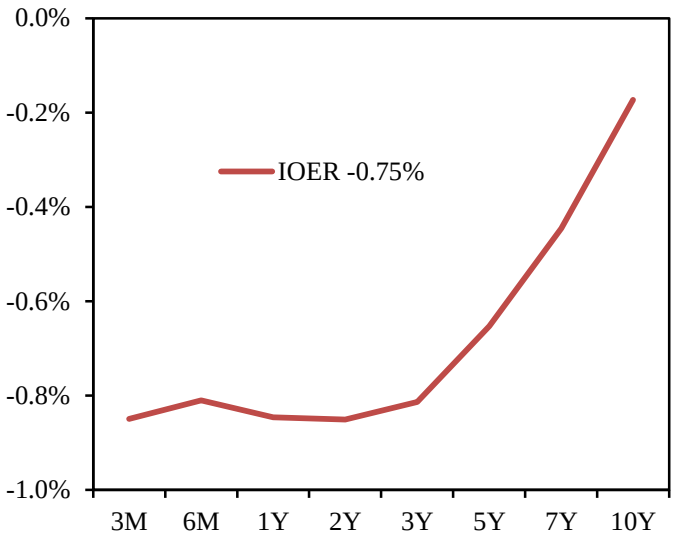
Background

- The total amount of fixed-rate sovereign debt trading at negative yields is $10.4 trillion ($7.3 trillion long term and $3.1 trillion short term) as of May 31 (Fitch, 2016).
- It had been assumed that nominal interest rates could not fall below zero as long as people could hold currency. Recent episodes, however, show that negative-yielding government bonds can coexist with currency.
- The power of arbitrage between government bonds and currency is not so strong as to forbid bonds' yields falling below zero, although it is proposed that arbitrage still works to the extent that there exists a negative limit that nominal interest rates cannot go beyond (Viñals et al. (2016), Witmer and Yang (2016)).

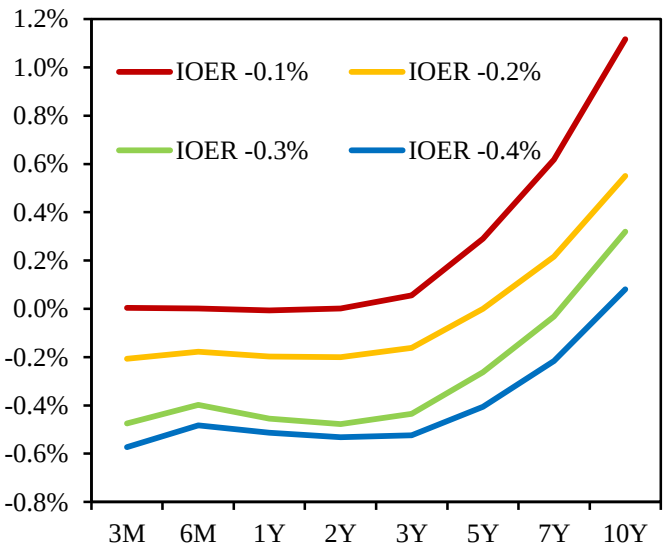
Background (cont.)

- After the introduction of the negative IOER, government bond yields not only in shorter terms but also in longer terms have fallen below zero. Furthermore, government bond yields in various terms have become more deeply negative than the IOER.

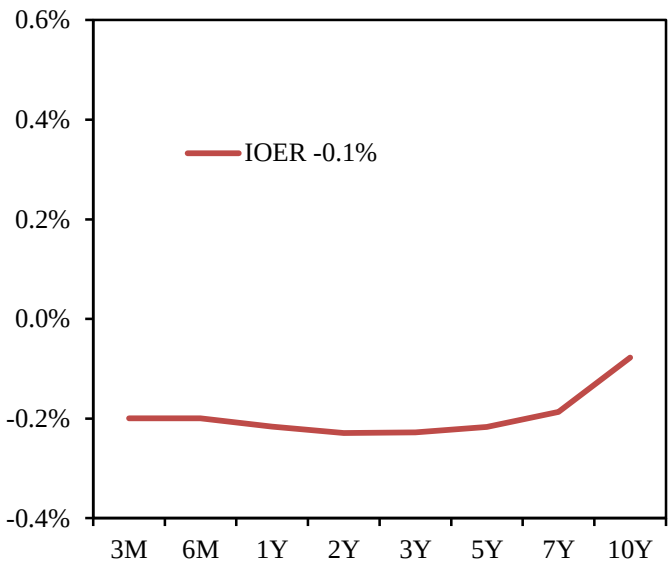
Switzerland



Germany



Japan



Background (cont.)

- The negative interest rate policy is conducted together with other unconventional monetary policy measures such as quantitative easing and/or forward guidance. This combination of unconventional policy measures causes some difficulties in evaluating the single effect of each policy measure.
- Besides, negative interest rates in nominal terms had been thought to be unreal, so theories and models to deal with negative interest rates are underdeveloped.

Contribution

- Develop a model to evaluate the effects of unconventional monetary policy measures including the negative interest rate policy on government bond term structures.
- Generalize two popular models, the Gaussian affine model and the Black model.
    - The main difference between the two popular models is how they deal with non-negativity of nominal interest rates, or the power of arbitrage between government bonds and cash.
    - Arbitrage between bonds and cash still works in the newly proposed model (Extended model).
    - But, it is not so powerful as to prohibit bond yields becoming lower than the interest rate on cash or reserves.

Contribution (cont.)

- Propose an efficient and accurate solution method able to apply to both the Black model and the Extended model.
- Show that the Extended model is superior to the Gaussian affine model and the Black model by estimation results using government bond term structure data from Switzerland, Germany and Japan.
- Quantify each effect of forward guidance, quantitative easing and the negative policy interest rate.
- Find that the power of arbitrage between money or reserves and government bonds moves in tandem with basis swap spreads.

Model

$$i_t = s_t 1_{\{s_t \geq y_t\}} + \{\varphi_t s_t + (1-\varphi_t) y_t\} 1_{\{s_t < y_t\}}$$

$$s_t = \rho x_t$$

$$dx_t = \kappa_x(\theta_x - x_t)dt + \sigma_x dW_{x,t}^Q$$

$$dy_t = \sigma_y dW_{y,t}^{\mathrm{Q}}$$

$$\varphi_t = \varphi_t^* 1_{\{0 \leq \varphi_t^* \leq 1\}} + 1_{\{1 \leq \varphi_t^*\}},$$

$$d\varphi_t^* = \sigma_{\varphi^*} dW_{\varphi^*,t}^Q.$$

$$\lambda_t = \lambda_0 + \lambda_1 x_t$$

$$dW_t^P = \lambda_t dt + dW_t^Q$$

Figure 1: Relationship between the nominal short rate and shadow rate

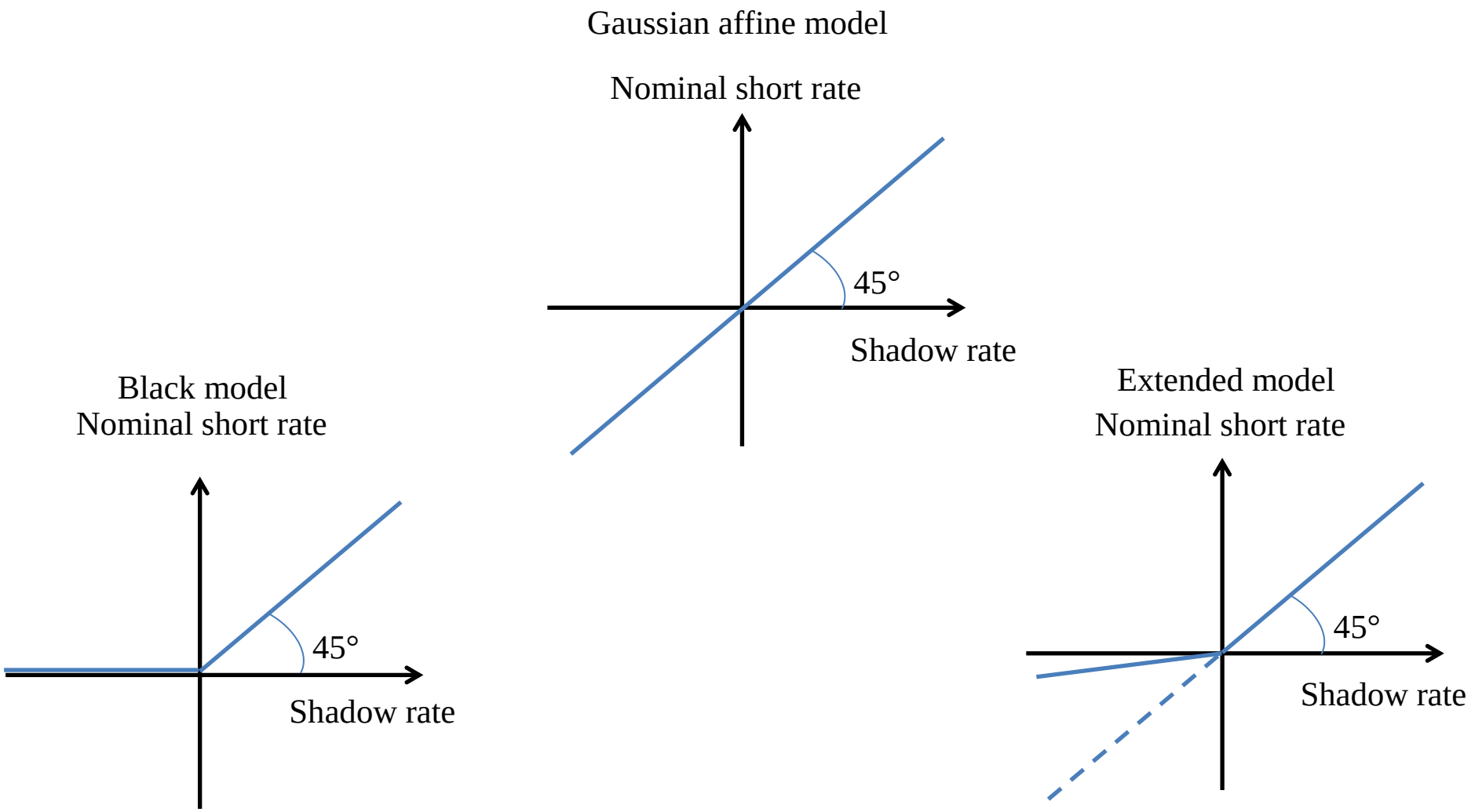
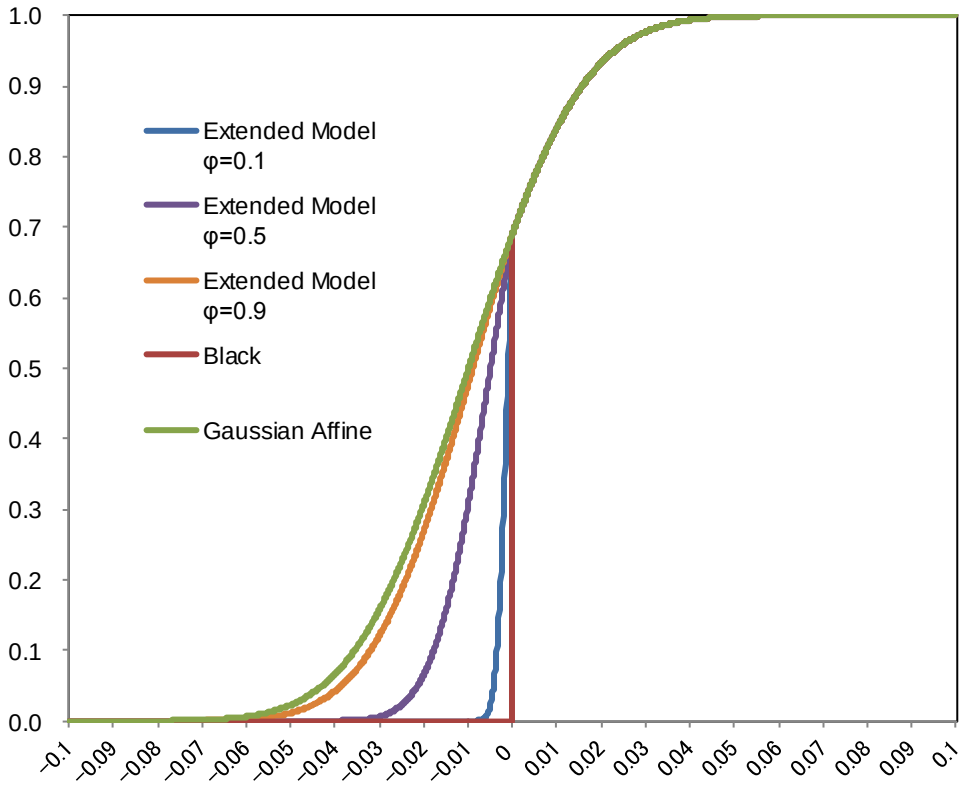


Figure 2: Cumulative probability distribution function of nominal short rate



Note: one-factor model, $s_t = 0,\ y_t = 0, E_t[s_\tau] = -0.01, Var_t[s_\tau] = 0.02^2$.

Approximation methods

- Government bond price, P_τ, and yield, R_τ, at maturity τ

$$P_\tau \equiv E^Q\left[exp\left(-\int_0^\tau i_t dt\right)\right],$$

$$R_\tau \equiv -\log(P_\tau)/\tau.$$

- Priebsch (2013): 2nd order approximation of bond yields;

$$R_\tau \approx \frac{1}{\tau}\left(E^Q\left[\int_0^\tau i_t dt\right] - 0.5Var^Q\left[\int_0^\tau i_t dt\right]\right),$$

Approximation methods (cont.)

- New approximation method

$$I_\tau \approx \widetilde{I_\tau} \equiv \alpha_0 + \alpha_1 i_{\frac{1}{4}\tau} + \alpha_2 i_{\frac{3}{4}\tau},$$

$$P_\tau \approx E^Q\left[exp\left(-\widetilde{I_\tau}\right)\right].$$

$$R_\tau \approx -\log\left(E^Q\left[exp\left(-\widetilde{I_\tau}\right)\right]\right)/\tau,$$

- The parameters α_0、α_1、α_2 are determined by minimizing the mean squared error as follows;

$$\min E^Q\left[\left(I_\tau - \widetilde{I_\tau}\right)^2\right]$$

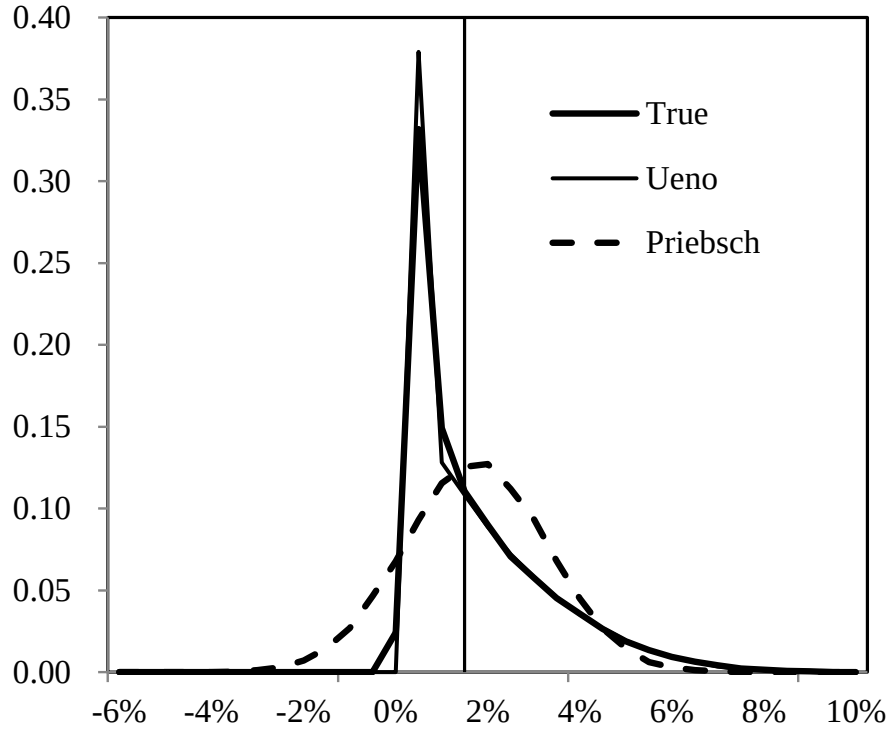
$$s.t.$$

$$E^Q[I_\tau] = E^Q\left[\widetilde{I_\tau}\right],$$

$$Var^Q[I_\tau] = Var^Q\left[\widetilde{I_\tau}\right].$$

Approximation methods (cont.)

Distribution of I_τ/τ and its approximations



Note: one-factor model, $x_0 = 0$, $y = 0$, $\theta = 0.01$、$\kappa = 0.1$、$\sigma = 0.2$、$\phi = 0$.
Maturity is 10 year. 100,000 paths are generated.

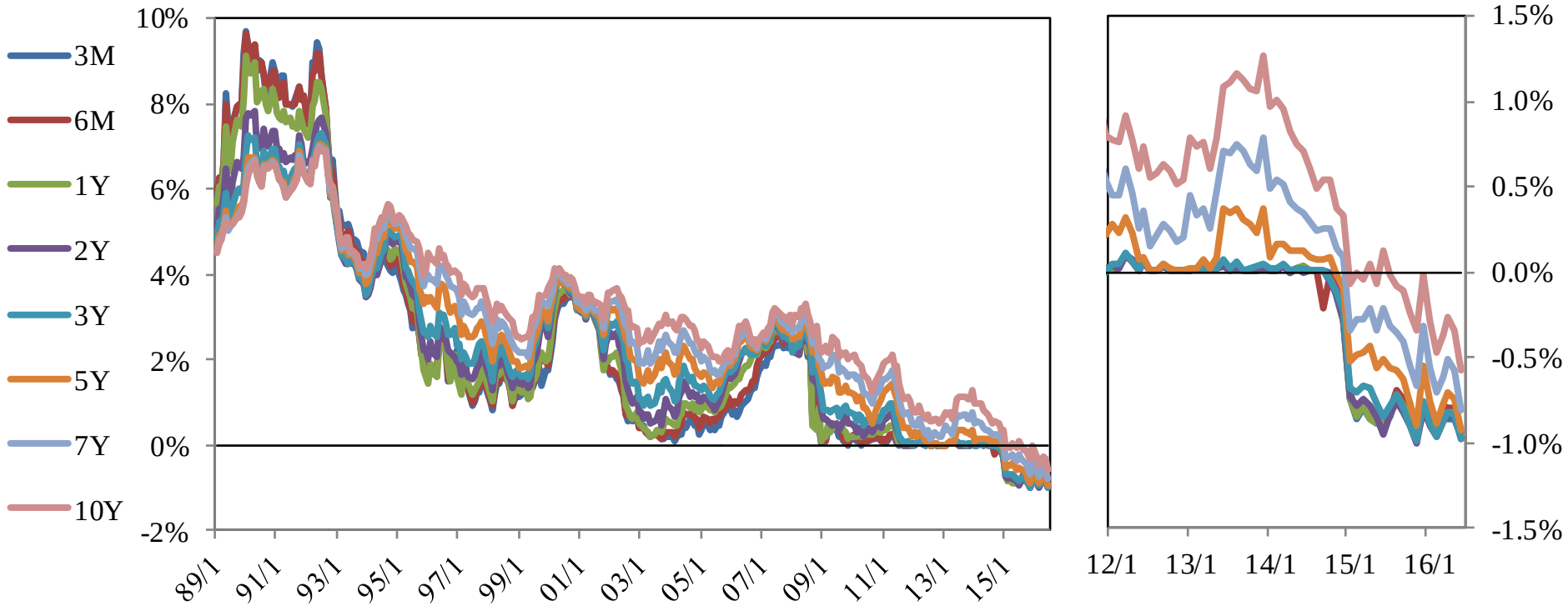
Approximation methods (cont.)

Shadow rate		Maturity 1Y	5Y	10Y	30Y
1%	Exat	0.98829	0.92449	0.84104	0.58363
	Priebsch(2013)	0.98829	0.92456	0.84178	0.59507
	Rate of deviation	0.00050%	0.00723%	0.08848%	1.95929%
	difference bps	0.050	0.145	0.884	6.468
	This paper	0.98829	0.92449	0.84101	0.58294
	Rate of deviation	0.00049%	-0.00011%	-0.00369%	-0.11872%
	difference bps	0.049	-0.002	-0.037	-0.396
0%	Exact	0.99463	0.94622	0.87124	0.61258
	Priebsch(2013)	0.99463	0.94628	0.87192	0.62391
	Rate of deviation	-0.00050%	0.00612%	0.07800%	1.84886%
	difference bps	-0.050	0.122	0.780	6.107
	This paper	0.99462	0.94622	0.87122	0.61189
	Rate of deviation	-0.00051%	0.00007%	-0.00234%	-0.11223%
	difference bps	-0.051	0.001	-0.023	-0.374

Note: one-factor model, $y = 0, \theta = 0.01$、$\kappa = 0.1$、$\sigma = 0.2$、$\phi = 0$.

Data

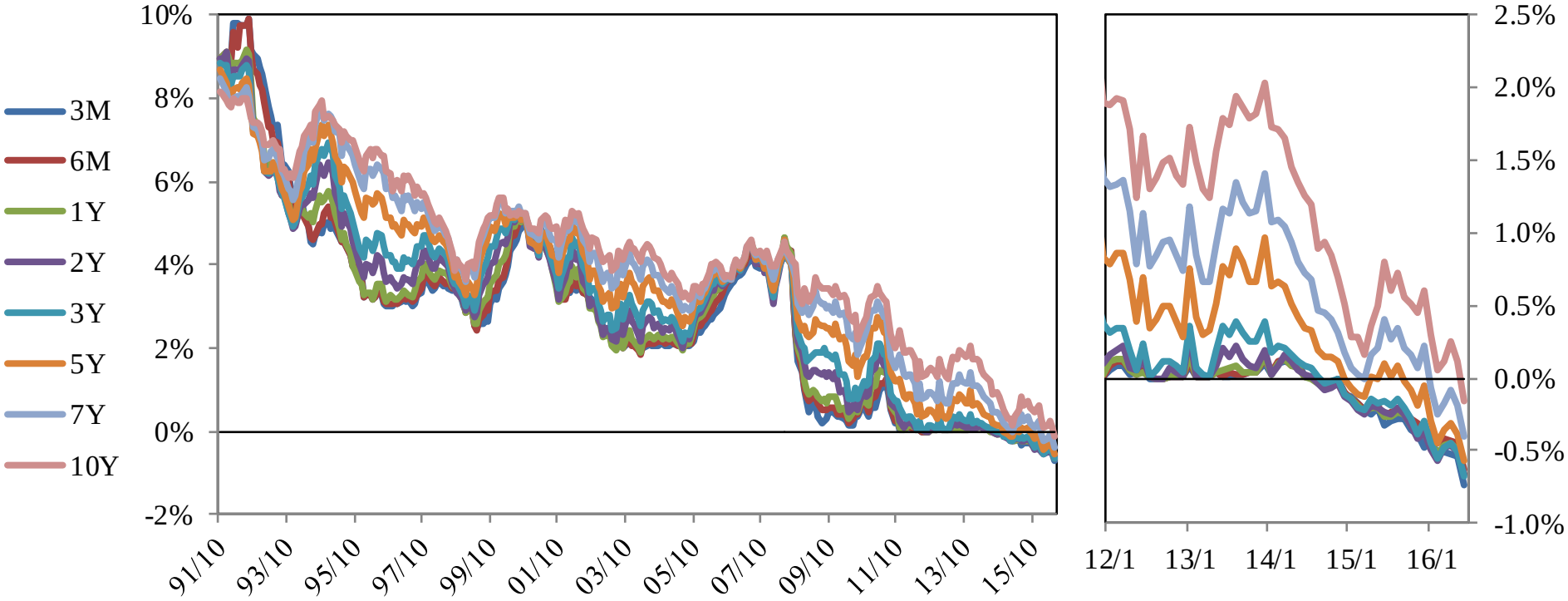
Switzerland



Source: Bloomberg

Data (cont.)

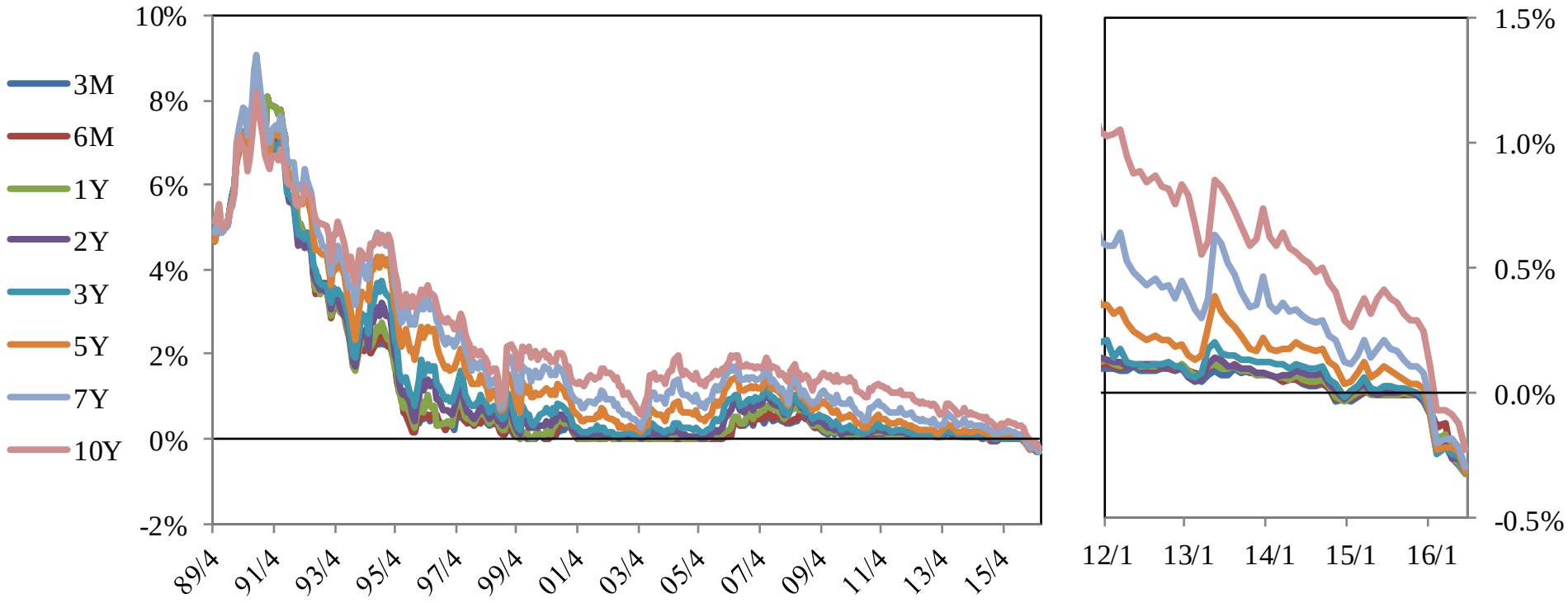
Germany



Source: Bloomberg

Data (cont.)

Japan



Source: Bloomberg

Estimation method

- Estimate four kinds of models by (quasi-) maximum likelihood estimation; the Gaussian affine model, the Black model, two versions of the Extended model.
    - The difference between the two versions is whether φ_t is constant or variable.
- Use the Single-Stage Iteration Filter (SSIF) for the Black model and Extended model and the Kalman filter for the Gaussian affine model as in Joslin et al. (2011) and others.
    - In the Black model and Extended model, the relationships between factors and bond yields are not linear, so non-linear filtering method should be used.
    - The Extended Kalman filter is used in many studies (Xia and Wu (2016) and others). Tanizaki (1996), however, shows by Monte Carlo simulations that estimation biases arise in the Extended Kalman filter if there is high non-linearity in estimated systems and propose the usage of other non-linear filtering methods including SSIF.

Estimation results: Parameters

	Switzerland				Germany				Japan			
	Gaussian affine	Black	Extended Fixed	Extended Variable	Gaussian affine	Black	Extended Fixed	Extended Variable	Gaussian affine	Black	Extended Fixed	Extended Variable
φ	1.0000	0.0000	0.0555	0.0000	1.0000	0.0000	0.0999	0.0039	1.0000	0.0000	0.0153	0.0000
σ_{φ}	0.0000	0.0000	0.0000	0.0504	0.0000	0.0000	0.0000	0.0511	0.0000	0.0000	0.0000	0.0164
Average of log likelihood	41.4711	41.5710	41.8567	41.9377	42.8840	42.7793	43.2187	43.3157	41.4112	42.4020	42.6489	42.7703
P-value (VS. Extended Fixed)	0.0000	0.0000	—	—	0.0000	0.0000	—	—	0.0000	0.0000	—	—
Pseudo P-value (VS. Extended Variable)	0.0000	0.0000	—	—	0.0000	0.0000	—	—	0.0000	0.0000	—	—
T	330	330	330	330	297	297	297	297	327	327	327	327

Estimation results: RMSE

Switzerland

bps

	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y	Average
Gaussian affine	7.86	7.44	11.33	5.77	6.88	6.27	4.48	5.75	6.97
Black	8.19	7.90	11.24	6.57	7.66	6.81	5.10	6.29	7.47
Extended Fixed	7.66	7.72	11.07	6.00	7.32	6.78	5.02	6.29	7.23
ExtendedVariable	7.64	7.37	11.15	6.00	7.08	6.37	4.54	5.85	7.00

Germany

bps

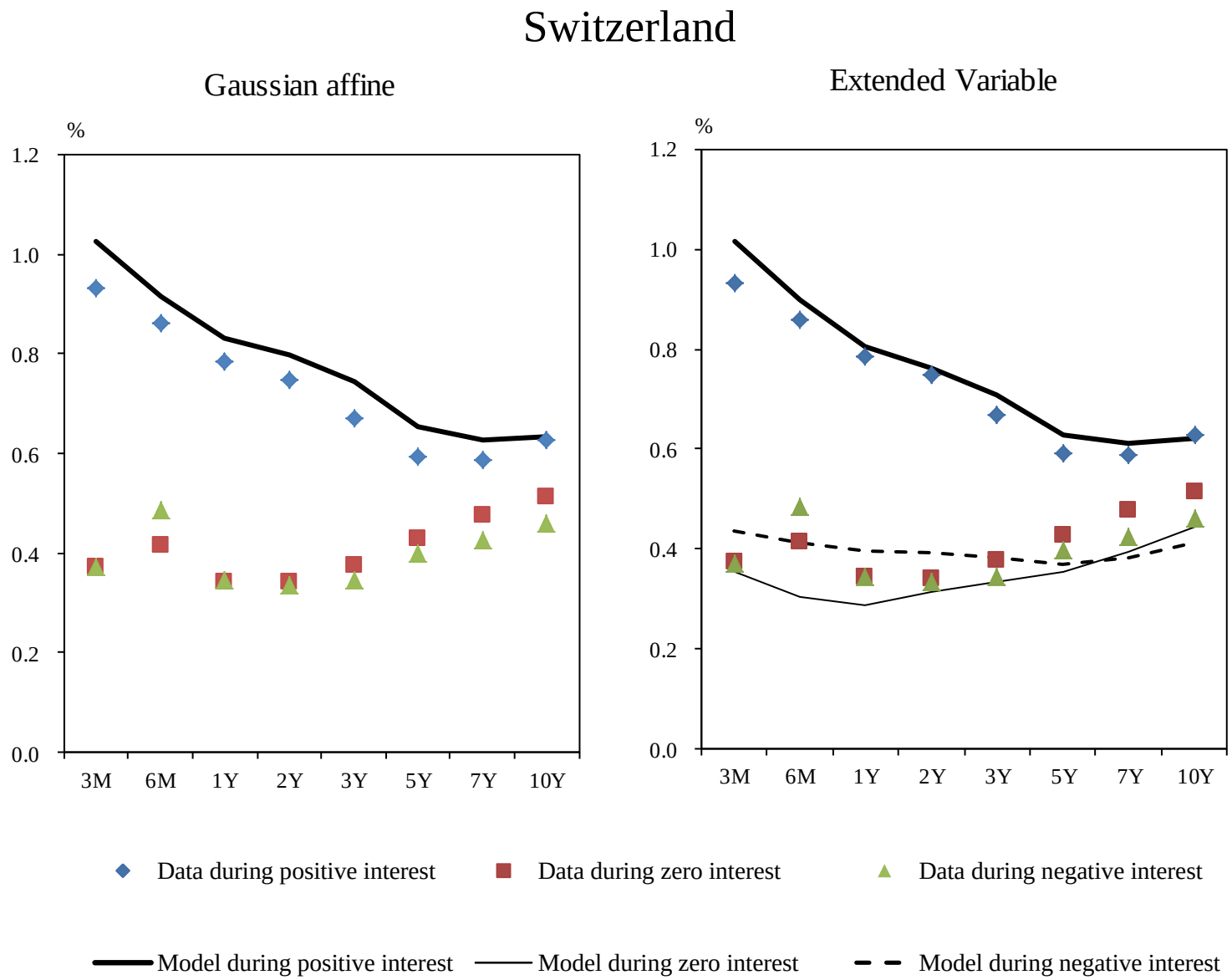
	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y	Average
Gaussian Affine	5.19	5.59	8.79	5.72	6.66	6.82	6.87	6.20	6.48
Black	8.88	8.58	10.44	6.62	6.91	6.05	5.96	5.09	7.32
Extended Fixed	4.37	5.33	8.36	4.92	5.71	6.14	6.81	5.55	5.90
ExtendedVariable	4.78	5.50	8.51	4.57	5.35	5.42	6.03	5.14	5.66

Japan

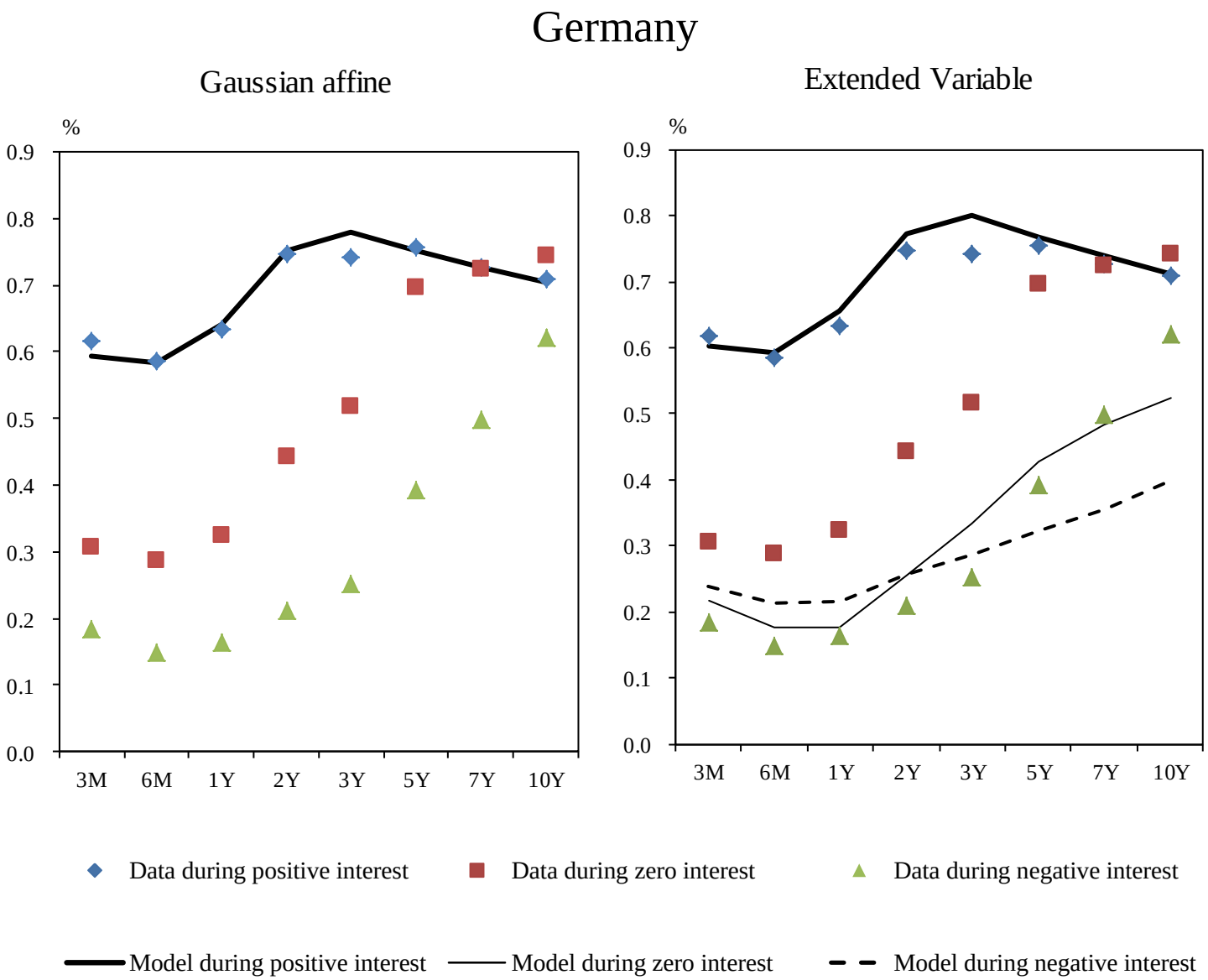
bps

	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y	Average
Gaussian Affine	3.99	3.15	7.50	6.49	6.50	7.48	12.63	10.74	7.31
Black	4.95	4.50	8.13	6.95	7.33	7.51	12.39	9.66	7.68
Extended Fixed	4.22	3.72	7.71	6.73	7.04	7.19	12.25	9.66	7.32
ExtendedVariable	3.96	3.46	7.62	6.26	6.83	7.08	12.13	9.57	7.11

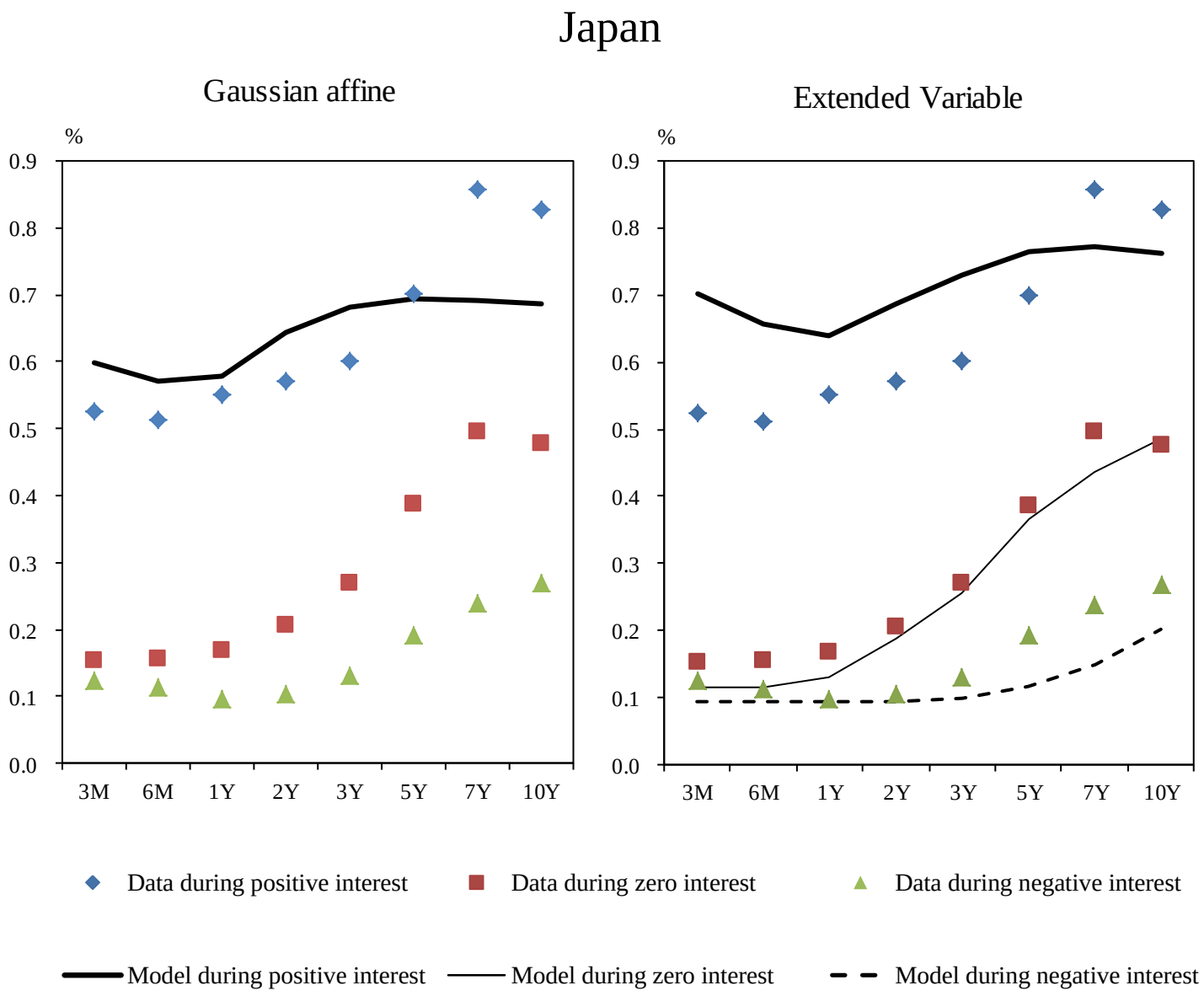
Estimation results: Volatility



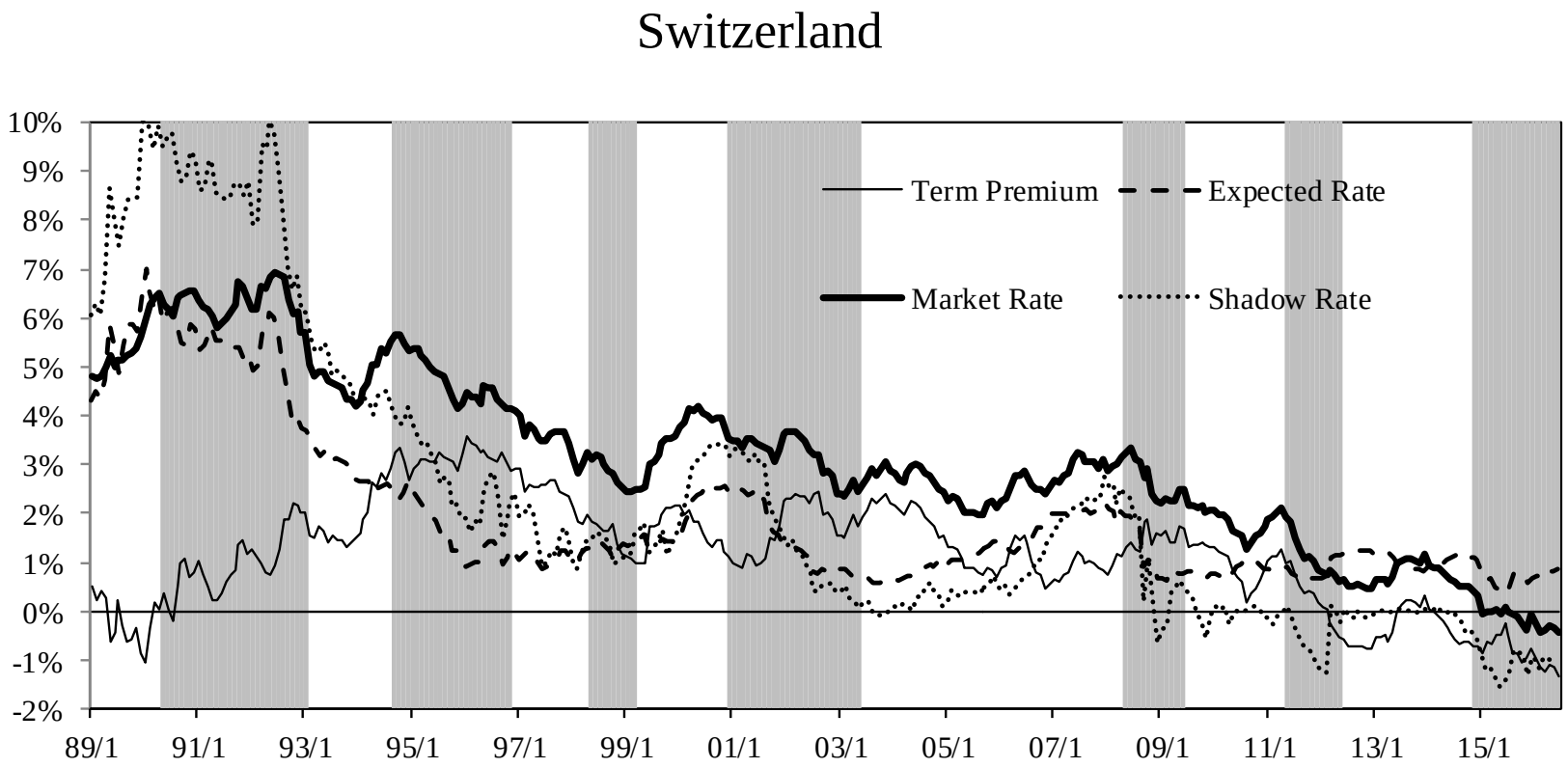
Estimation results: Volatility (cont.)



Estimation results: Volatility (cont.)

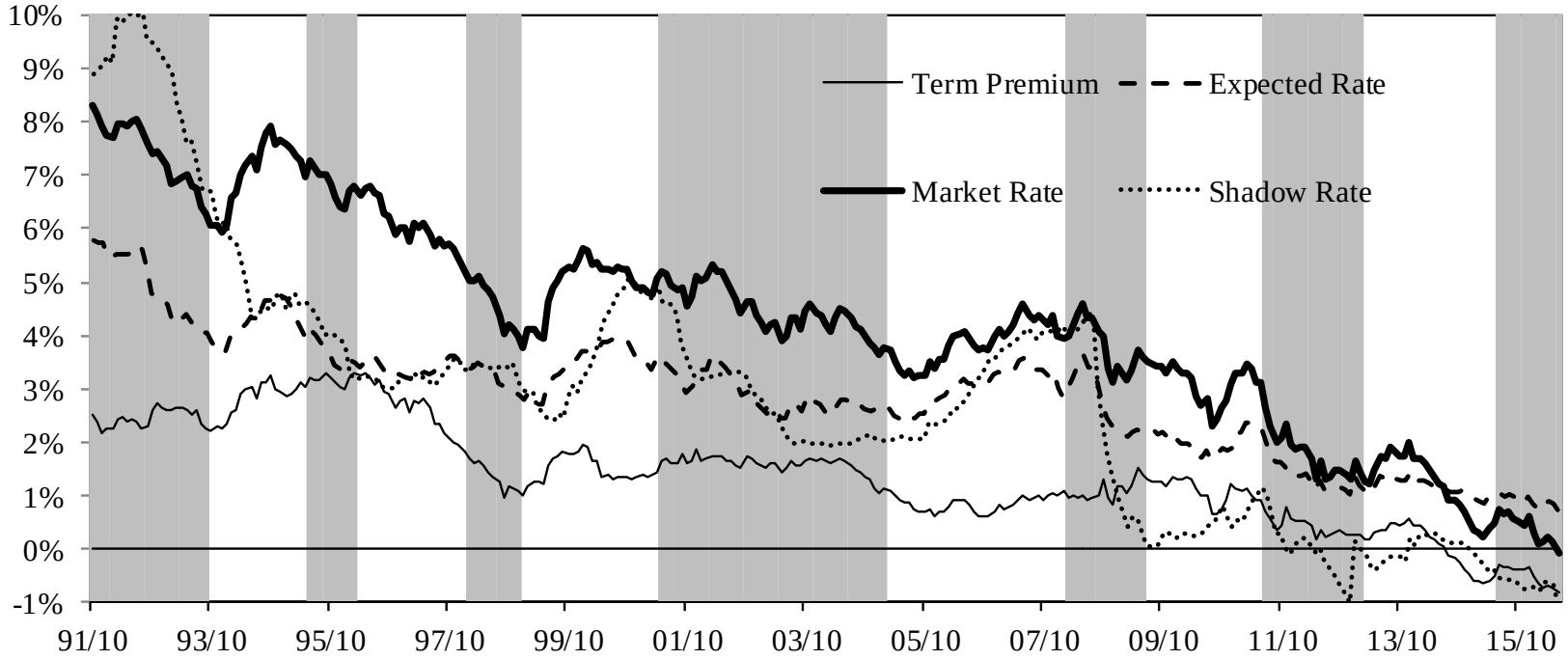


Estimation results: 10 year expected rate and term premium (cont.)



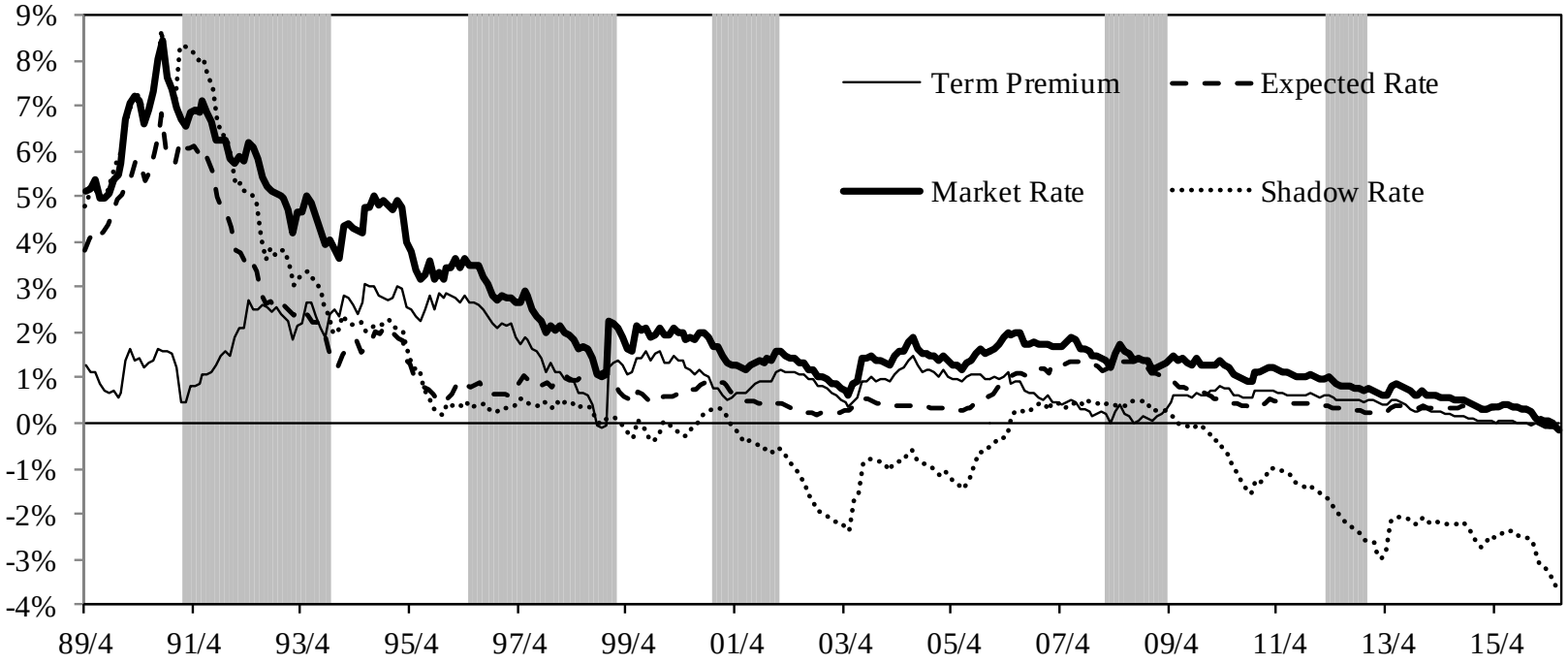
Estimation results: 10 year expected rate and term premium (cont.)

Germany



Estimation results: 10 year expected rate and term premium (cont.)

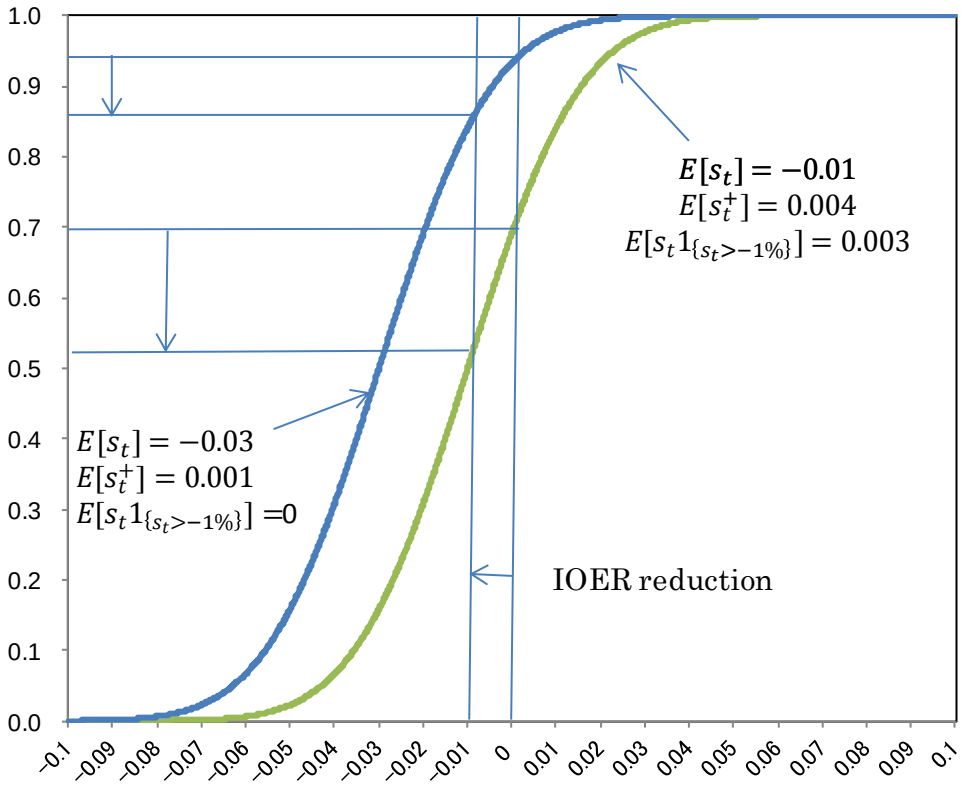
Japan



Sensitivity of yield curves to the IOER

- Simple Example: $E[i_t] = \mathrm{E}\left[s_t 1_{\{s_t \geq y\}}\right] + y \times P(s_t < y)$

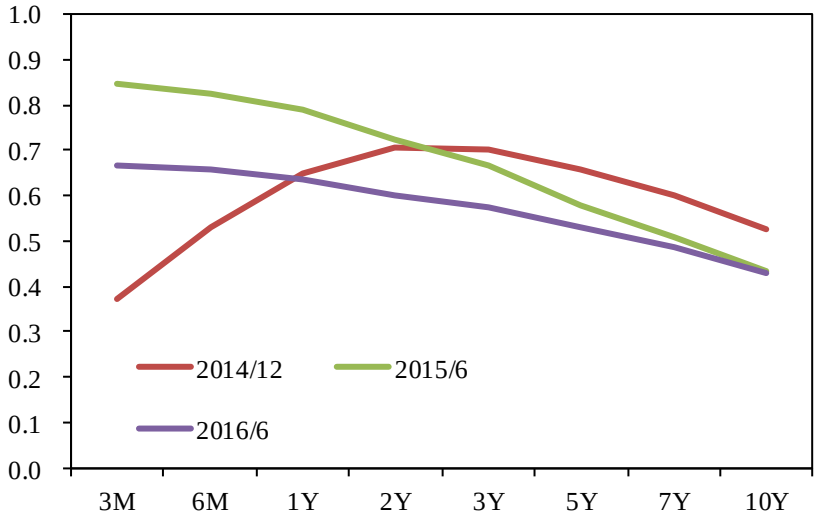
$P(s_t < y)$



$E[i_t]$	$y = 0\%$	$y = -1\%$	Diff.
$\mathrm{E}[s_t] = -1\%$	0.40%	-0.20%	-0.60%P
$\mathrm{E}[s_t] = -3\%$	0.10%	-0.85%	-0.95%P

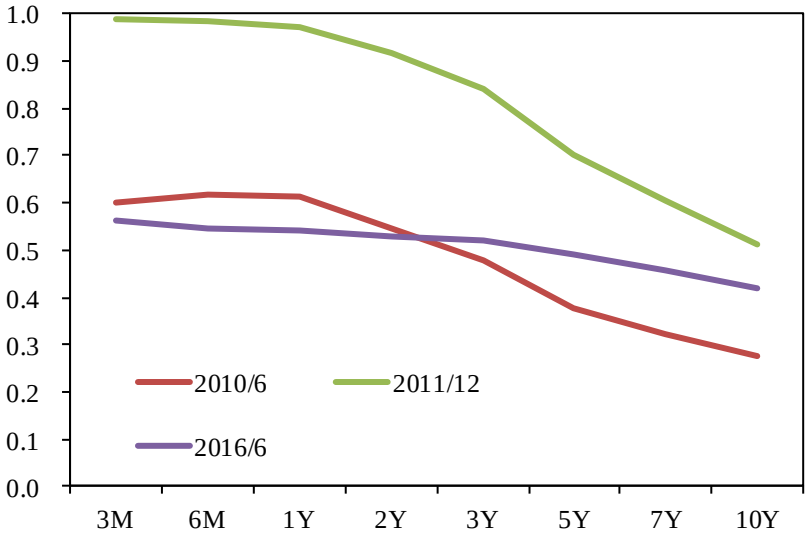
Sensitivity of yield curves to the negative interest rate policy (cont.)

Switzerland



	$s(x_t)$	φ_t
2014/12	0.0%	0.03
2015/6	-0.5%	0.12
2016/6	-1.0%	0.33

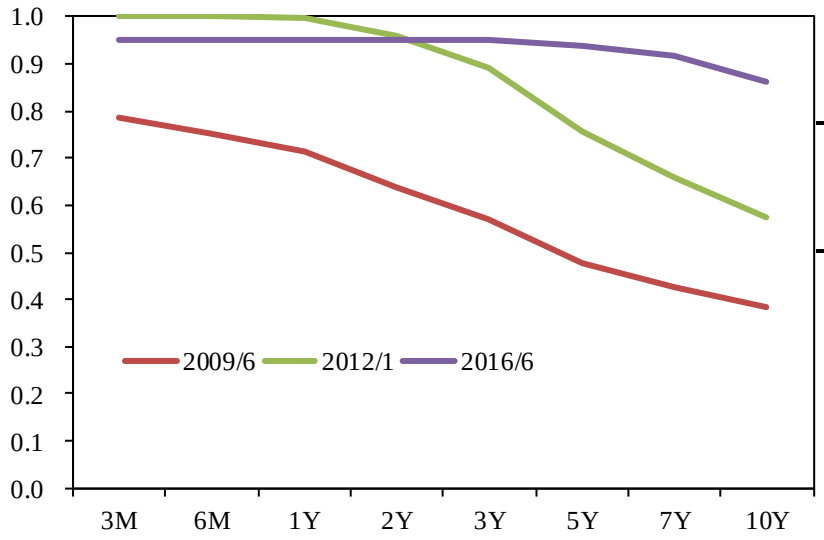
Germany



	$s(x_t)$	φ_t
2010/6	0.0%	0.00
2011/12	-1.1%	0.00
2016/6	-1.0%	0.32

Sensitivity of yield curves to the negative interest rate policy (cont.)

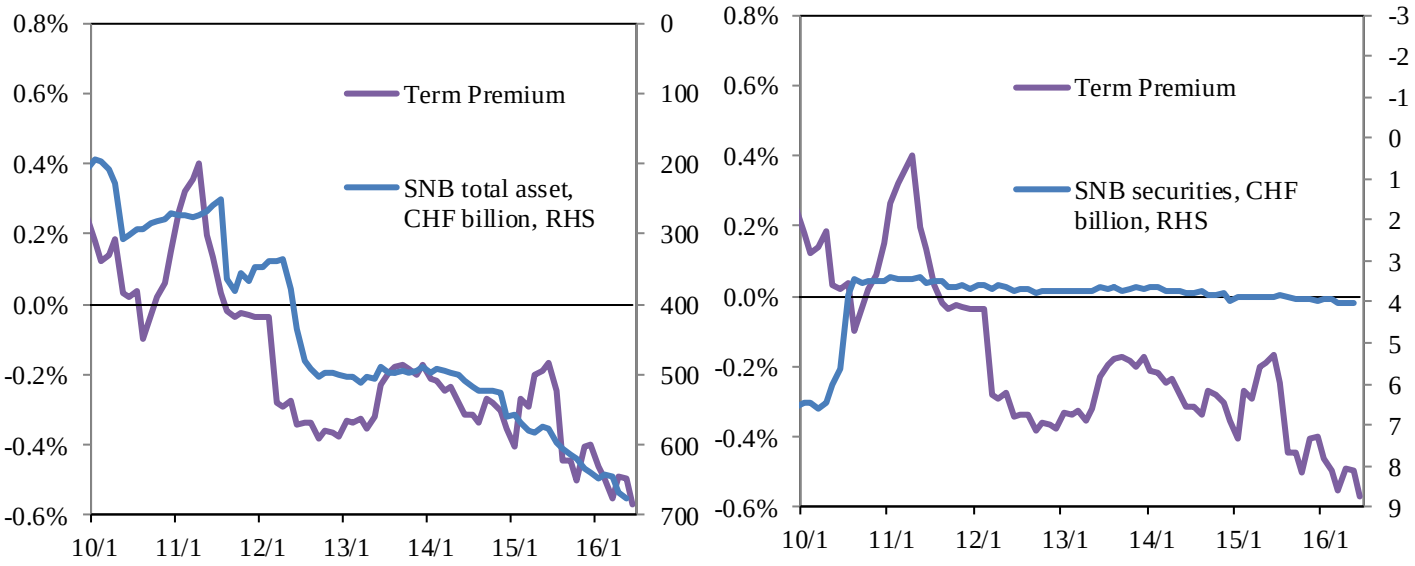
Japan



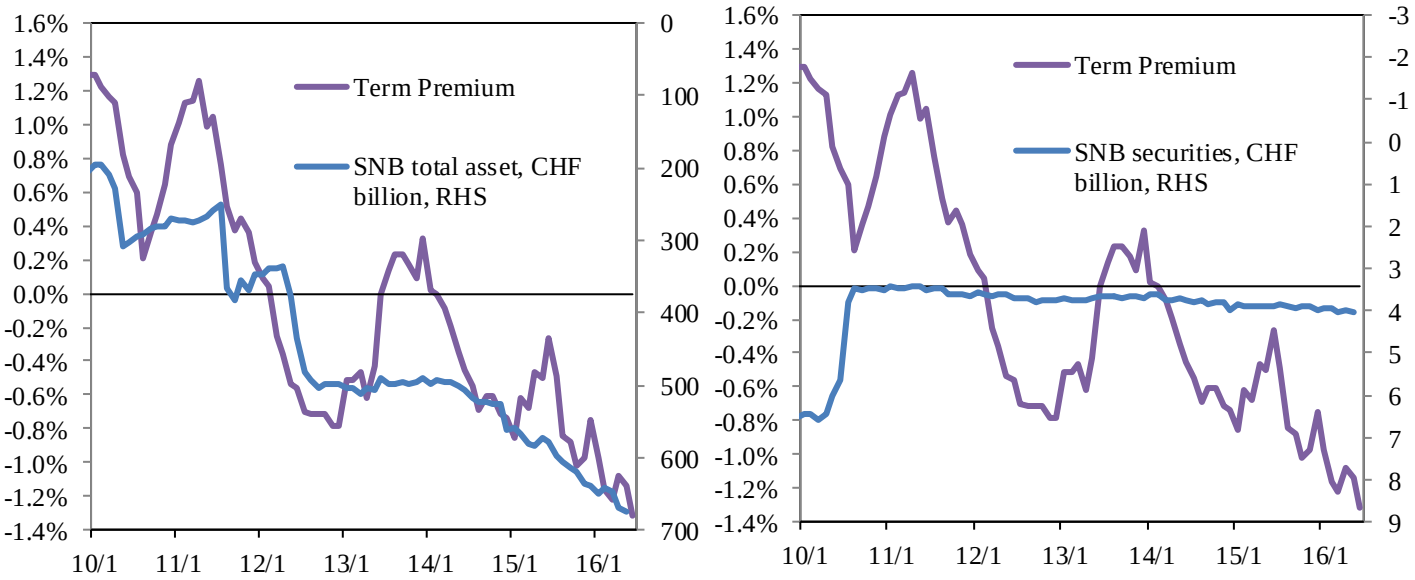
	$s(x_t)$	φ_t
2009/6	0.0%	0.00
2012/1	-1.5%	0.02
2016/6	-3.7%	0.05

Relationship between quantitative easing and yield term premia: Switzerland

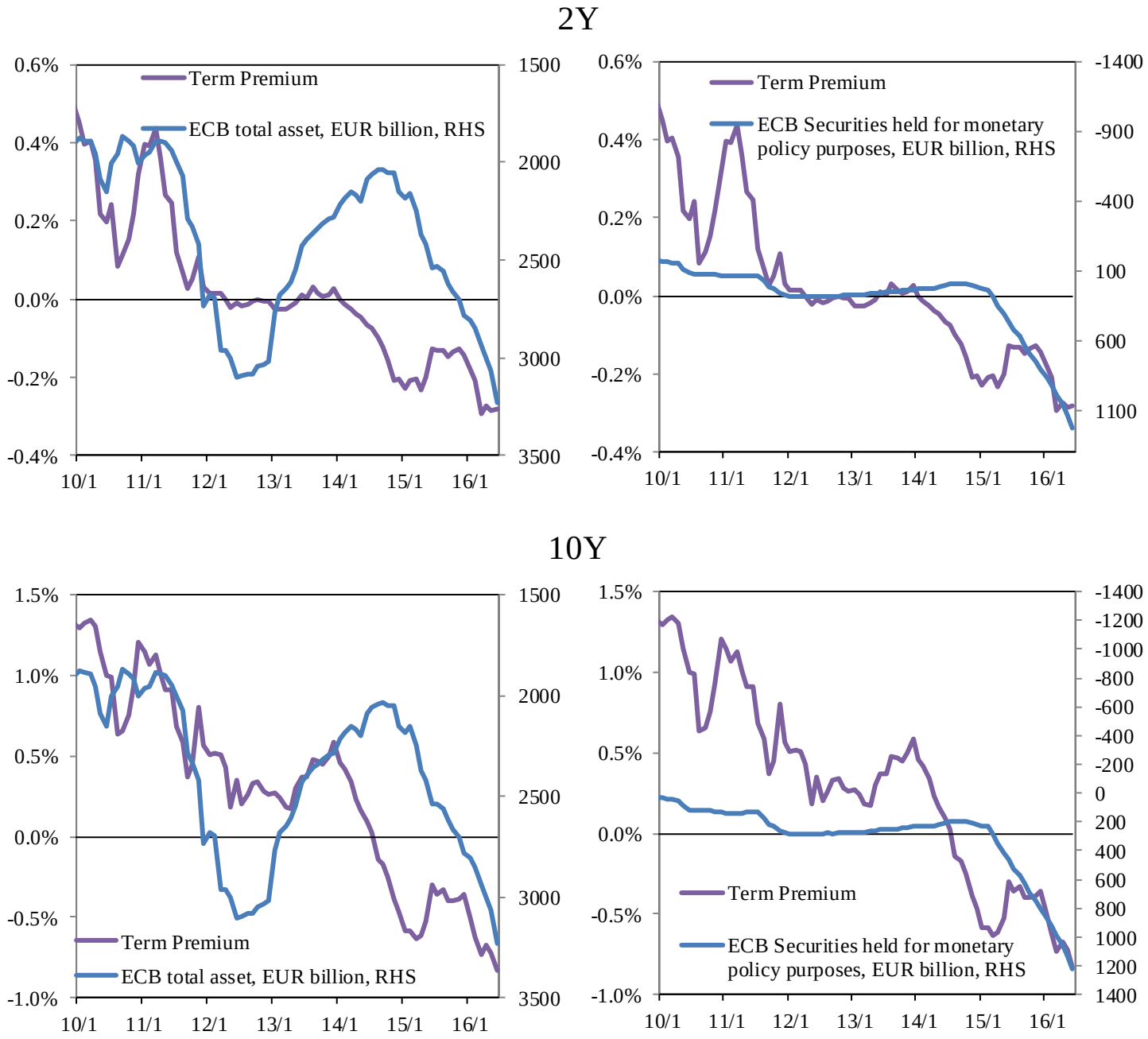
2Y



10Y

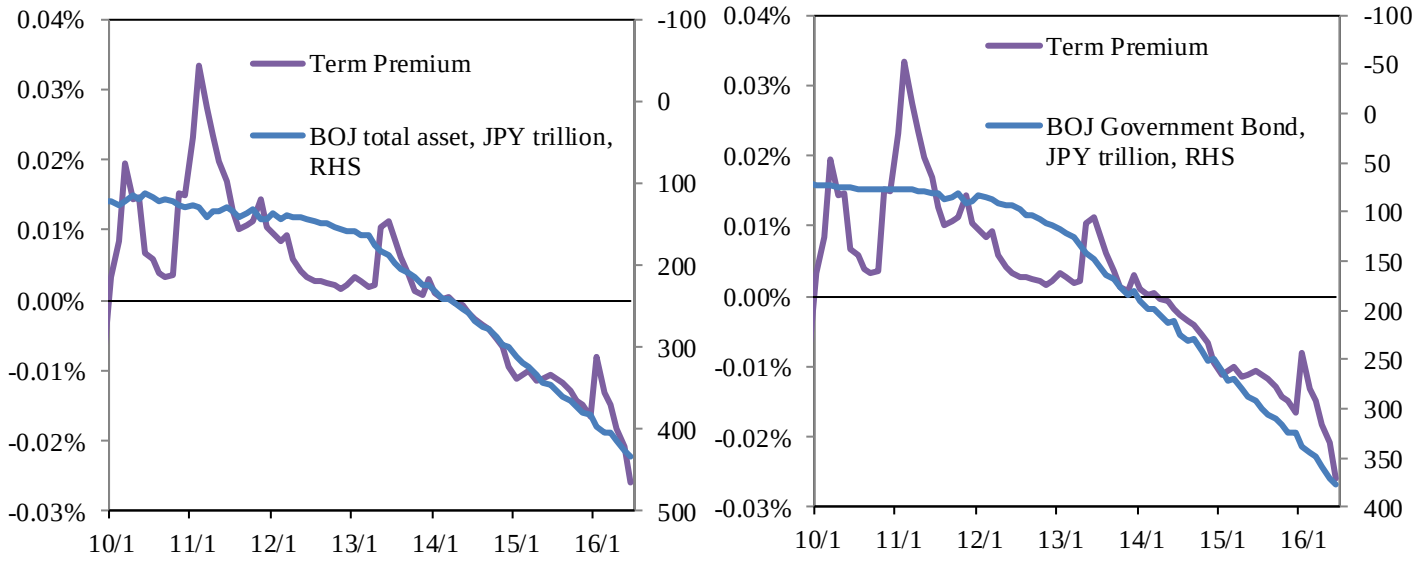


Relationship between quantitative easing and yield term premia: Germany

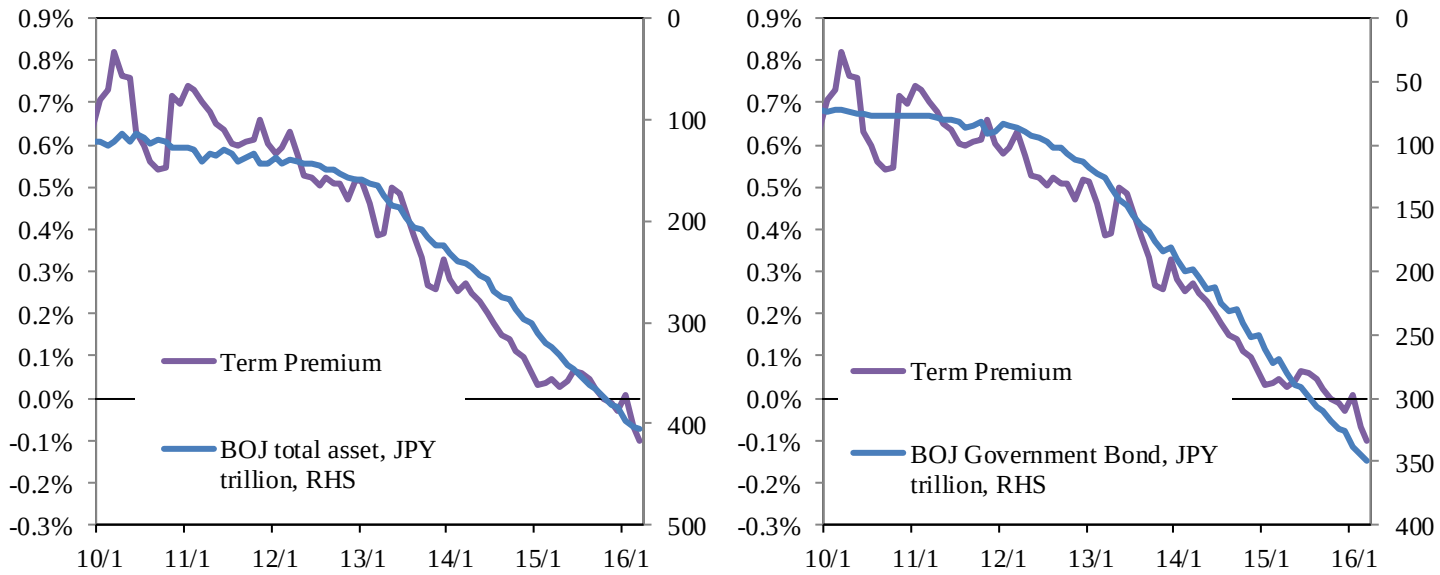


Relationship between quantitative easing and yield term premia: Japan

2Y



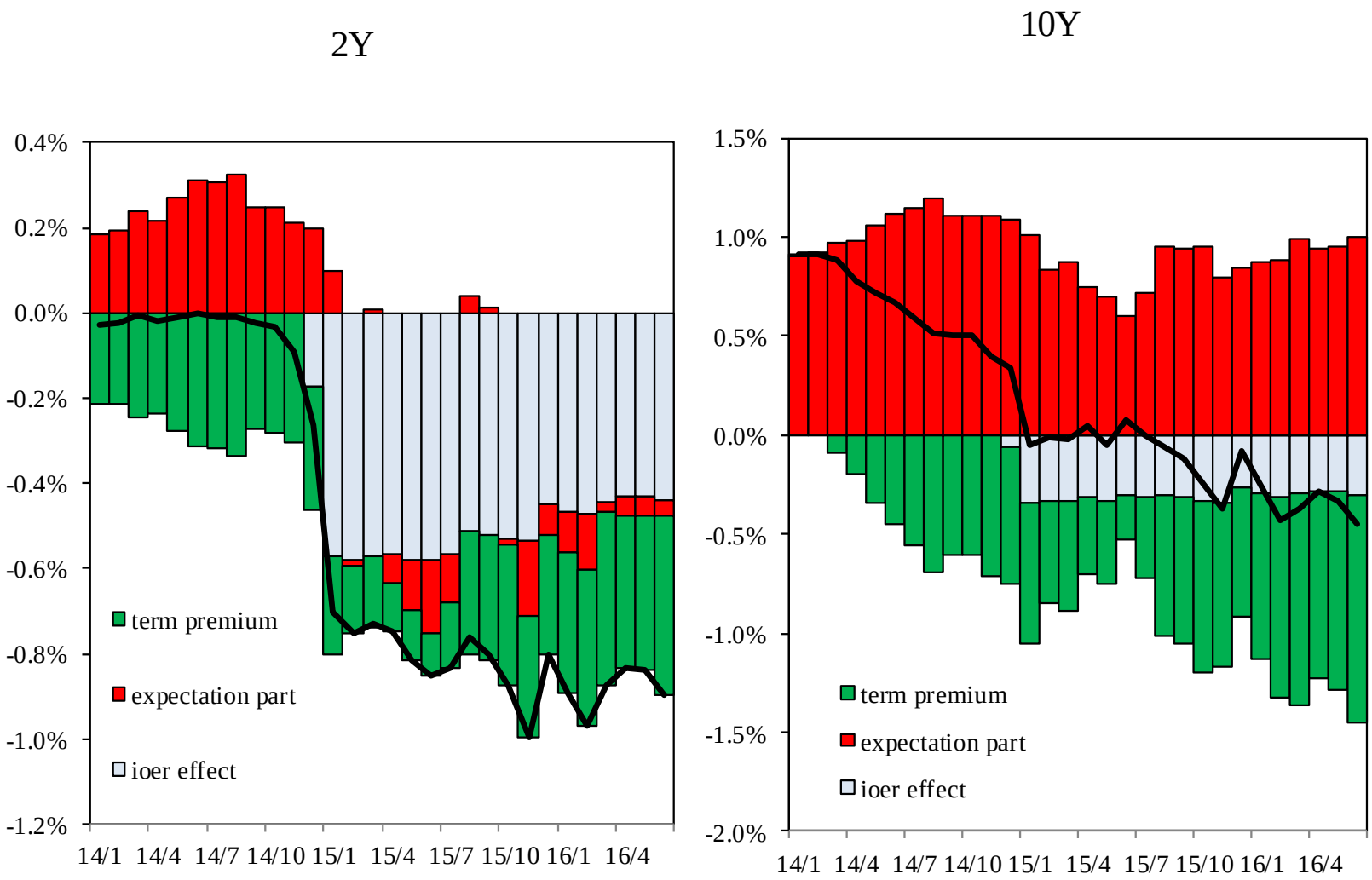
10Y



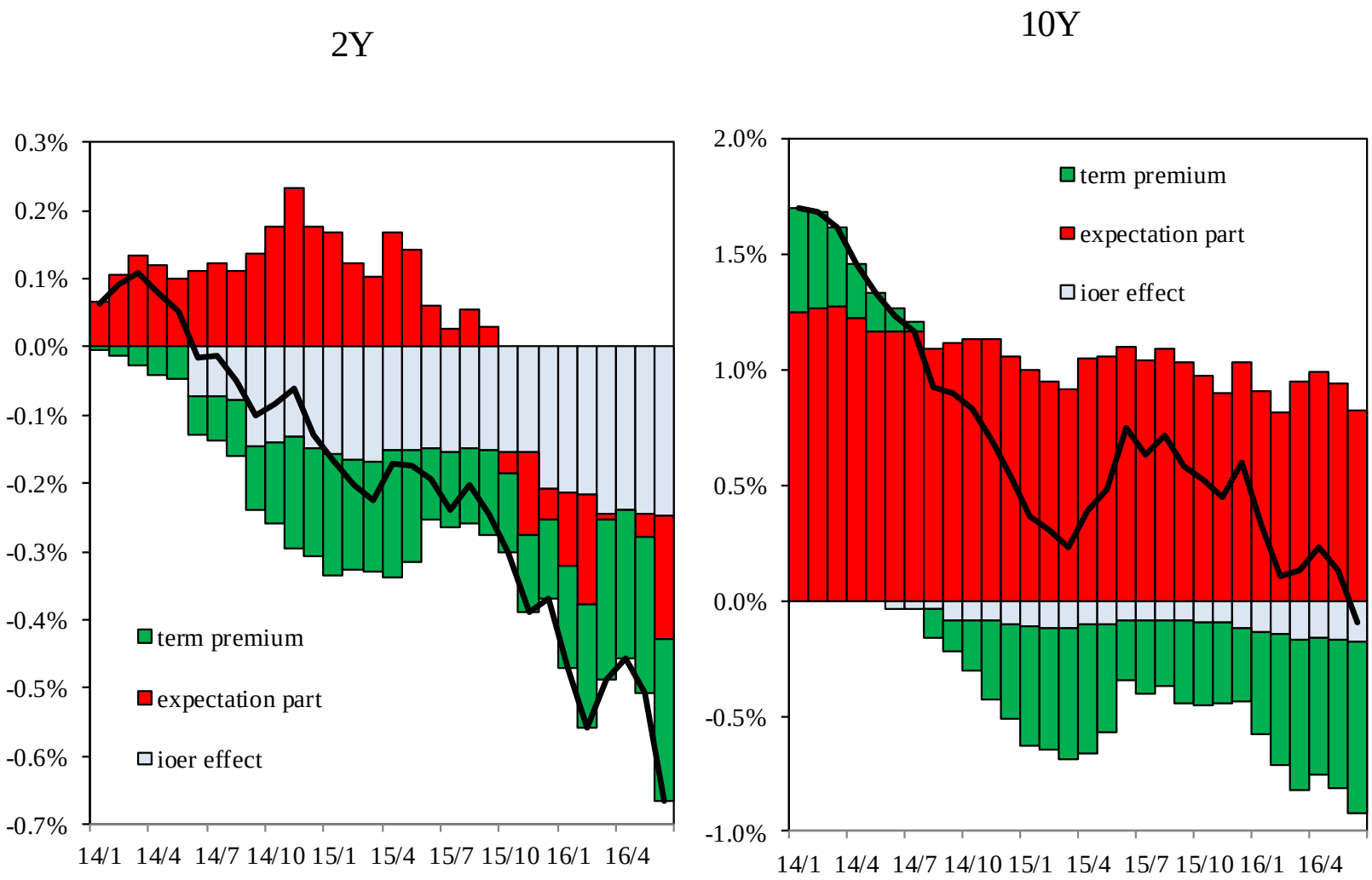
Decomposition of the yield curves

$$
\begin{aligned}
R_\tau &\equiv -\log(P_\tau)/\tau \\
&\equiv -\log\frac{\left(E^Q\left[exp\left(-\int_0^\tau i(x_t, y_t, \varphi_t)dt\right)\right]\right)}{\tau} \\
&\equiv f^Q(x_t, y_t, \varphi_t) \\
&= E^P\left[\int_0^\tau i(x_t, 0, \varphi_t)dt\right] + \left\{f^Q(x_t, 0, \varphi_t) - E^P\left[\int_0^\tau i(x_t, 0, \varphi_t)dt\right]\right\} \\
&+\{f^Q(x_t, y_t, \varphi_t) - f^Q(x_t, 0, \varphi_t)\}\,.
\end{aligned}
$$

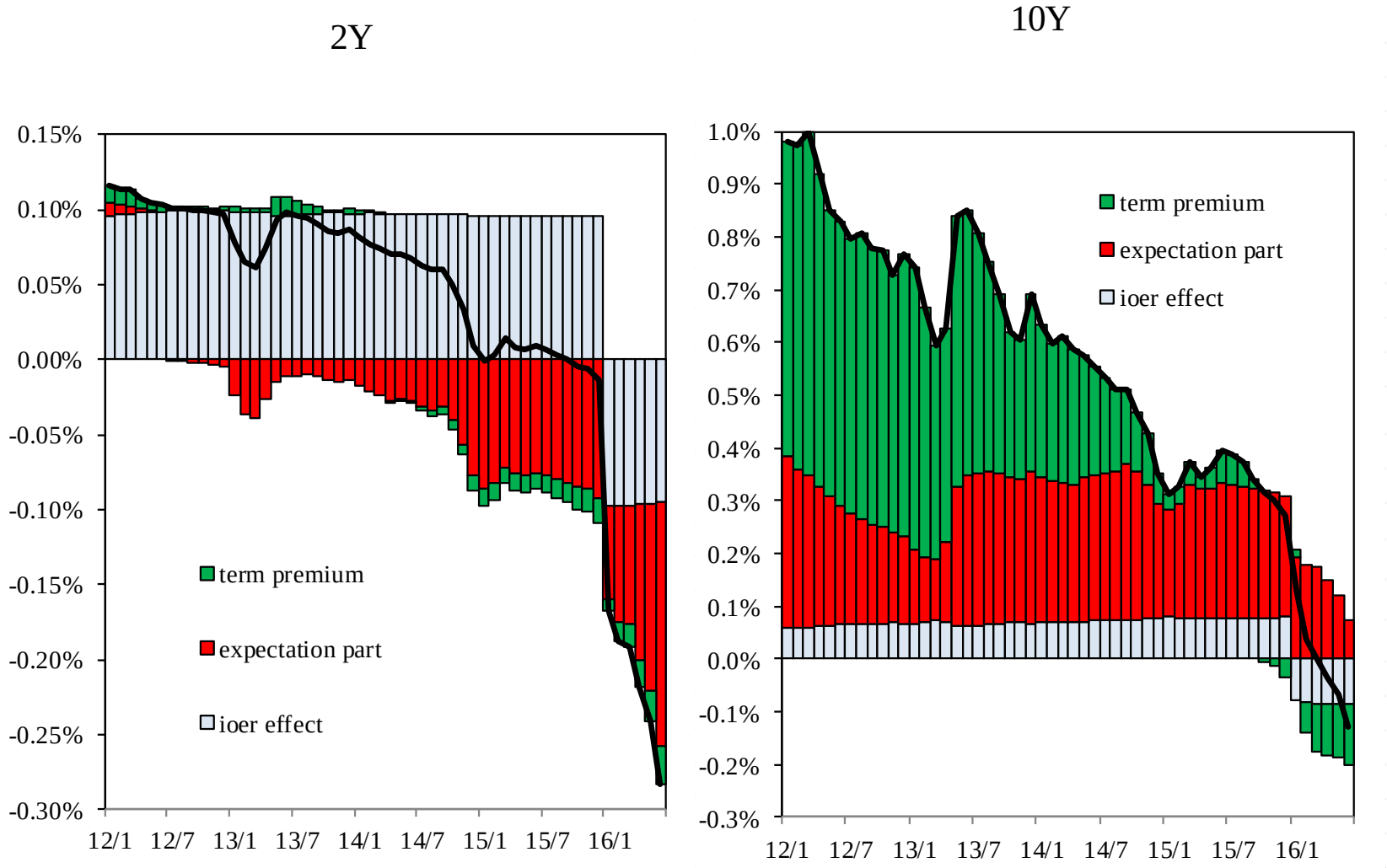
Decomposition of the yield curves: Switzerland



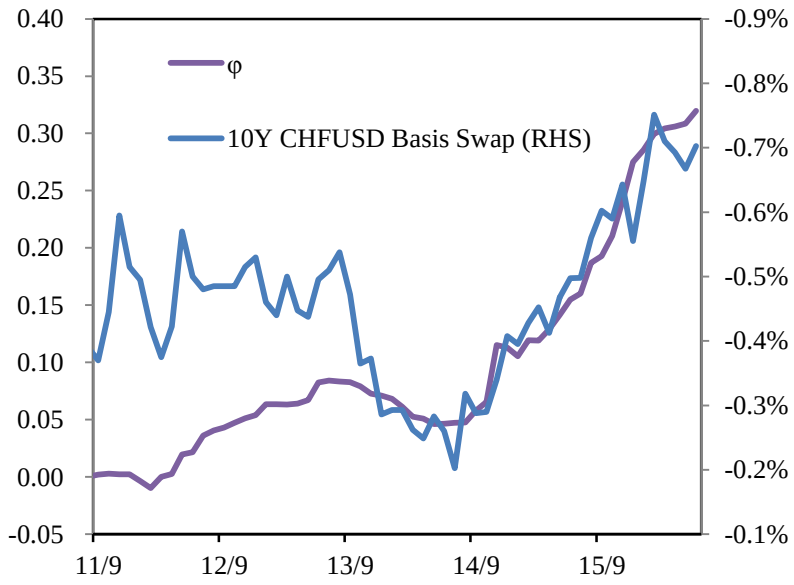
Decomposition of the yield curves: Germany

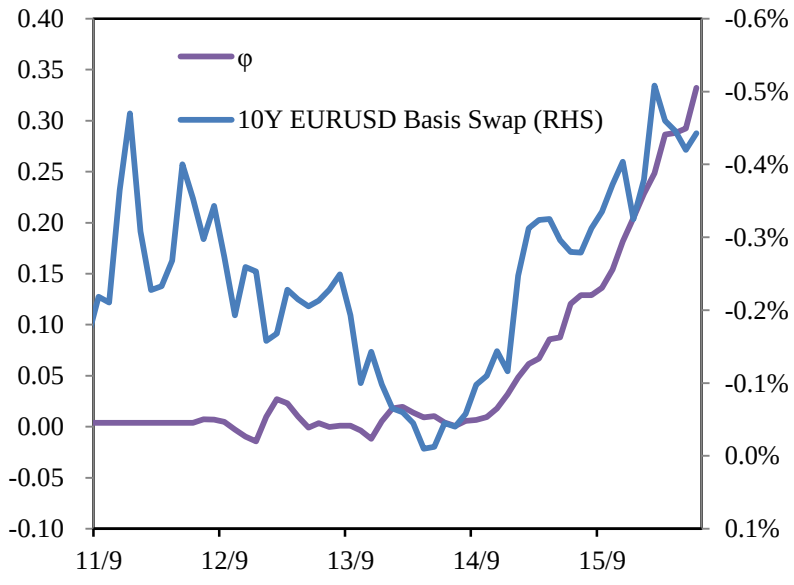


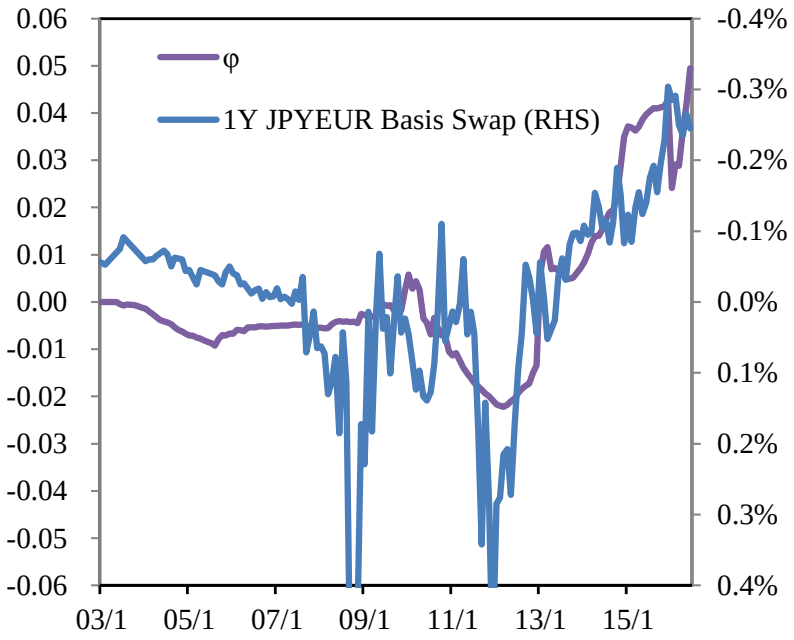
Decomposition of the yield curves: Japan

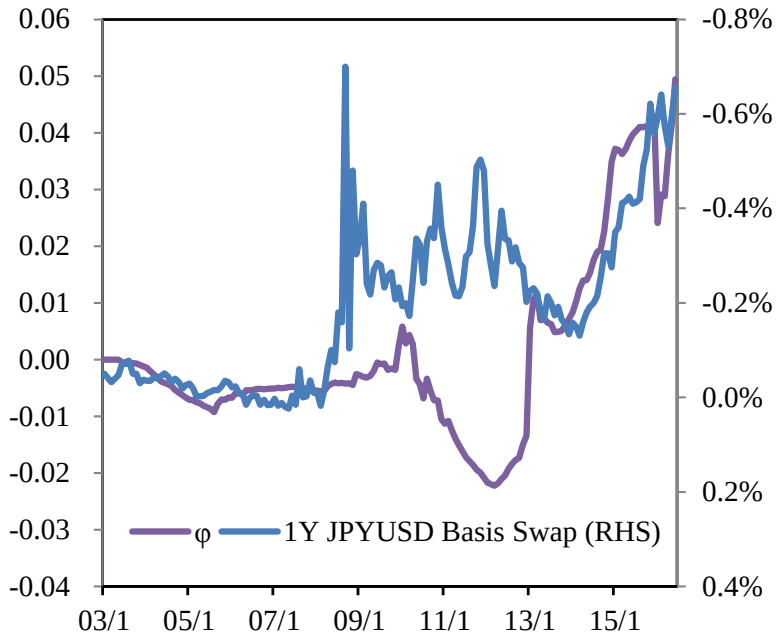


What is behind the movements of the power of arbitrage?









A cross-currency basis swap

- A cross-currency basis swap is an agreement between two counterparties trading floating rate payments in their respective currencies.
- Without frictions in financial markets, a cross-currency swap should have a zero value with no spread on either side. But, there are relative funding costs in the different currencies over the lifetime of the swap. The market is charging a premium for transferring assets or liabilities from one currency to another. The cost is reflected as a spread on the floating leg in the foreign currency.
- U.S. investors can convert the cash flows from foreign government bonds in foreign currency to those in U.S. dollar through the basis swap market receiving premia. Even if the foreign government bond yields are negative, the large enough basis swap spreads can attract U.S. investors to investing the foreign government bonds.

Conclusion

- Develop a model to evaluate the effects of unconventional monetary policy measures including the negative interest rate policy on government bond term structures.
- Propose an efficient and accurate solution method applicable to both the Black model and the Extended model.
- Show that the Extended model is superior to the other models using data from Switzerland, Germany and Japan.
- Quantify each effect of FG, QE and the NIRP.
- Find that the power of arbitrage between money or reserves and government bonds moves in tandem with basis swap spreads.
  - A two country term structure model is to be invented and empirically examined in future research.