Inflation Stabilization and Default Risk in a Currency Union

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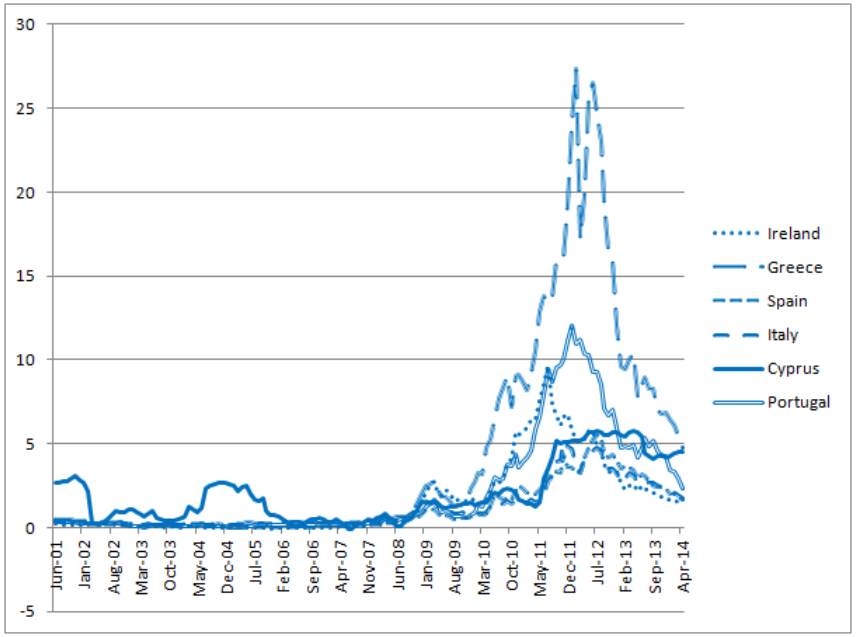
1 Introduction

- How do we conduct monetary policy in a currency union amid sovereign risk premiums?
- How do the monetary and fiscal authorities behave in this difficult situation?
- What clues do we have for removing the trade-off between the prevention of default risk and stabilizing inflation?
- In this paper, we show that there is not necessarily a trade-off between the prevention of default risk and stabilizing inflation.
- Policy authorities, namely, the central bank and the government, should without hesitation, conduct optimal monetary and fiscal policies, which is equivalent to stabilizing inflation.

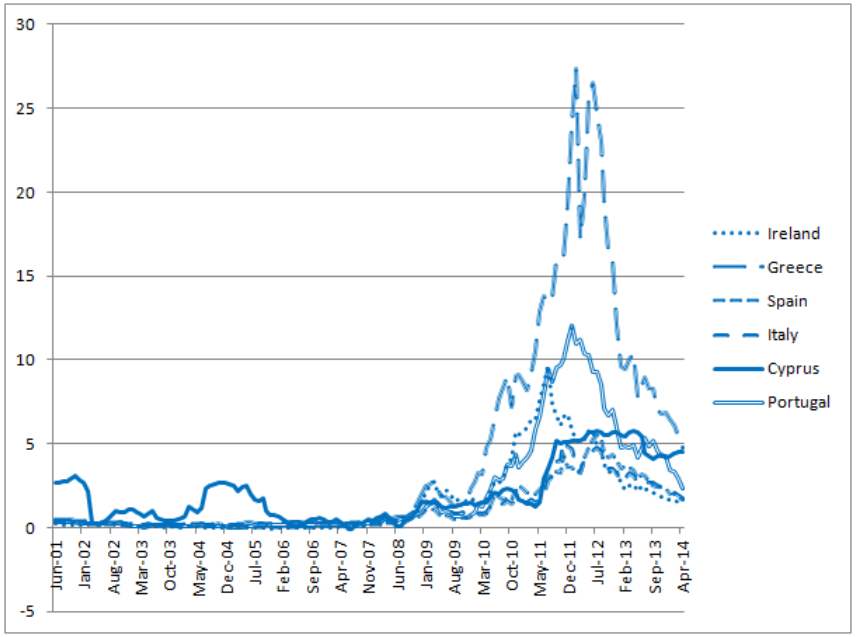
Background and Facts

Fig. 1: Deviation in 10-Year Government Bond Yields from Germany

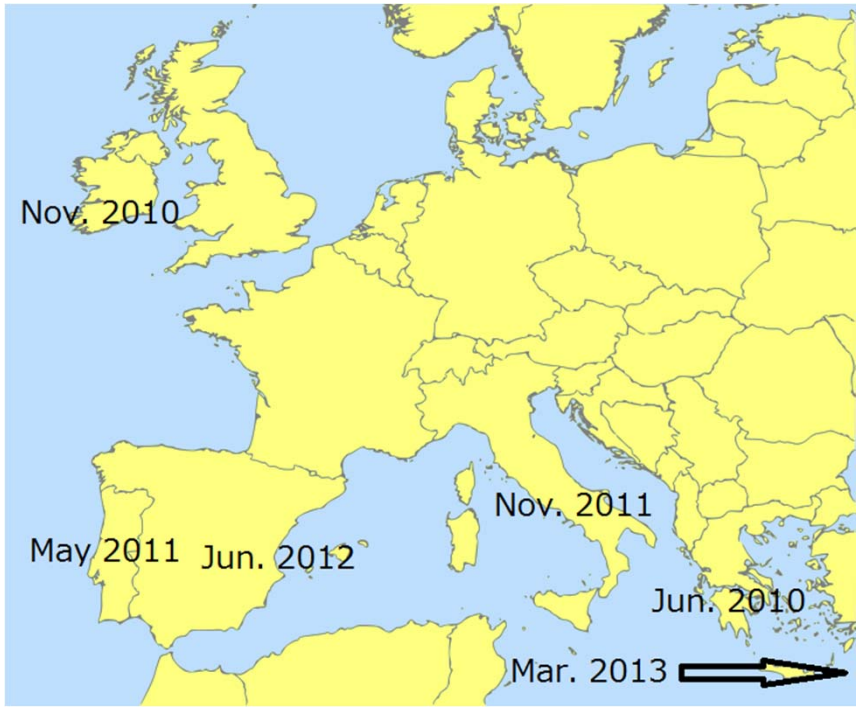


- In October 2009, there was a change of government in Greece and this revealed the presence of a severe fiscal deficit in that country. This incident was the trigger for a general European debt crisis and the yields on sovereign bonds in Greece began to deviate substantially from those prevailing in Germany at the time.
- Leading up to the bankruptcy of Lehman Brothers in September 2008 (Fig.1), the difference in yields on 10-year government bonds in Greece from those in Germany ranged from 13 to 67 basis points.

Fig. 1: Deviation in 10-Year Government Bond Yields from Germany

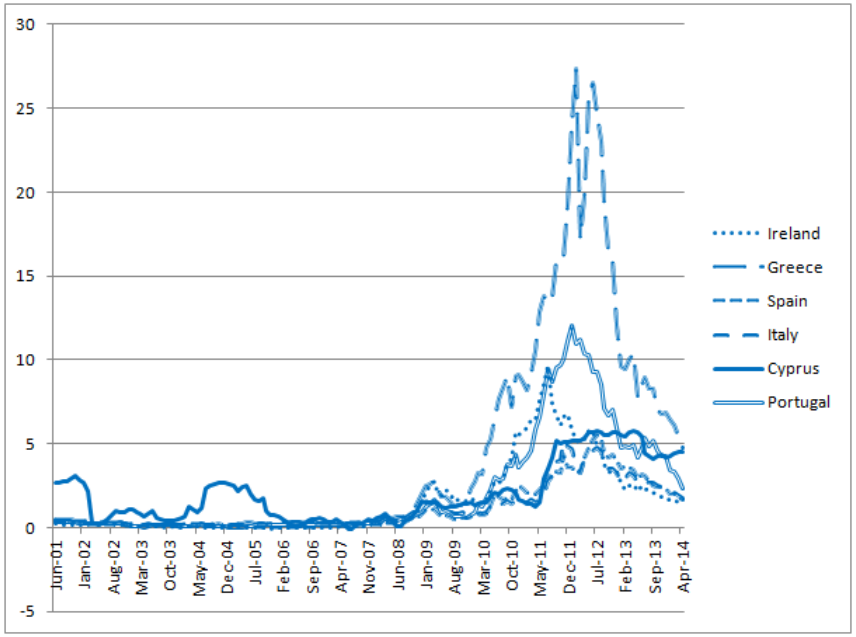


- Following the change of government, however, the difference between the yields on Greek and German sovereign bonds rapidly increased, ultimately topping 904 basis points in September 2010.
- This led to the establishment, in June 2010, of the European Financial System Facility (EFSF) to support the Greek economy.



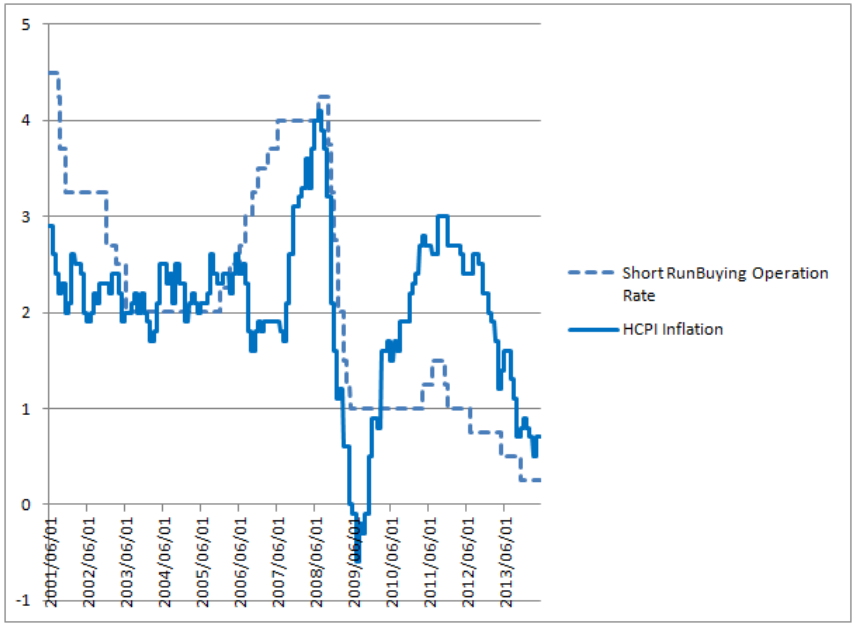
- However, sovereign risk premiums then began to rise sharply in not only Greece, but also elsewhere in the Eurozone.
- Even with the support of the EFSF, the economic situation in Greece did not improve, and an additional program of support was set in place in February 2012, with increasing concerns in May 2012 that Greece would break from the European Monetary Union (EMU).

Fig. 1: Deviation in 10-Year Government Bond Yields from Germany



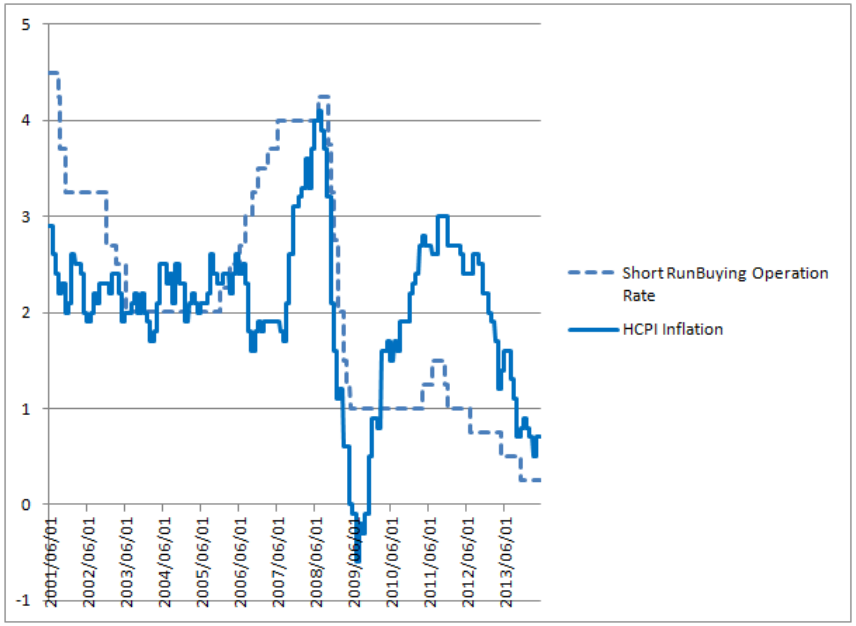
- The difference in 10-year government bond yields of the countries discussed over those in Germany now appears more stable than from 2010 to 2013, the deviation remains at a high level of some 144 to 474 basis points PIIGS.
- However, before crisis, the deviation was 0 to -6 basis in PIIGS in Jul., 2007.
- Therefore, we cannot say that the European sovereign debt crisis has ended by any means, and it continues to smolder to the present day.

Fig. 2: Buying Operation Rate and HCPI Inflation



- At first impression, the ECB appears to face a trade-off between inflation stabilization and the suppression of debt default.
- Before Feb. 2009, the ECB's policy rate was never less than the HCPI except for about 21 months.
- It seems that the ECB attempted to stabilize by keeping in mind a simple monetary model comprising the NKIS and NKPC introduced by Woodford (2003), together with the Taylor principle.

Fig. 2: Buying Operation Rate and HCPI Inflation



- While HCPI inflation reached 3%, the policy rate remained at 1.5% and the ECB no longer sought to increase the policy rate.
- As of May 2014, the ECB policy rate is just 0.25%, still less than HCPI inflation rate of 0.7%.
- Therefore, it would seem that the ECB has given up on any attempt to stabilize inflation and is instead apparently more concerned with combating the still smoldering sovereign default risk.

- In this paper, we analyze how policy authorities cope with stabilizing inflation and suppressing sovereign risk by developing a class of DSGE models with nominal rigidities.
- To develop our model, we drew upon earlier work in this area by Uribe (2006) and Benigno (2001, 2009).

Papers Giving us a Hint

- Uribe (2006)

1. Analyzing monetary policy in a closed economy with sovereign risk.
2. Default depends on the ratio of the net present value of the real fiscal surplus in terms of the marginal utility of consumption to real government debt with interest payments in terms of the marginal utility of consumption.
3. The lower the ratio, the higher the probability of default.

- In our model, one of the two countries, which we refer to as 'country F' defaults on its financial obligations following the mechanism in Uribe (2006).

- Benigno (2001, 2009)

1. Developing a micro-founded portfolio balance model.
2. Assuming two countries and the situation where households face difficulties in purchasing foreign assets.
3. Households obtain less remuneration from the purchase of foreign assets, an idea we include in our model.

- We introduce Benigno's (2001) idea on households' budget constraint into our model.

- By combining Uribe (2006) and Benigno (2001), in this analysis we are able to replicate the current European debt crisis.
- That is, the higher the fiscal deficit, the greater the default rate via the decrease in the ratio of the real fiscal surplus to real government debt with interest payments.
- We analyze both the Taxation and the Non-taxation regime and compare each other to clarify the role of tax gap (percentage deviation of tax rate from its steady state value), fiscal policy tool to conduct optimal fiscal policy, which is synonim of inflation stabilization policy.
- To minimize welfare costs, under the taxation, both countries' government change tax rate although just country *H*'s government changes tax rate under the non-taxation.

Our Findings

1. Monetary policy aimed at minimizing welfare cost, namely, stabilizing inflation, and the prevention of default are not inconsistent.
2. CPI inflation under the taxation regime is stable as a result, unlike under the non-taxation regime.
3. The default volatility under the non-taxation regime is higher than under the taxation regime.

- Policy authorities in EMU should stabilize inflation without hesitation, and that the PIIGS, especially Greece, should strengthen their collection of taxes.

Related Papers

1. Corsetti, Kuester, Meier and Mueller (2012)

- Analyzing the impact of strained government finances on macroeconomic stability and the transmission of fiscal policy.
- Studying a 'sovereign risk channel' through which sovereign default risk raises funding costs in the private sector and show that fiscal retrenchment can help curtail the risk of macroeconomic instability and, in extreme cases, even stimulate economic activity.
- Unlike this paper, they do not analyze it from the viewpoint of welfare costs.

2. Uribe (2006)

- There is a trade-off between stabilizing inflation and preventing default and that default is inevitable in stabilizing inflation.
- The fiscal deficit is an exogenous shock, such that there is a no way to improve the fiscal deficit to avoid sovereign default, and controlling the price level is the only way to affect the default rate.
- This assumption generates differences on implication between him and us.

The Remainder of the Paper

- Section 2 develops the model.
- Section 3 solves the linear-quadratic (LQ) problem, shows the first-order necessary conditions (FONCs) for the policy authorities, and discusses the taxation and non-taxation regimes.
- Section 4 calibrates the model under both regimes.
- Section 5 concludes the paper.

2 The Model

- We derive a model basically based on Okano (2014).
- The currency union consists of countries *H* and *F*, which together organize a monetary union. The households on the interval $[0, \alpha)$ belong to country *H* while those on the interval $[\alpha, 1]$ belong to country *F*.
- There is a default risk in country *F* and the default mechanism follows that in Uribe (2006).
- We follow Benigno (2001) to clarify the households' choice of risky assets.

2.1 Households

- Preferences

$$\mathcal{U}_H \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_{H,t} \right) \; ; \mathcal{U}_F \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_{F,t} \right), \tag{1}$$

with $U_{H,t} \equiv \ln C_t - \frac{1}{1+\varphi} N_{H,t}^{1+\varphi}$ and $U_{F,t} \equiv \ln C_t - \frac{1}{1+\varphi} N_{F,t}^{1+\varphi}$.

- Consumption Index

$$C_t \equiv \left[\alpha^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

- CPI

$$P_t = \left[\alpha P_{H,t}^{1-\theta} + (1-\alpha) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{3}$$

- CPI Inflation

$$\pi_t = \alpha \pi_{H,t} + (1-\alpha) \pi_{F,t}, \tag{4}$$

- Total Demands for Goods

$$C_{H,t} = \alpha \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \; ; \; C_{F,t} = (1-\alpha) \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t. \tag{7}$$

- Households Budget Constraint

$$B^n_{H,t-1} + B^n_{F,t-1} + W_{H,t} N_{H,t} + PR_{H,t} \geq P_t C_t + R_t^{-1} \left[B^n_{H,t} + \frac{B^n_{F,t}}{\mathrm{E}_t \left(1 - \delta_{t+1}\right) \Gamma \left(\widetilde{df}_{F,t} \right)} \right],$$

$$B^n_{H,t-1} + B^n_{F,t-1} + W_{F,t} N_{F,t} + PR_{F,t} \geq P_t C_t + R_t^{-1} \left[B^n_{H,t} + \frac{B^n_{F,t}}{\mathrm{E}_t \left(1 - \delta_{t+1}\right) \Gamma \left(\widetilde{df}_{F,t} \right)} \right], \tag{8}$$

- Plugging $\mathrm{E}_t \left(\delta_{t+1}\right) = 0$ into the first equality of Eq.(8) yields:

$$B^n_{H,t-1} + B^n_{F,t-1} + W_{H,t} N_{H,t} + PR_{H,t} \geq P_t C_t + R_t^{-1} \left[B^n_{H,t} + \frac{B^n_{F,t}}{\Gamma \left(\widetilde{df}_{F,t} \right)} \right],$$

On Interest Rate Multiplier $\Gamma\left(\tilde{df}_{F,t}\right)$

- $\Gamma\left(\tilde{df}_{F,t}\right)$ is the interest rate multiplier for holding country *F*'s government debt with : $\tilde{df}_{F,t} \equiv \frac{\frac{B^n_{F,t}}{P_t} - \Pi_t^{-1} R_{t-1} \frac{B^n_{F,t-1}}{P_{t-1}}}{B_F(1-R)} - 1$
- Thus, $R_t\Gamma\left(\tilde{df}_{F,t}\right)$ is paid for households holding country *F*'s debt although R_t is paid for households holding risk-free assets, country *H*'s debt.
- Benigno (2001) assumes that households in country *H* face a burden in international financial markets. As lenders, they will receive remuneration lower than the foreign interest rate. Thus, $\Gamma'(\cdot) < 0$ in his setting.
- We assume that the higher country *F*'s fiscal deficit, the higher the remuneration for holding country *F*'s government debt because of default. Thus, $\Gamma'(\cdot) > 0$.
- Note that Γ is function of state contingent claim issued by households in country *F* in Benigno (2001).

- Optimality Conditions

$$\beta \mathrm{E}_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}} \right) = \frac{1}{R_t}, \tag{9}$$

$$C_t N_{H,t}^{\varphi} = \frac{W_{H,t}}{P_t} \; ; \; C_t N_{F,t}^{\varphi} = \frac{W_{F,t}}{P_t}, \tag{10}$$

- Eqs.(9) and (10) are standard inter-temporal and intra-temporal optimality conditions.
- Eq.(11) shows that increases in the expected default rate and in country *F*'s government debt increase the interest rate multiplier.

- Optimality Conditions (cont.)

$$\frac{1}{R_t}\left[\frac{1}{\mathrm{E}_t\left(1-\delta_{t+1}\right)\Gamma\left(\widetilde{df}_t\right)}-1\right] = \mathrm{E}_t\left\{\frac{1}{R_{t+1}\left(1-\delta_{t+2}\right)\Gamma\left(\widetilde{df}_{t+1}\right)}B_{F,t+1}\frac{\Gamma'\left(\widetilde{df}_{t+1}\right)}{\Gamma\left(\widetilde{df}_{t+1}\right)} - \frac{1}{R_t\left(1-\delta_{t+1}\right)\Gamma\left(\widetilde{df}_t\right)}B_{F,t}\frac{\Gamma'\left(\widetilde{df}_t\right)}{\Gamma\left(\widetilde{df}_t\right)}\right\}\frac{1}{SP} \quad (11)$$

- Eq.(11) shows that increases in the expected payoff for holding country F's government debt increase the discount rate for holding country F's debt.
- That is, households do not request high return on holding current country F's government debt, if the expected payoff for holding country F's government debt is relatively high.

- Log-linearizing Eq.(11) yields:

$$\mathrm{E}_t\left(\delta_{t+1}\right) = \omega_b b_{F,t} + \frac{1-\beta}{\phi\beta}\delta_t + (1-\beta)\hat{r}_t - (\gamma - 2\phi)\pi_t - (\gamma - 2\phi)\hat{r}_{t-1} - (\gamma - 2\phi)b_{F,t-1} \quad (12)$$

- Following Benigno (2001), we define $\phi \equiv \Gamma'(0)$, refer to it as the interest rate spread for country *F*'s government debt in the steady state.
- $\gamma \equiv \frac{\Gamma''(0)}{\Gamma'(0)}$ denotes the elasticity of the interest rate spread to a one percent change in the fiscal deficit in the steady state.
- We assume $|\Gamma'(\cdot)| < |\Gamma''(\cdot)|$ thus $\gamma > 1$. Our assumption implies that changes in the fiscal deficit alter the interest rate spread in the steady state, although this pressure is larger than interest rate spread itself. By envisaging that the yields on risky assets increase at an increasing rate, we can accept this assumption.
- When $\phi = 0$, Eq.(12) boils down to $\hat{\delta}_t = 0$ implying that there is no default risk.

$$\mathrm{E}_t\left(\delta_{t+1}\right) = \omega_b b_{F,t} + \frac{1-\beta}{\phi\beta}\delta_t + (1-\beta)\,\hat{r}_t - (\gamma - 2\phi)\,\pi_t - (\gamma - 2\phi)\,\hat{r}_{t-1} - (\gamma - 2\phi)\,b_{F,t-1} \quad (12)$$

1. An Increase in country *F*'s government debt:

   Increases Expected Default Rate.

2. An Increase in the Nominal Interest Rate:

   Increases Expected Default Rate.

3. An Increase in CPI Inflation:

   Decreases Default Rate.

2.2 Firms

- Firms' Technology

$$Y_t(h) = A_{H,t} N_{H,t}(h), \quad ; \quad Y_t(f) = A_{F,t} N_{F,t}(f),$$

- FONCs for Firms

$$\begin{aligned} \mathrm{E}_t\left[\sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{H,t+k}\left(\tilde{P}_{H,t} - \frac{\varepsilon}{\varepsilon-1} P_{H,t+k} MC_{H,t+k}\right)\right] &= 0, \\ \mathrm{E}_t\left[\sum_{k=0}^{\infty} \theta^k \beta^k \tilde{Y}_{F,t+k}\left(\tilde{P}_{F,t} - \frac{\varepsilon}{\varepsilon-1} P_{F,t+k} MC_{F,t+k}\right)\right] &= 0, \end{aligned} \tag{16}$$

- Log-linearized FONCs for Firms

$$\begin{aligned} \pi_{H,t} &= \beta \mathrm{E}_t(\pi_{H,t+1}) + \kappa mc_{H,t}, \\ \pi_{F,t} &= \beta \mathrm{E}_t(\pi_{F,t+1}) + \kappa mc_{F,t}, \end{aligned} \tag{17}$$

- Marginal Costs

$$\begin{aligned} MC_{H,t} &= \frac{P_t}{P_{H,t}} \frac{C_t N_{H,t}^{\varphi}}{(1-\tau_{H,t}) A_{H,t}}, \\ MC_{F,t} &= \frac{P_t}{P_{F,t}} \frac{C_t N_{F,t}^{\varphi}}{(1-\tau_{F,t}) A_{F,t}}, \end{aligned} \tag{18}$$

2.3 Government

- Government Budget Constraint

$$\begin{aligned} B_{H,t}^{n} &= R_{t-1}B_{H,t-1}^{n} - P_{H,t}\left(\tau_{H,t}Y_{H,t} - G_{H,t}\right), \\ B_{F,t}^{n} &= R_{t-1}\Gamma\left(\widetilde{df}_{F,t-1}\right)\left(1-\delta_{t}\right)B_{F,t-1}^{n} - P_{F,t}\left(\tau_{F,t}Y_{F,t} - G_{F,t}\right). \end{aligned} \tag{21}$$

- Iterated Forward Government Budget Constraint in Country H

$$1 = \frac{\sum_{k=0}^{\infty}\beta^{k}\mathrm{E}_{t}\left(C_{t+k}^{-1}SP_{H,t+k}\right)}{C_{t}^{-1}R_{t-1}\frac{B_{H,t-1}^{n}}{P_{t}}}, \tag{22}$$

- with $SP_{H,t} \equiv \frac{P_{H,t}}{P_{t}}\left(\tau_{H,t}Y_{H,t} - G_{H,t}\right)$, which is quite similar to the central equation of the FTPL.

- Eq.(23) can be rewritten as:

$$1 = \frac{\sum_{k=0}^{\infty} \beta^k \mathrm{E}_t \left(C_{t+k}^{-1} SP_{H,t+k} \right)}{C_t^{-1} R_{t-1} B_{H,t-1} \Pi_t^{-1}},$$

- Lower the fiscal surplus, the higher the inflation.
- Eq.(23) can be also rewritten as a second-order differential equation, which can be log-linearized:

$$c_t = \mathrm{E}_t \left(c_{t+1} \right) - \hat{r}_t + \mathrm{E}_t \left(\pi_{t+1} \right) - b_{H,t} + \frac{1}{\beta} b_{H,t-1} + \frac{1}{\beta} \hat{r}_{t-1} - \frac{1}{\beta} \pi_t - \frac{1-\beta}{\beta} sp_{H,t}. \tag{23}$$

- Iterated Forward Government Budget Constraint in Country F

$$1-\delta_t=\frac{\sum_{k=0}^{\infty}\beta^k \mathrm{E}_t\left(C_{t+k}^{-1}SP_{F,t+k}\right)}{C_t^{-1}R_{t-1}\Gamma\left(\widetilde{df}_{F,t-1}\right)\frac{B_{F,t-1}^n}{P_t}}. \tag{24}$$

- Eq. (25) can be rewritten as:

$$1-\delta_t=\frac{\sum_{k=0}^{\infty}\beta^k \mathrm{E}_t\left(C_{t+k}^{-1}SP_{F,t+k}\right)}{C_t^{-1}R_{t-1}\Gamma\left(\widetilde{df}_{F,t-1}\right)B_{F,t-1}\Pi_t^{-1}}.$$

- Lower the fiscal surplus, the higher the inflation or the higher the default rate.
- Uribe (2006) shows the trade-off between inflation stabilization and suppressing default probability (or rate).
- However, we have to pay attention his exogenous setting on fiscal surplus.

- Eq.(24) can be rewritten as a second-order differential equation, which can be log-linearized:

$$\begin{aligned} c_t &= \mathrm{E}_t\left(c_{t+1}\right) - \hat{r}_t + \mathrm{E}_t\left(\pi_{t+1}\right) + \mathrm{E}_t\left(\delta_{t+1}\right) - \frac{\omega_\phi}{1-\beta} b_{F,t} + \frac{\omega_\phi}{(1-\beta)\beta} b_{F,t-1} + \frac{\varphi}{(1-\beta)\beta} b_{F,t-2} \\ &+ \frac{\tilde{\omega}_\phi}{(1-\beta)\beta} \hat{r}_{t-1} + \frac{\phi}{(1-\beta)\beta} \hat{r}_{t-2} - \frac{\tilde{\omega}_\phi}{(1-\beta)\beta} \pi_t - \frac{\phi}{(1-\beta)\beta} \pi_{t-1} - \frac{1}{\beta} \delta_t \\ &- \frac{1-\beta}{\beta} sp_{F,t}, \end{aligned} \tag{25}$$

- When we assume $\delta_t = 0$ for all t and $\phi = 0$, Eq. (25) corresponds to Eq.(23) with the exception that $b_{F,t}$ and $sp_{F,t}$ replaces $b_{H,t}$ and $sp_{H,t}$.

- Log-linearized Fiscal Surplus

$$\begin{aligned} sp_{H,t} &= -(1-\alpha) s_t + \frac{\beta\tau}{(1-\beta)\varsigma_B} y_{H,t} + \frac{\beta\tau}{(1-\beta)\varsigma_B} \hat{\tau}_{H,t} - \frac{\beta\varsigma_G}{(1-\beta)\varsigma_B} g_{H,t} \\ sp_{F,t} &= \alpha s_t + \frac{\beta\tau}{(1-\beta)\varsigma_B} y_{F,t} + \frac{\beta\tau}{(1-\beta)\varsigma_B} \hat{\tau}_{F,t} - \frac{\beta\varsigma_G}{(1-\beta)\varsigma_B} g_{F,t}. \end{aligned} \tag{26}$$

An increase in the tax gap suppresses the default rate via an increase in fiscal surplus.

2.4 Equilibrium

- Market Clearing Condition

$$\begin{aligned} Y_t(h) &= C_t(h) + C_t^*(h) + G_t(h), \\ Y_t(f) &= C_t(f) + C_t^*(f) + G_t(f), \end{aligned}$$

- Aggregated Market Clearing Condition with Optimal Allocation for Goods

$$\begin{aligned} Y_{H,t} &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + G_{H,t}, \\ Y_{F,t} &= \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t + G_{F,t}. \end{aligned} \tag{27}$$

2.5 Welfare Costs

- Welfare Criterion

$$\mathcal{W}^W = -\mathcal{L}^W + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3\right),$$

with $\mathcal{W}^W \equiv \alpha \sum_{t=0}^{\infty} \beta^t \left(U_{H,t} - U\right) + (1-\alpha) \sum_{t=0}^{\infty} \beta^t \left(U_{F,t} - U\right)$.

- Welfare Costs

$$\mathcal{L}^W \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t L_t^W$$

with:

$$L_t^W \equiv \frac{\alpha}{2}\Lambda_\pi \pi_{H,t}^2 + \frac{1-\alpha}{2}\Lambda_\pi \pi_{F,t}^2 + \frac{1}{2}\Lambda_x x_t^2 + \frac{\alpha(1-\alpha)}{2}\Lambda_z z_t^2 \tag{31}$$

2.6 Welfare Relevant Output Gap and Dynamics

- NKISs

$$x_t = \frac{\varsigma_B}{\omega_1}\mathrm{E}_t\left(x_{t+1}\right) - \frac{\left(1-\varsigma_G\right)}{\omega_1}\hat{r}_t + \frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}\mathrm{E}_t\left(\pi_{t+1}\right) - \frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}b_{H,t} + \frac{\left(1-\varsigma_G\right)\varsigma_B}{\beta\omega_1}\hat{r}_{t-1} + \frac{\left(1-\varsigma_G\right)\varsigma_B}{\beta\omega_1}b_{H,t-1} + \frac{\left(1-\varsigma_G\right)\varsigma_B}{\beta\omega_1}\pi_t - \frac{\left(1-\alpha\right)\left(1-\varsigma_G\right)}{\beta\omega_1}z_t - \frac{\left(1-\varsigma_G\right)\tau}{\omega_1}\hat{\tau}_{H,t} + \psi_{H,t}, \quad (32)$$

$$x_t = \frac{\varsigma_B}{\omega_1}\mathrm{E}_t\left(x_{t+1}\right) - \frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}\hat{r}_t + \frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}\mathrm{E}_t\left(\pi_{t+1}\right) + \frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}\mathrm{E}_t\left(\delta_{t+1}\right) - \frac{\left(1-\varsigma_G\right)\varsigma_B\tilde{\omega}_\phi}{\omega_1\left(1-\beta\right)}b_{F,t} + \frac{\left(1-\varsigma_G\right)\varsigma_B\hat{\omega}_\phi}{\omega_1\left(1-\beta\right)\beta}b_{F,t-1} + \frac{\left(1-\varsigma_G\right)\varsigma_B\phi}{\omega_1\left(1-\beta\right)}b_{F,t-2} + \frac{\left(1-\varsigma_G\right)\varsigma_B\tilde{\omega}_\phi}{\omega_1\left(1-\beta\right)\beta}\hat{r}_{t-1} + \frac{\left(1-\varsigma_G\right)\varsigma_B\phi}{\omega_1\left(1-\beta\right)\beta}\hat{r}_{t-2} - \frac{\left(1-\varsigma_G\right)\varsigma_B\tilde{\omega}_\phi}{\omega_1\left(1-\beta\right)\beta}\pi_t - \frac{\left(1-\varsigma_G\right)\varsigma_B\phi}{\omega_1\left(1-\beta\right)\beta}\pi_{t-1} - \frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1\beta}\delta_t + \frac{\alpha\left(1-\varsigma_G\right)\omega_2}{\beta\omega_1}z_t - \frac{\left(1-\varsigma_G\right)\tau}{\omega_1}\hat{\tau}_{F,t} + \psi_{F,t}, \quad (33)$$

- If $\phi = 0$ and $\delta_t = 0$, Eq.(33) is analogous to Eq.(32).

- NKPCs

$$\pi_{H,t} = \beta \mathrm{E}_t \left(\pi_{H,t+1}\right) + \kappa \left(1+\varphi\right) x_t + \kappa \left(1-\alpha\right)\left(1+\varphi\right) z_t + \frac{\kappa\tau}{1-\tau}\hat{\tau}_{H,t} + \epsilon_{H,t}, \tag{34}$$

$$\pi_{F,t} = \beta \mathrm{E}_t \left(\pi_{F,t+1}\right) + \kappa \left(1+\varphi\right) x_t - \kappa\alpha \left(1+\varphi\right) z_t + \frac{\kappa\tau}{1-\tau}\hat{\tau}_{F,t} + \epsilon_{F,t}, \tag{35}$$

  which has less feature because there are no novel friction on FONCs for firms.

- Because of distorted steady state, cost push shocks appear.

- TOT Gap

$$\Delta z_t = \pi_{F,t} - \pi_{H,t} + \frac{\omega_{21}}{\left(1-\tau\right)\omega_{18}}\Delta\hat{\tau}_{H,t} - \frac{\omega_{21}}{\left(1-\tau\right)\omega_{18}}\Delta\hat{\tau}_{F,t} + \Delta\xi_t, \tag{36}$$

3 The LQ Problem

- To clarify the role of the tax gap as a policy instrument, we analyze the taxation regime and the non-taxation regime.

1. Taxation regime: Both countries' government change the tax rate (gap) to minimize welfare costs
2. Non-taxation regime: Country F's government does not change the tax rate.

- Non-taxation regime depicts the actual situation of taxation failure in Greece, as discussed.
- The welfare costs are minimized by stabilizing the fluctuation in GDP inflation, the welfare-relevant output gap and the TOT gap.
- The weight on GDP inflation, the weight on the welfare-relevant output gap and the weight on the TOT gap are 151.08, 4.28 and 45.22, respectively.
- Clearly, optimal policy is consistent with inflation-stabilization policy.

- We discuss how GDP inflation stabilized to minimize welfare costs.
- FONCs for the Welfare Relevant Output Gap and GDP Inflation:

$$\begin{aligned} x_t &= -\frac{1}{\Lambda_x}\mu_{1,t} - \frac{1}{\Lambda_x}\mu_{2,t} + \frac{\kappa\left(1+\varphi\right)}{\Lambda_x}\mu_{3,t} + \frac{\kappa\left(1+\varphi\right)}{\Lambda_x}\mu_{4,t} - \frac{\tau\tilde{\phi}_1}{\Lambda_x\varsigma_B\bar{\omega}_\phi}\mu_{5,t} - \frac{\tau\tilde{\phi}_0}{\Lambda_x\varsigma_B\bar{\omega}_\phi}\mu_{6,t} \\ &\quad + \frac{\varsigma_B}{\beta\Lambda_x\omega_1}\mu_{1,t-1} + \frac{\varsigma_B}{\beta\Lambda_x\omega_1}\mu_{2,t-1}, \end{aligned} \tag{37}$$

- FONC for the GDP Inflation

$$\alpha\pi_{H,t} = -\frac{1}{\Lambda_\pi}\left(\mu_{3,t} - \mu_{3,t-1}\right) - \frac{1}{\Lambda_\pi}\mu_{7,t} + \frac{\alpha}{\Lambda_\pi}\mu_{8,t} \tag{39}$$

$$\left(1-\alpha\right)\pi_{F,t} = -\frac{1}{\Lambda_\pi}\left(\mu_{4,t} - \mu_{4,t-1}\right) + \frac{1}{\Lambda_\pi}\mu_{7,t} + \frac{1-\alpha}{\Lambda_\pi}\mu_{8,t}, \tag{40}$$

- Eqs.(37), (39) and (40) implies that the central bank and governments must respond by driving welfare relevant output gap in the face of inflationary pressures.
- This mechanism is familiar in literature in optimal monetary policy.

- There are two ways to stabilize GDP inflation:

1. Hiking Nominal Interest Rate
2. Hiking Tax Gap

- FONC for the Nominal Interest Rate:

$$\begin{aligned}\mu_{1,t}+\mu_{2,t} &= \frac{(1-\beta)\,\omega_1}{(1-\varsigma_G)\,\varsigma_B}\mu_{9,t}+\mathrm{E}_t\left(\mu_{1,t+1}\right)+\frac{\tilde{\omega}_\phi}{1-\beta}\mathrm{E}_t\left(\mu_{2,t+1}\right)+\frac{\omega_1}{(1-\varsigma_G)\,\varsigma_G}\mathrm{E}_t\left(\mu_{5,t+1}\right)\\ &\quad-\frac{\beta\,(\gamma-2\phi)\,\omega_1}{(1-\varsigma_G)\,\varsigma_G}\mathrm{E}_t\left(\mu_{9,t+1}\right),\end{aligned} \tag{41}$$

- Eq.(41) implies that an increase in the nominal interest rate increases the expected default rate.

- FONC for the Tax Gap

$$\mu_{1,t} = \frac{\kappa\omega_1}{(1-\varsigma_G)(1-\tau)}\mu_{3,t} - \frac{\alpha\omega_1}{(1-\varsigma_G)\varsigma_B}\mu_{5,t} - \frac{\omega_1}{(1-\varsigma_G)\varsigma_B}\mu_{6,t} + \frac{\omega_1\omega_{21}}{(1-\varsigma_G)\tau(1-\tau)\omega_{18}}\mu_{7,t} - \frac{\beta\omega_1\omega_{21}}{(1-\varsigma_G)\tau(1-\tau)\omega_{18}}\mu_{7,t+1}, \tag{42}$$

$$\mu_{2,t} = \frac{\kappa\omega_1}{(1-\varsigma_G)(1-\tau)}\mu_{4,t} - \frac{(1-\alpha)(1-\beta)\omega_1}{(1-\varsigma_G)\varsigma_B\bar{\omega}_\phi}\mu_{5,t} + \frac{(1-\alpha)(1-\beta)\omega_1}{(1-\varsigma_G)\varsigma_B\bar{\omega}_\phi}\mu_{6,t} - \frac{\omega_1\omega_{21}}{(1-\varsigma_G)\tau(1-\tau)\omega_{18}}\mu_{7,t} + \frac{\beta\omega_1\omega_{21}}{(1-\varsigma_G)\tau(1-\tau)\omega_{18}}\mu_{7,t+1}, \tag{43}$$

- Eqs.(42) and (43) implies that an increase in the tax gap decreases GDP inflation.
- An increase in the tax gap improve the fiscal balance. Thus, there is not necessarily a trade-off between stabilizing inflation and suppressing the default.

4 Numerical Analysis

- We calibrate the model following mainly former works.
- We calculate IRFs, volatility and correlations.

4.1 Parametrization

Following the analysis of optimal monetary and fiscal policy in a currency union in Okano (2014), we set:

1. The Ealues for the Subjective Discount Factor β: 0.99
2. The Elasticity of Substitution across Goods ε : 11
3. The Inverse of the Labor Supply Elasticity φ : 3
4. The Steady-state Share of Government Debt to Output σ_B : 2.4
5. The Steady-state Share of Government Expenditure to Output σ_G : 0.276
6. The Steady-state Tax Rate τ : 0.3
7. Price Stickiness θ : 0.75
8. The Elasticity of Substitution between Goods Produced in Countries H and F η: 4.5
9. The persistence of productivity shocks and the persistence of government spending shocks: 0.705 and 0.8

- Following the GDP share of the PIIGS among the 17 countries in the Euro area in 2013Q4, we set:

1. Population in Country H α : 0.68

- Following Benigno's (2001) parametrization, we set:

1. The interest rate spread for country F's government debt ϕ : 0.1

- Although, we cannot find any empirical results, we set:

1. The elasticity of the interest rate spread to a one percent change in the fiscal deficit in Country F γ : 2
2. This value implies that the interest rate spread increases by 200 basis points when the fiscal deficit increases by 1% in the steady state.

4.2 Macroeconomic Dynamics

Table 1: Macroeconomic Volatility

Variable	Taxation	Non-taxation
x_t	1.05	1.08
z_t	0.53	1.05
π_t	0.21	0.84
$\pi_{H,t}$	0.14	1.15
$\pi_{F,t}$	0.50	0.50
$\hat{r}_t$	0.49	0.71
$\hat{\tau}_{H,t}$	9.53	49.59
$\hat{\tau}_{F,t}$	26.87	NA
$b_{H,t}$	28.41	45.10
$b_{F,t}$	0.34	0.35
δ_t	1.85	3.14
$sp_{H,t}$	123.73	604.81
$sp_{F,t}$	323.31	37.77

Note: Standard Deviation in Percentages.

- The volatility of the default rate under the taxation model is lower than under non-taxation.
- In addition, the volatility of GDP and CPI inflation under taxation are lower than under non-taxation.
- This means that there is not necessarily a trade-off between stabilizing inflation and suppressing the default rate.

Table 2: Correlation of Selected Variables under Taxation

Variable	Regime	π_t	$\pi_{H,t}$	$\pi_{F,t}$	δ_t	$sp_{H,t}$	$sp_{F,t}$
x_t	Taxation	0.14	0.27	0.06	0.02	-0.72	-0.04
	Non-taxation	0.00	-0.02	0.25	-0.06	-0.15	-0.43
z_t	Taxation	0.53	0.73	0.34	-0.07	-0.08	0.71
	Non-taxation	-0.43	-0.41	-0.36	0.44	-0.57	-0.74
π_t	Taxation	1.00	0.79	0.95	-0.84	0.22	0.93
	Non-taxation	1.00	0.99	0.42	-0.99	0.96	0.55
$\pi_{H,t}$	Taxation	0.79	1.00	0.54	-0.41	0.20	0.83
	Non-taxation	1.00	1.00	0.34	-0.99	0.96	0.55
$\pi_{F,t}$	Taxation	0.95	0.54	1.00	-0.82	0.20	0.82
	Non-taxation	0.42	0.34	1.00	-0.34	0.43	0.34
$\hat{r}_t$	Taxation	-0.79	-0.49	-0.82	0.91	-0.17	-0.76
	Non-taxation	-0.92	-0.91	-0.34	0.85	-0.96	-0.61
$\hat{\tau}_{H,t}$	Taxation	0.22	0.23	0.17	-0.03	1.00	0.33
	Non-taxation	0.96	0.95	0.43	-0.91	1.00	0.70
$\hat{\tau}_{F,t}$	Taxation	0.93	0.82	0.82	-0.77	0.33	1.00
	Non-taxation	NA	NA	NA	NA	NA	NA
$b_{H,t}$	Taxation	0.08	0.13	0.04	-0.11	0.11	0.01
	Non-taxation	-0.02	-0.03	0.02	0.00	0.04	0.51
$b_{F,t}$	Taxation	-0.69	-0.88	-0.48	0.22	-0.39	-0.75
	Non-taxation	-0.96	-0.94	-0.51	0.90	-0.98	-0.66
δ_t	Taxation	-0.84	-0.41	-0.93	1.00	-0.07	-0.72
	Non-taxation	-0.99	-0.99	-0.34	1.00	-0.91	-0.49
$sp_{H,t}$	Taxation	0.22	0.20	0.20	-0.07	1.00	0.33
	Non-taxation	0.96	0.96	0.43	-0.91	1.00	0.69
$sp_{F,t}$	Taxation	0.93	0.83	0.82	-0.72	0.33	1.00
	Non-taxation	0.56	0.55	0.34	-0.49	0.69	1.00

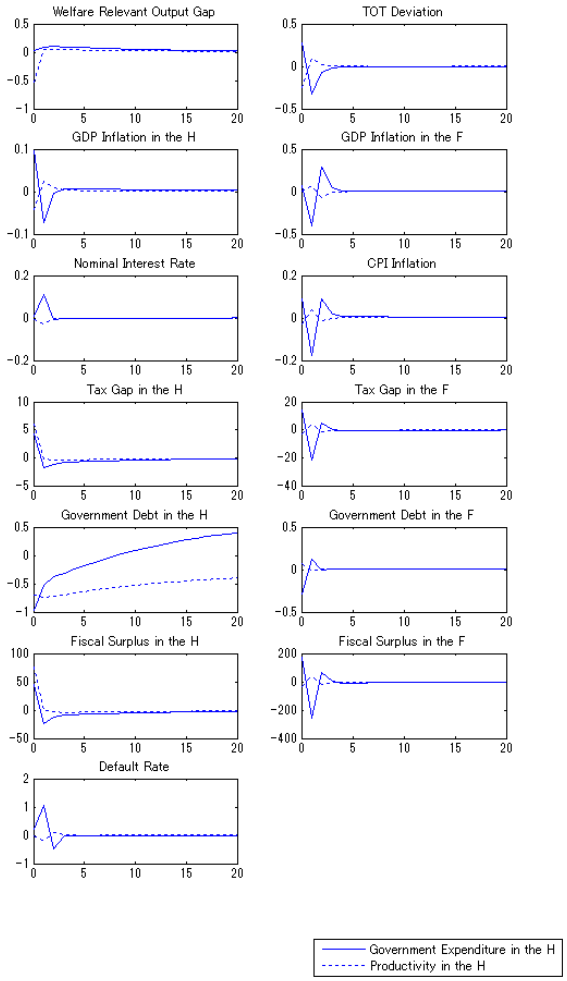
- The correlation between CPI inflation in both countries and the default rate is -0.84 under the taxation. This intimates that the higher is inflation, the lower is the default rate and vice versa.
- However, the correlation between the fiscal surplus in country F and the default rate is -0.72 under the taxation. This shows that the fiscal surplus channel cannot be ignored.

Table 2: Correlation of Selected Variables under Taxation

Variable	Regime	π_t	$\pi_{H,t}$	$\pi_{F,t}$	δ_t	$sp_{H,t}$	$sp_{F,t}$
x_t	Taxation	0.14	0.27	0.06	0.02	-0.72	-0.04
	Non-taxation	0.00	-0.02	0.25	-0.06	-0.15	-0.43
z_t	Taxation	0.53	0.73	0.34	-0.07	-0.08	0.71
	Non-taxation	-0.43	-0.41	-0.36	0.44	-0.57	-0.74
π_t	Taxation	1.00	0.79	0.95	-0.84	0.22	0.93
	Non-taxation	1.00	0.99	0.42	-0.99	0.96	0.55
$\pi_{H,t}$	Taxation	0.79	1.00	0.54	-0.41	0.20	0.83
	Non-taxation	1.00	1.00	0.34	-0.99	0.96	0.55
$\pi_{F,t}$	Taxation	0.95	0.54	1.00	-0.82	0.20	0.82
	Non-taxation	0.42	0.34	1.00	-0.34	0.43	0.34
$\hat{r}_t$	Taxation	-0.79	-0.49	-0.82	0.91	-0.17	-0.76
	Non-taxation	-0.92	-0.91	-0.34	0.85	-0.96	-0.61
$\hat{\tau}_{H,t}$	Taxation	0.22	0.23	0.17	-0.03	1.00	0.33
	Non-taxation	0.96	0.95	0.43	-0.91	1.00	0.70
$\hat{\tau}_{F,t}$	Taxation	0.93	0.82	0.82	-0.77	0.33	1.00
	Non-taxation	NA	NA	NA	NA	NA	NA
$b_{H,t}$	Taxation	0.08	0.13	0.04	-0.11	0.11	0.01
	Non-taxation	-0.02	-0.03	0.02	0.00	0.04	0.51
$b_{F,t}$	Taxation	-0.69	-0.88	-0.48	0.22	-0.39	-0.75
	Non-taxation	-0.96	-0.94	-0.51	0.90	-0.98	-0.66
δ_t	Taxation	-0.84	-0.41	-0.93	1.00	-0.07	-0.72
	Non-taxation	-0.99	-0.99	-0.34	1.00	-0.91	-0.49
$sp_{H,t}$	Taxation	0.22	0.20	0.20	-0.07	1.00	0.33
	Non-taxation	0.96	0.96	0.43	-0.91	1.00	0.69
$sp_{F,t}$	Taxation	0.93	0.83	0.82	-0.72	0.33	1.00
	Non-taxation	0.56	0.55	0.34	-0.49	0.69	1.00

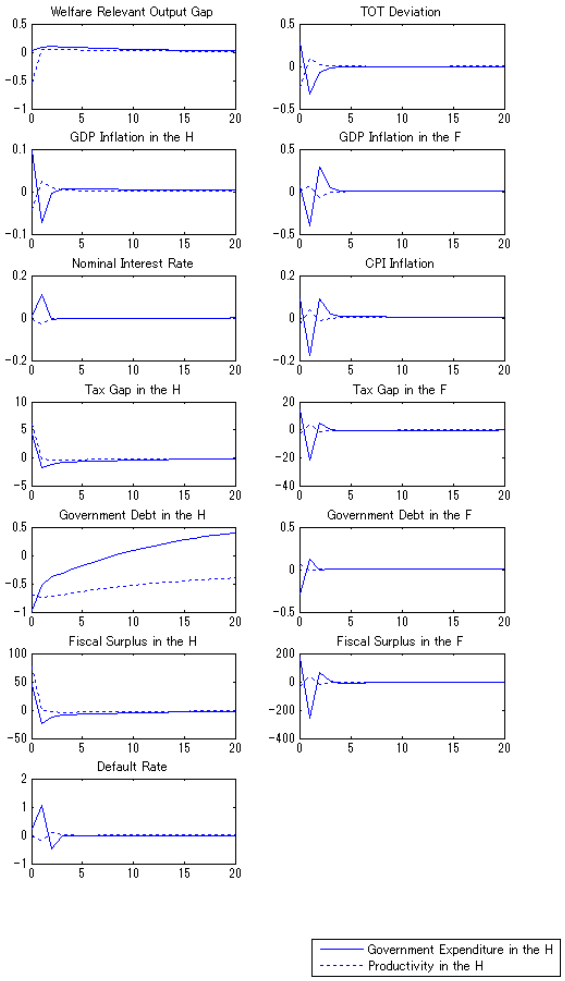
- An increase in the tax gap then decreases GDP inflation on NKPCs, while this increase decreases the default rate via an increase in the fiscal surplus in country F.
- Uribe's (2006) policy implication is not applicable in our model and the fiscal surplus channel, overcomes the CPI inflation channel.
- At the least, in our setting stabilizing inflation is then consistent with suppressing the default rate.

Figure 3: IRFs to Shocks in Country H under Taxation



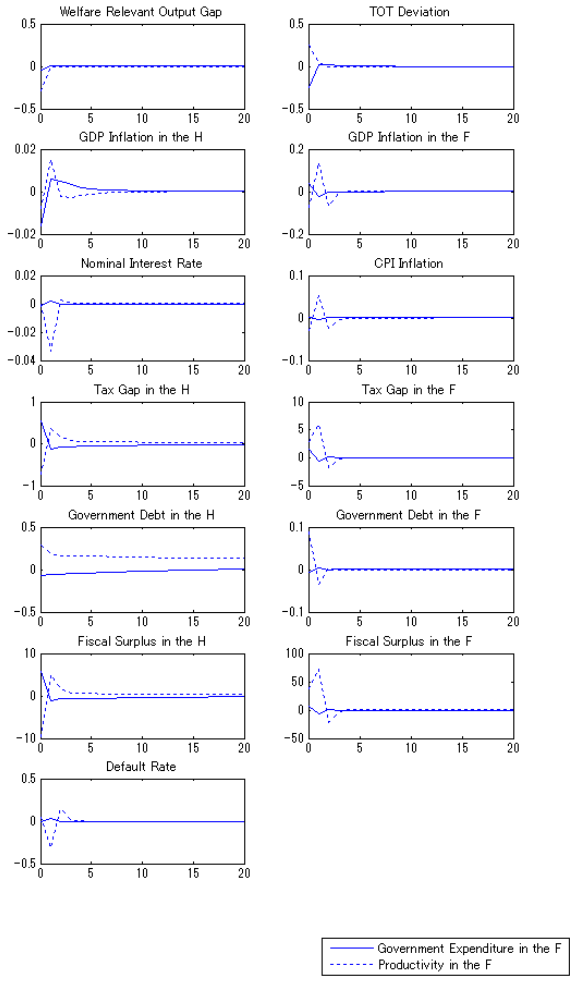
- When government expenditure in country *H* increases, CPI inflation decreases via an increase in the nominal interest rate.
- Tax gap in both countries decrease to mitigate deflation pressure stemming from an increase in the nominal interest rate.
- The default rate increases.
- However, this is just exceptional case.

Figure 3: IRFs to Shocks in Country H under Taxation



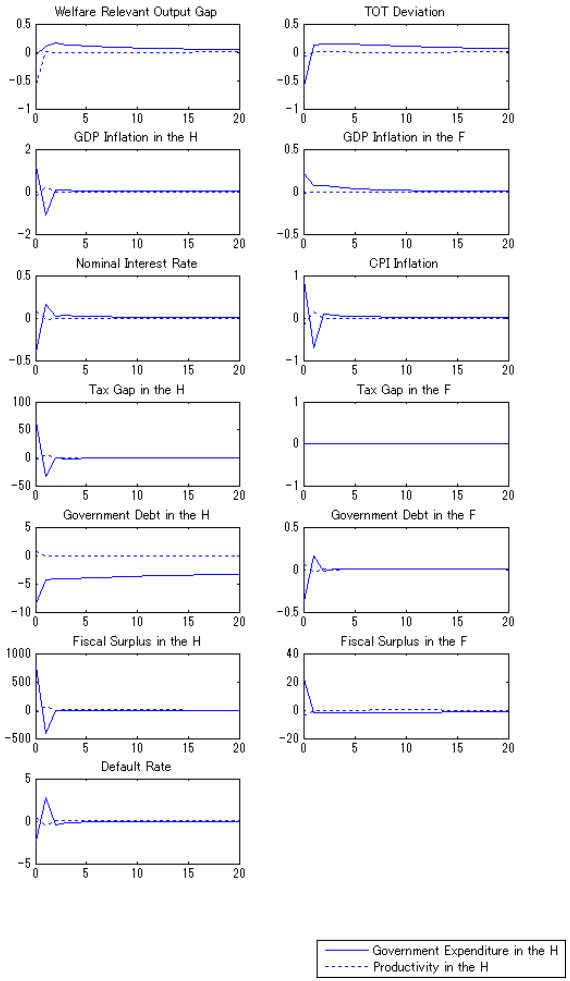
- When productivity in country H rises, the tax gaps are hiked and the CPI inflation is stabilized via stabilization in the GDP inflation.
- The fiscal surplus in country F rises and the default rate becomes negative.

Figure 4: IRFs to Shocks in Country *F* under Taxation



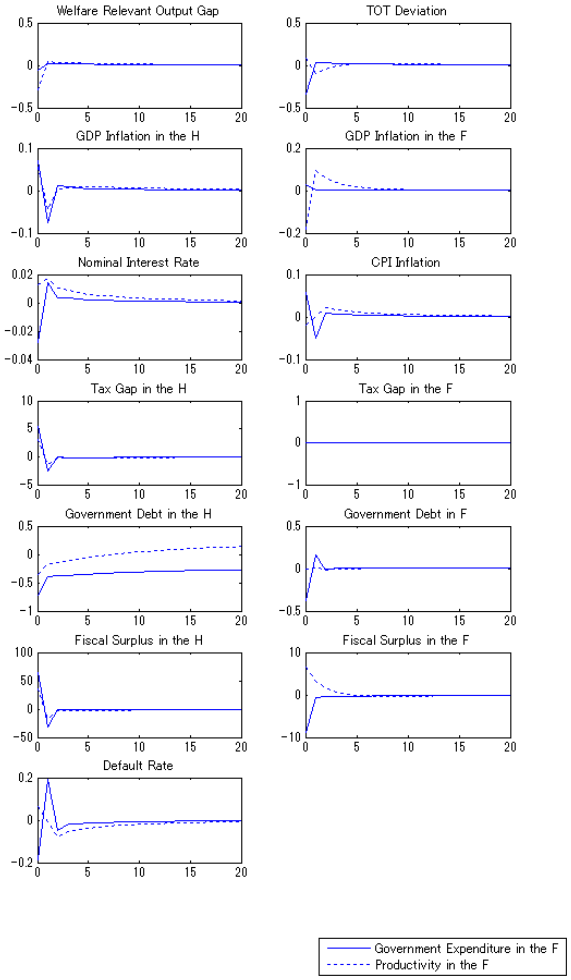
- An increase in government expenditure in country F provides pressure to increase GDP inflation in country F and the tax gap in country F is hiked to stabilize this increase.
- The CPI inflation is stabilized via stabilization in the GDP inflation.
- The tax gap slightly rises.
- An increase in productivity in country F increases the tax gap in country F and the default rate becomes negative.

Figure 5: IRFs to Shocks in Country H under Non-taxation



- Mechanism behind IRFs under Non-taxation is similar to one under Taxation.
- However, IRFs take much time to settle because of the lack of policy tool.

Figure 6: IRFs to Shocks in Country F under Non-taxation



- Mechanism behind IRFs under Non-taxation is similar to one under Taxation.
- However, IRFs take much time to settle because of the lack of policy tool.

- Welfare Costs

$$\mathcal{L}^W = \frac{\alpha}{2}\Lambda_\pi \text{var}\left(\pi_{H,t}\right) + \frac{1-\alpha}{2}\Lambda_\pi \text{var}\left(\pi_{F,t}\right) + \frac{1}{2}\Lambda_x \text{var}\left(x_t\right) + \frac{\alpha\left(1-\alpha\right)}{2}\Lambda_z \text{var}\left(z_t\right).$$

- The welfare costs under taxation and non-taxation are 10.79 and 81.90, respectively.
- The welfare costs under non-taxation are then about 7.6 times much as under taxation.
- Combined with the low default rate, there is then no reason to avoid conducting optimal monetary and fiscal policy, which is equivalent to conducting inflation stabilization policy via optimal taxation.

5 Conclusion

- While sovereign risk continues to smolder throughout the Eurozone, the ECB is hesitating to stabilize inflation.
- However, as we show in our analysis, optimal monetary and fiscal policy itself, namely stabilizing inflation, does not worsen the default rate.
- Because the central bank's role is limited in a currency union, in an asymmetric two-country model the role of fiscal policy as a tool for minimizing welfare cost is greater than that in a simple currency union setting.
- In term of policy recommendations, governments should address taxation failure and policy authorities should not hesitate to stabilize inflation.