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Inflation Stabilization and Default Risk in a Currency Union *

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Abstract

By developing a class of dynamic stochastic general equilibrium models with nominal rigidities and assuming a two-country currency union with sovereign risk, we show that there is not necessarily a trade-off between the prevention of default risk and stabilizing inflation. Under optimal monetary and fiscal policy, comprising a de facto inflation stabilization policy, the tax rate as an optimal fiscal policy tool plays an important role in stabilizing inflation, although not completely because of the distorted steady state. Changes in the tax rate to minimize welfare costs via stabilizing inflation then improve the fiscal surplus, and because of this and the incompletely stabilized inflation, the default rate does not increase as much.

Keywords: Sovereign Risk; European Crisis; Optimal Monetary Policy; Fiscal Theory of the Price Level; Currency Union

JEL Classification: E52; E60; F41; F47

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1 Introduction

How do we conduct monetary policy in a currency union amid sovereign risk premiums? How do the monetary and fiscal authorities behave in this difficult situation? What clues do we have for removing the trade-off between the prevention of default risk and stabilizing inflation? In this paper, we show that there is not necessarily a trade-off between the prevention of default risk and stabilizing inflation. Policy authorities, namely, the central bank and the government, should without hesitation, conduct optimal monetary and fiscal policies, which is equivalent to stabilizing inflation.

In October 2009, there was a change of government in Greece and this revealed the presence of a severe fiscal deficit in that country. This incident was the trigger for a general European debt crisis and the yields on sovereign bonds in Greece began to deviate substantially from those prevailing in Germany at the time. Leading up to the bankruptcy of Lehman Brothers in September 2008 (Fig.1), the difference in yields on 10-year government bonds in Greece from those in Germany ranged from 13 to 67 basis points. The difference temporarily soared to 285 basis points amid the economic turmoil stemming from the Lehman Brothers' bankruptcy, after which it decreased to 121 basis points in August 2009, shortly before the change of political regime in Greece. Following the change of government, however, the difference between the yields on Greek and German sovereign bonds rapidly increased, ultimately topping 904 basis points in September 2010. This led to the establishment, in June 2010, of the European Financial System Facility (EFSF) to support the Greek economy.

However, sovereign risk premiums then began to rise sharply in not only Greece, but also elsewhere in the Eurozone. In November 2010, Ireland received support from the EFSF, followed by Portugal in May 2011. In November 2011, economic instability stemming from the expanding sovereign risk crisis brought about the resignation of Italian president Silvio Berlusconi. However, even with the support of the EFSF, the economic situation in Greece did not improve, and an additional program of support was set in place in February 2012, with increasing concerns in May 2012 that Greece would break from the European Monetary Union (EMU). By then, the sovereign risk crisis had extended to Spain, with the country seeking European Union (EU) support in June 2012.

Since then, the European sovereign risk crisis has widened even further, with Cyprus receiving support through the EFSF in March 2013. However, while the difference in 10-year government bond yields of the countries discussed over those in Germany now appears more stable than from 2010 to 2013, the deviation remains at a high level of some 144 to 474 basis points across Portugal, Ireland, Italy, Greece, and Spain (the so-called PIIGS). In addition, although the maximum yield differences decreased to 411 basis points in September 2013, down from 580 basis points in April 2013, they have now started to increase again, rising to 454 basis points by April 2014. It is germane that the deviation in basis points for 10-year bond yields from those Germany was only -5 in Ireland in October 2005, 13 in Greece in January 2005, 0 in Spain in August 2005, 14 in Italy in September 2003 through to February and March 2005, and 0 in Portugal in January and March 2005. Indeed, there was only a -6 basis point difference in bond yields in Cyprus in July 2007 over those in Germany, even before Cyprus had adopted the Euro. Therefore, we cannot say that the European sovereign debt crisis has ended by any means, and it continues to smolder to the present day.

At first impression, the European Central Bank (ECB) appears to face a trade-off between stabilization and the suppression of debt default. From January 1999 when the Euro was introduced until February 2009, the ECB's official policy rate was never less than the Harmonized Consumer Price Index (HCPI) measure of inflation, except for about 21 months.[1] On this basis, we assume that the ECB attempted to stabilize inflation in the Eurozone by keeping in mind a simple monetary model comprising the New Keynesian IS (NKIS) and Phillips curve (NKPC) introduced by Woodford[12], together with the Taylor principle. Amid the Greek crisis in March 2010, the ECB's policy rate began to fall below HCPI inflation and remained low. While HCPI inflation reached 3%, the policy rate remained at 1.5% and the ECB no longer sought to increase the policy rate. As of May 2014, the ECB policy rate is just 0.25%, still less than HCPI inflation rate of 0.7%. Therefore, it would seem that the ECB has given up on any attempt to stabilize inflation in the Eurozone, and is instead apparently more concerned with combating the still smoldering sovereign default risk. Furthermore, the ECB would even appear to welcome inflation, as this would help mitigate the burden associated with redeeming existing government debt.

In this paper, we develop a class of dynamic stochastic general equilibrium (DSGE) models with nominal rigidities and assume a currency union consisting of two countries. To develop our model, we drew upon earlier work in this area by Uribe[11] and Benigno[2]. Following the analysis in Uribe[11] of monetary policy in a closed economy with sovereign risk, we introduce government into our model. Uribe[11] assumes that default depends on the ratio of the net present value of the real fiscal surplus in terms of the marginal utility of consumption to real government debt with interest payments in terms of the marginal utility of consumption, namely, the burden of government debt redemption, such that the lower the ratio, the higher the probability of default. In our model, one of the two countries, which we refer to as 'country F' defaults on its financial obligations following the mechanism in Uribe[11]. In the other country, referred to as country H, this ratio is strictly unity, and thus 'country H' never defaults on its debt. In our model, the default mechanism is fully endogenous, although Corsetti, Kuester, Meier and Mueller[5], who analyze the impact of strained government finances on macroeconomic stability and the transmission of fiscal policy, assume default depending on an exogenous probability function. In assuming just two countries, we in effect consider that the Eurozone currently consists of countries without sovereign risk and those with sovereign risk.

In the other related study, Benigno[2] develops a micro-founded portfolio balance model, based on the somewhat conventional models in the field of international money and finance. Similar to our analysis, he assumes two countries in his model and the situation where households face difficulties in purchasing foreign assets. Because of this, households obtain less remuneration from the purchase of foreign assets, an idea we include in our model.[2] As mentioned, country F's government debt may fall into default. Because of this, country F's government has to pay additional interest. Formally, we introduce an interest rate multiplier as an increasing function of the fiscal deficit. By combining Uribe[11] and Benigno[2], in this analysis we are able to replicate the current European debt crisis, that is, the higher the fiscal deficit, the higher the level of government debt. Furthermore, the higher the level of government debt, the greater the default rate via the decrease in the ratio of the real fiscal surplus to real government debt with interest payments. In our model, we are able to replicate this actual scenario.

Our novel and important finding is that monetary policy aimed at minimizing welfare cost,

[1] August, September, and November 2003, April to December 2004, February, April, and June to November 2005, and January, February, and May 2005.

[2] If anything, our model is closer to Benigno[1]than Benigno[2].

namely, stabilizing inflation, and the prevention of default are not inconsistent. Note that the variances of these variables consist of our welfare cost function and optimal policy here is a policy that minimizes that function.[3] We compare a taxation regime where the governments in both countries in the currency union conduct an optimal 'fiscal' policy, in which the tax rate is a fiscal policy tool, with a non-taxation regime where only country H conducts optimal fiscal policy. Intuitively, inflation is stable and welfare costs minimized under the former rather than under the latter. Indeed, our numerical results show that GDP inflation, the welfare-relevant output gap, which is the deviation between the actual and target level output, and the terms of trade (TOT) deviation, which is the deviation between actual TOT and its efficient level, are stable under the taxation regime but not the non-taxation regime. Because CPI inflation is the weighted average of GDP inflation in our setting, CPI inflation under the taxation regime is stable as a result, unlike under the non-taxation regime. The tax rate, to be exact the tax gap (that is the percentage deviation of the tax rate from its steady state value in our model) works effectively, even though the nominal interest rate, one of the policy tools in our model, does not work hard. In fact, the volatility of the nominal interest rate is lower than that for the tax gap, similar to Ferrero[8] who also analyze optimal monetary and fiscal policy tools in a currency union. In addition, the volatility of the tax gap in country H under the non-taxation regime is higher than it would be under the taxation regime. That is, the tax gap in country H under non-taxation shoulders the tax gap in country F, which has no role in minimizing welfare costs given non-taxation.

If we assume that the ECB welcomes inflation from viewpoint of preventing default, is the ECB's policy stance correct? Now, we need to pay attention to the fact that default risk depends not only on CPI inflation, but also on a fiscal surplus as discussed. Although the evidence should be verified, both Bloomberg[4] and Ekathimerini[7] report that the Greek debt crisis is rooted in the failure of taxation in Greece. That is, taxation failure worsens a fiscal deficit, and this could have accounted for the Greek debt crisis. Our analysis under the non-taxation regime attempts to replicate the current situation for a number of Euro countries. Here, country F, in not conducting optimal fiscal policy and with a constant tax gap depicts the PIIGS most notably represented by Greece, while country H, in conducting optimal fiscal policy depicts Euro countries without any concerns with sovereign risk. As discussed, our results are that the default volatility under the non-taxation regime is higher than under the taxation regime. The volatilities of inflation and welfare costs are also lower under the taxation regime than under the non-taxation regime. Our policy implication is then that the policy authority should stabilize inflation without hesitation, and that the PIIGS, especially Greece, should strengthen their collection of taxes.

Finally, we discuss the relationship between our results and those in Corsetti, Kuester, Meier and Mueller[5] and Uribe[11]. To start with, our result is consistent with Corsetti, Kuester, Meier and Mueller[5] who analyze the impact of strained government finances on macroeconomic stability and the transmission of fiscal policy. Using a variant of the model by Curdia and Woodford[6], they study a 'sovereign risk channel' through which sovereign default risk raises funding costs in the private sector and show that fiscal retrenchment can help curtail the risk of macroeconomic instability and, in extreme cases, even stimulate economic activity. However, unlike this paper, they do not analyze it from the viewpoint of welfare costs.

Elsewhere, Uribe[11] shows that there is a trade-off between stabilizing inflation and preventing

[3] We derive our welfare cost function following Benigno and WoodfordBenignoWoodford05 because of the distorted steady state.

default and that default is inevitable in stabilizing inflation. This result and the associated policy implication are quite different from ours. This difference stems from differences in the fiscal deficit settings. In our model, because of the use of a DSGE model, the fiscal deficit is endogenous and depends on output and the tax rate. However, Uribe [11] assumes an endowment economy and no firm activity. The fiscal deficit is then an exogenous shock, such that there is a no way to improve the fiscal deficit to avoid sovereign default, and controlling the price level is the only way to affect the default rate. Thus, there is a simple trade-off between stabilizing the price level, namely stabilizing CPI inflation, and suppressing the rate of default. Although our model provides two ways to suppress default rate, namely, abandoning the stabilization of inflation and improving the fiscal deficit, Uribe's[11] analysis provides just a single policy, namely, abandoning inflation stabilization. Thus, there is the possibility with our analysis that we can simultaneously stabilize both inflation and the default rate if we adopt an endogenized fiscal deficit, which is a natural setting.

The remainder of the paper is organized as follows. Section 2 develops the model. Section 3 solves the linear-quadratic (LQ) problem, shows the ?rst-order necessary conditions (FONCs) for the policy authorities, and discusses the taxation and non-taxation regimes. Section 4 calibrates the model under both regimes. Section 5 concludes the paper.

2 The Model

We derive a model basically based on Okano[10], who also analyzed optimal monetary and fiscal policy in a currency union.[4] The currency union consists of countries H and F, which together organize a monetary union. The households on the interval $[0, \alpha)$ belong to country H while those on the interval $[\alpha, 1]$ belong to country F. We assume that there is a default risk in country F and the default mechanism follows that in Uribe[11]. In addition, we follow Benigno[1] (an earlier working paper version of Benigno[2]) to clarify the households' choice of risky assets.

2.1 Households

A representative household's preferences are given by:

$$\mathcal{U}_H \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_{H,t} \right) ; \mathcal{U}_F \equiv \mathrm{E}_0 \left(\sum_{t=0}^{\infty} \beta^t U_{F,t} \right), \tag{1}$$

where $U_{H,t} \equiv \ln C_t - \frac{1}{1+\varphi} N_{H,t}^{1+\varphi}$ and $U_{F,t} \equiv \ln C_t - \frac{1}{1+\varphi} N_{F,t}^{1+\varphi}$ denote the period utility in countries H and F, respectively, E_t is the expectation conditional on the information set at period t, $\beta \in (0, 1)$ is the subjective discount factor, C_t is the consumption index, $N_{H,t} \equiv \int_0^{\alpha} N_{H,t}(h)\, dh$ and $N_{F,t} \equiv \int_{\alpha}^{1} N_{H,t}(f)\, df$ are the hours of labor in countries H and F, respectively, and φ is the inverse of the elasticity of labor supply.

The consumption index is defined as follows:

$$C_t \equiv \left[\alpha^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

[4]Okano[10] expands Gali and Monacelli's [9] small economy model to a currency union model consisting of two countries following Ferrero[8]

where $C_{H,t} \equiv \left[\left(\frac{1}{\alpha}\right)^{\frac{1}{\varepsilon}} \int_0^{\alpha} C_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $C_{F,t} \equiv \left[\left(\frac{1}{1-\alpha}\right)^{\frac{1}{\varepsilon}} \int_{\alpha}^{1} C_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}}$ denote consumption subindexes of the continuum of differentiated goods produced respectively in countries H and F, $C_t(h)$ and $C_t(f)$ are generic goods produced in countries H and F, respectively, $\eta > 0$ is the elasticity of substitution between goods produced in countries H and F and $\varepsilon > 1$ is the elasticity of substitution across goods.

Total consumption expenditures are given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$, where $P_{H,t} \equiv \left[\left(\frac{1}{\alpha}\right) \int_0^{\alpha} P_t(h)^{1-\varepsilon} dh\right]^{\frac{1}{1-\varepsilon}}$, and $P_{F,t} \equiv \left[\left(\frac{1}{1-\alpha}\right) \int_{\alpha}^{1} P_t(f)^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}}$ denote the producer price indexes (PPIs) in countries H and F, respectively. In addition:

$$P_t = \left[\alpha P_{H,t}^{1-\theta} + (1-\alpha) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3}$$

denotes the consumer price index (CPI). By log-linearizing (3) and taking the first differential, we have:

$$\pi_t = \alpha\pi_{H,t} + (1-\alpha)\pi_{F,t}, \tag{4}$$

where $\pi_t \equiv p_t - p_{t-1}$ denotes CPI inflation and $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ and $\pi_{F,t} \equiv p_{F,t} - p_{F,t-1}$ denote GDP inflation in countries H and F, respectively. Note that we define $v_t \equiv \frac{dV_t}{V}$, which is the percentage deviation from an arbitrary variable's steady-state value where V_t is an arbitrary variable and V is the arbitrary variable's steady-state value.

We define the terms of trade (TOT) as $\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}}$, which can be log-linearized as $s_t = p_{F,t} - p_{H,t}$. Combining this log-linearized equality and the definition of PPI inflation yields:

$$\Delta s_t = \pi_{F,t} - \pi_{H,t}, \tag{5}$$

where $\Delta v_t \equiv v_t - v_{t-1}$ denotes changes in an arbitrary variable v_t. Eq.(5) implies that changes in the TOT corresponds to PPI inflation differential between countries H and F.

By solving cost-minimization problems for households, we have the optimal allocation of expenditures as follows:

$$C_t(h) = \frac{1}{\alpha}\left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}\ ;\ C_t(f) = \frac{1}{1-\alpha}\left(\frac{P_t(f)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}. \tag{6}$$

Hence, the total demand for goods produced in countries H and F is given by:

$$C_{H,t} = \alpha\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\ ;\ C_{F,t} = (1-\alpha)\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t. \tag{7}$$

By aggregating households' budget constraints, we have the aggregated budget constraints in countries H and F as follows:

$$B_{H,t-1}^n + B_{F,t-1}^n + W_{H,t}N_{H,t} + PR_{H,t} \geq P_tC_t + R_t^{-1}\left[B_{H,t}^n + \frac{B_{F,t}^n}{\mathrm{E}_t(1-\delta_{t+1})\Gamma\left(\widetilde{df}_{F,t}\right)}\right],$$
$$B_{H,t-1}^n + B_{F,t-1}^n + W_{F,t}N_{F,t} + PR_{F,t} \geq P_tC_t + R_t^{-1}\left[B_{H,t}^n + \frac{B_{F,t}^n}{\mathrm{E}_t(1-\delta_{t+1})\Gamma\left(\widetilde{df}_{F,t}\right)}\right], \tag{8}$$

where $R_t \equiv 1 + r_t$ denotes the gross risk-free nominal interest rate, r_t is the net interest rate , $B_{H,t}^n$ and $B_{F,t}^n$ are the nominal government debts issued by countries H's and F's government, respectively, $B_{H,t} \equiv \frac{B^n_{H,t}}{P_t}$ and $B_{H,t} \equiv \frac{B^n_{F,t}}{P_t}$ are the real government debts issued by countries H's and F's government, respectively, $W_{H,t}$ and $W_{F,t}$ are the nominal wage in countries H and F, respectively, $PR_{H,t}$ and $PR_{F,t}$ denote profits from the ownership of the firms in countries H and F, respectively, δ_t is the default rate and $\widetilde{df}_{F,t} \equiv \frac{\frac{B^n_{F,t}}{P_t} - \Pi_t^{-1} R_{t-1} \frac{B^n_{F,t-1}}{P_{t-1}}}{B_F(1-R)} - 1$ is the percentage deviation of changes in (real) government debt with interest payments, excluding the risk premium from its steady state value.

Eq.(8) is an unfamiliar but plausible depiction of the budget constraint in a country with sovereign risk. We drew upon Benigno[1] and formulate Eq.(8). Benigno[1] analyzes price stability with imperfect financial integration using a two-country DSGE model. He assumes that households in a country H face a burden in international financial markets. As borrowers, households in a country will be charged a premium on the foreign interest rate; as lenders, they will receive remuneration lower than the foreign interest rate. We replicate Benigno's[1] idea by assuming that the expected default rate is zero, namely $\mathrm{E}_t(\delta_{t+1}) = 0$ in our model. Plugging $\mathrm{E}_t(\delta_{t+1}) = 0$ into the first equality in Eq.(8), we have:

$$B^n_{H,t-1} + B^n_{F,t-1} + W_{H,t}N_{H,t} + PR_{H,t} \geq P_tC_t + R_t^{-1}\left[B^n_{H,t} + \frac{B^n_{F,t}}{\Gamma\left(\widetilde{df}_{F,t}\right)}\right],$$

which is close to the households budget constraint in country H in Benigno[1].

Now we discuss $\Gamma\left(\widetilde{df}_{F,t}\right)$, the interest rate multiplier for holding country F's government bond where $\widetilde{df}_{F,t} \equiv \frac{\frac{B^n_{F,t}}{P_t} - \Pi_t^{-1} R_{t-1} \frac{B^n_{F,t-1}}{P_{t-1}}}{B_F(1-R)} - 1$ denotes the percentage deviation of changes in (real) government debt with interest payments excluding the risk premium from its steady state value. The numerator of $\widetilde{df}_{F,t}$ corresponds to the (real) fiscal deficit if there is no default risk. Thus, we interpret $\widetilde{df}_{F,t}$ as the percentage deviation of the fiscal deficit from its steady state value if there is no default risk and we refer simply to $\widetilde{df}_{F,t}$ as the similar fiscal deficit in country F. The function $\Gamma\left(\widetilde{df}_{F,t}\right)$ captures the costs of households holding country F's government debt with default risk. As lenders, they will receive remuneration from the government besides the risk-free nominal interest rate.

This kind of cost is different from that in Benigno's[1] setting. As mentioned, Benigno[1] assumes that households in country H face a burden in international financial markets. As borrowers, households in this country will be charged a premium on the foreign interest rate; as lenders, they will receive remuneration lower than the foreign interest rate. Following his setting, Benigno[1] assumes $\Gamma'(\cdot) < 0$, which implies that the higher country F's government debt, the lower the remuneration for holding country F's government debt.[5] However, on the contrary, our setting implies that the higher country F's government debt via the worsening of the fiscal deficit, the lower the remuneration for holding country F's government debt because of default. Thus, country F has to pay additional remuneration for holding country F's government debt, which provides households with a motivation for holding country F's government debt. Thus, we assume that $\Gamma'(\cdot) > 0$. The representative household in a currency union maximizes Eq.(1) subject to Eq.(8).

[5]Benigno[1] mentions that this function, which depends only on the level of real government bonds in his setting, captures the costs of undertaking positions in the international asset market or the existence of intermediaries in the foreign asset market.

The optimality conditions are given by:

$$\beta \mathrm{E}_t \left(\frac{P_t C_t}{P_{t+1} C_{t+1}} \right) = \frac{1}{R_t}, \tag{9}$$

which is the standard intertemporal optimality condition and:

$$C_t N_{H,t}^{\varphi} = \frac{W_{H,t}}{P_t} \; ; \; C_t N_{F,t}^{\varphi} = \frac{W_{F,t}}{P_t}, \tag{10}$$

which are standard intratemporal optimality conditions.

Because we introduce default risk for country F's government, households are not indifferent between holding country H's government debt and holding country F's debt. By holding government H's debt, the condition that the marginal utility of nominal income is constant over time if the nominal interest rate remains at its steady state value is applied and we have Eq.(12), the Euler equation. By holding country F's debt, Eq.(12) still applies, although it requires an additional condition, which depicts households' motivation to hold country F's government debt, stemming from one of the FONCs for households, which can be obtained by differentiating the Lagrangian by country F's government nominal debt. That condition is given by:

$$\begin{aligned} \frac{1}{R_t}\left[\frac{1}{\mathrm{E}_t\left(1-\delta_{t+1}\right)\Gamma\left(\widetilde{df}_t\right)}-1\right] &= \mathrm{E}_t\left\{\frac{1}{R_{t+1}\left(1-\delta_{t+2}\right)\Gamma\left(\widetilde{df}_{t+1}\right)}B_{F,t+1}\frac{\Gamma'\left(\widetilde{df}_{t+1}\right)}{\Gamma\left(\widetilde{df}_{t+1}\right)}\right. \\ &\quad \left. -\frac{1}{R_t\left(1-\delta_{t+1}\right)\Gamma\left(\widetilde{df}_t\right)}B_{F,t}\frac{\Gamma'\left(\widetilde{df}_t\right)}{\Gamma\left(\widetilde{df}_t\right)}\right\}\frac{1}{SP} \end{aligned} \tag{11}$$

where $SP \equiv \tau Y - G$ denotes the fiscal surplus in the steady state and note that variables without time script denotes their steady state value. Eq.(11) implies that the differences on return on between holding safety and risky assets depends on an expected change on real payoff of holding risky assets.

Log-linearizing Eq.(11) yields:

$$\mathrm{E}_t\left(\delta_{t+1}\right) = \omega_b b_{F,t} + \frac{1-\beta}{\phi\beta}\delta_t + (1-\beta)\hat{r}_t - (\gamma-2\phi)\pi_t - (\gamma-2\phi)\hat{r}_{t-1} - (\gamma-2\phi) b_{F,t-1} \tag{12}$$

with $\omega_b \equiv \gamma - 2\phi - (1-\beta)$ and $\hat{r}_t \equiv \frac{dR_t}{R}$. Following Benigno[1], we define $\phi \equiv \Gamma'(0)$, refer to it as the interest rate spread for country F's government debt in the steady state and assume $\Gamma(0) = 1$ following Benigno[1] who calibrates his model by assuming ten basis point spread for the nominal interest rate. Unfamiliar parameter, $\gamma \equiv \frac{\Gamma''(0)}{\Gamma'(0)}$ denotes the elasticity of the interest rate spread to a one percent change in the fiscal deficit in the steady state. We assume $\mid \Gamma'(\cdot) \mid < \mid \Gamma''(\cdot) \mid$ thus $\gamma > 1$. Our assumption implies that changes in the fiscal deficit alter the interest rate spread in the steady state, although this pressure is larger than interest rate spread itself. By envisaging that the yields on risky assets increase at an increasing rate, we can accept this assumption.

Eq.(12) provides us with some additional economic implications of Eq.(11). The first term on the right-hand side RHS in Eq.(12) shows that an increase in country F's government debt increases the expected default rate decreases. The third implies that an increase in the nominla interest rate increases the expected default rate via increasing the burden of redemption. The fourth term shows that an increase in CPI inflation decreases the expected default rate. An increase in CPI inflation decreases fiscal deficits via a decrease in the real value of past government debt. Thus, te expected default rate decreases. In addition, when $\phi = 0$ implting that there are no interest rate spreads,

$$\hat{\delta}_t = 0$$

is applied and the default risk disappears.

2.2 Firms

This subsection depicting the production, price setting and marginal cost and features of the firms is quite similar to Gali and Monacelli[9], although here tax is levied on firm sales and is not constant.[6]

A typical firm in each country produces a differentiated good with a linear technology represented by the production function:

$$Y_t(h) = A_{H,t} N_{H,t}(h), \quad ; \quad Y_t(f) = A_{F,t} N_{F,t}(f),$$

where $Y_{H,t}(h)$ and $Y_{F,t}(f)$ denote the output of a generic good in countries H and F, respectively, and $A_{H,t}$ and $A_{F,t}$ denote the productivity in countries H and F, respectively.

Analogous to consumption indexes, we define $Y_{H,t} \equiv \left[\left(\frac{1}{\alpha}\right) \int_0^\alpha Y_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $Y_{F,t} \equiv \left[\left(\frac{1}{1-\alpha}\right) \int_\alpha^1 Y_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}}$. Combining these definitions and the PPI indexes, we have:

$$Y_t(h) = \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} Y_{H,t} \; ; \; Y_t(f) = \left(\frac{P_t(f)}{P_{F,t}}\right)^{-\varepsilon} Y_{F,t}. \tag{13}$$

By combining the production technology in the currency union and Eq.(13), we have an aggregate production function relating to aggregate employment as follows:

$$N_{H,t} = \frac{Y_{H,t} Z_{H,t}}{A_{H,t}} \; ; \; N_{F,t} = \frac{Y_{F,t} Z_{F,t}}{A_{F,t}}, \tag{14}$$

where $Z_{H,t} \equiv \int_0^\alpha \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\varepsilon} dh$ and $Z_{F,t} \equiv \int_\alpha^1 \left(\frac{P_t(f)}{P_{F,t}}\right)^{-\varepsilon} df$ denote the price dispersions in countries H and F, respectively.

Log-linearizing Eq.(14) yields:

$$n_{H,t} = y_{H,t} - a_{H,t} \; ; \; n_{F,t} = y_{F,t} - a_{F,t}. \tag{15}$$

Notice that $Z_{H,t}$ and $Z_{F,t}$ disappear in Eq.(15) because these are $o\left(\|\xi\|^2\right)$.

Each firm is a monopolistic producer of one of the differentiated goods. Each firm sets their prices $P_t(h)$ and $P_t(f)$ taking as given P_t, $P_{H,t}$, $P_{F,t}$ and C_t. We assume that firms set prices in a staggered fashion in the Calvo–Yun style, according to which each seller has the opportunity to change its price with a given probability $1-\theta$, where an individual firm's probability of reoptimizing in any given period is independent of the time elapsed since it last reset its price. When a firm has the opportunity to set a new price in period t, it does so in order to maximize the expected

[6] Unlike our setting, Gali and Monacelli[9] assume that constant employment subsidies and monopolistic power completely disappear.

discounted value of its net profits. The FONCs for firms are given by:

$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}\theta^k\beta^k\tilde{Y}_{H,t+k}\left(\tilde{P}_{H,t}-\frac{\varepsilon}{\varepsilon-1}P_{H,t+k}MC_{H,t+k}\right)\right] = 0,$$
$$\mathrm{E}_t\left[\sum_{k=0}^{\infty}\theta^k\beta^k\tilde{Y}_{F,t+k}\left(\tilde{P}_{F,t}-\frac{\varepsilon}{\varepsilon-1}P_{F,t+k}MC_{F,t+k}\right)\right] = 0, \tag{16}$$

where $MC_{H,t} \equiv \frac{W_{H,t}}{(1-\tau_{H,t})P_{H,t}A_{H,t}}$ and $MC_{F,t} \equiv \frac{W_{F,t}}{(1-\tau_{F,t})P_{F,t}A_{F,t}}$ denote the real marginal costs in countries H and F, respectively, $\tilde{Y}_{H,t+k} \equiv \left(\frac{\tilde{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon}Y_{H,t+k}$ and $\tilde{Y}_{F,t+k} \equiv \left(\frac{\tilde{P}_{F,t}}{P_{F,t+k}}\right)^{-\varepsilon}Y_{F,t+k}$ denote the demand for goods produced in countries H and F, respectively, when firms choose a new price, $\tilde{P}_{H,t}$ and $\tilde{P}_{F,t}$ denote the newly set prices in countries H and F, respectively, and $\tau_{H,t}$ and $\tau_{F,t}$ denote the tax rates in countries H and F, respectively.

By log-linearizing Eq.(16), we have:

$$\pi_{H,t} = \beta\mathrm{E}_t\left(\pi_{H,t+1}\right)+\kappa mc_{H,t},$$
$$\pi_{F,t} = \beta\mathrm{E}_t\left(\pi_{F,t+1}\right)+\kappa mc_{F,t}, \tag{17}$$

with $\kappa \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}$ being the slop of New Keynesian Philips curve (NKPC). Eq.(17) is fundamental equality of our NKPC.

Substituting Eq.(10) into the definition of the real marginal cost yields:

$$MC_{H,t} = \frac{P_t}{P_{H,t}}\frac{C_tN_{H,t}^{\varphi}}{(1-\tau_{H,t})A_{H,t}},$$
$$MC_{F,t} = \frac{P_t}{P_{F,t}}\frac{C_tN_{F,t}^{\varphi}}{(1-\tau_{F,t})A_{F,t}}, \tag{18}$$

which can be rewritten as:

$$C_t^{-1}MC_{H,t}(1-\tau_{H,t}) = \frac{P_t}{P_{H,t}}\frac{N_{H,t}^{\varphi}}{A_{H,t}},$$
$$C_t^{-1}MC_{F,t}(1-\tau_{F,t}) = \frac{P_t}{P_{F,t}}\frac{N_{F,t}^{\varphi}}{A_{H,t}},$$

which implies that monopolistic power is no longer completely dissolved by optimal taxation. Note that the marginal cost in the steady state, which is the inverse of a constant markup, is smaller than one, while the gross tax rate is definitely smaller than one. In such a case, the steady-state wedge between the marginal product of labor and the marginal utility of consumption is not unity. That is, monopolistic power remains because it is unable to be completely absorbed through taxation. As we discuss later, we need to derive our welfare criteria following Benigno and Woodford[3] because monopolistic power is no longer removed completely and the steady state is distorted.

Log-linearizing Eq.(18) yields:

$$mc_{H,t} = c_t+\varphi y_{H,t}+(1-\alpha)s_t+\frac{\tau}{1-\tau}\hat{\tau}_{H,t}-(1+\varphi)a_{H,t},$$
$$mc_{F,t} = c_t+\varphi y_{F,t}-\alpha s_t+\frac{\tau}{1-\tau}\hat{\tau}_{F,t}-(1+\varphi)a_{F,t}, \tag{19}$$

where $\hat{\tau}_{H,t} \equiv \frac{d\tau_{H,t}}{\tau_H}$ and $\hat{\tau}_{F,t} \equiv \frac{d\tau_{F,t}}{\tau_F}$ denote the percentage deviation of the tax rate from its steady-state value in countries H and F, respectively, and $s_t \equiv p_{F,t}-p_{H,t}$ denotes the (logarithmic) terms of trade. We simply refer to the percentage deviation of the tax rate from its steady-state value $\hat{\tau}_{H,t}$ and $\hat{\tau}_{F,t}$ as the tax gap.

2.3 Government

Government plays an important role in the model because fiscal policy is an important policy tool used to minimize welfare costs.

Similar to private consumption, government expenditure is a Dixit–Stiglitz aggregator defined by:

$$G_{H,t} \equiv \left[\left(\frac{1}{\alpha} \right) \int_0^\alpha G_t(h)^{\frac{\varepsilon-1}{\varepsilon}} dh \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad ; \quad G_{F,t} \equiv \left[\left(\frac{1}{1-\alpha} \right) \int_\alpha^1 G_t(f)^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $G_{H,t}$ and $G_{F,t}$ denote the indexes of government expenditure in countries H and F, respectively. For simplicity, we assume that each government allocates a level of government expenditure only among domestic goods. Each government implies the following demands for the generic goods h and f:

$$G_t(h) = \left(\frac{P_t(h)}{P_{H,t}} \right)^{-\varepsilon} G_{H,t} \; ; \; G_t(f) = \left(\frac{P_t(f)}{P_{F,t}} \right)^{-\varepsilon} G_{F,t}. \tag{20}$$

Substituting Eq.(20) into flow government budget constraints yields:

$$\begin{aligned} B^n_{H,t} &= R_{t-1} B^n_{H,t-1} - P_{H,t} \left(\tau_{H,t} Y_{H,t} - G_{H,t} \right), \\ B^n_{F,t} &= R_{t-1} \Gamma \left(\widetilde{df}_{F,t-1} \right) (1-\delta_t) B^n_{F,t-1} - P_{F,t} \left(\tau_{F,t} Y_{F,t} - G_{F,t} \right). \end{aligned} \tag{21}$$

The first and second equalities in Eq.(21) are the government budget constraints in countries H and F, respectively. The first terms on the RHS correspond to the amount of redemption with the nominal interest payment. In particular, the first term on the RHS in the second equality shows that country F's government pays higher interest payments because of sovereign risk.

The appropriate transversality condition for country H's government debt is given by:

$$\lim_{j \to \infty} \beta^{t+j+1} \mathrm{E}_t \left(R_{t+j} \frac{B^n_{H,t+j}}{P_{t+j+1}} \right) = 0.$$

Iterating Eq.(21) forward, and imposing the appropriate transversality condition for country H's government debt, we have:

$$1 = \frac{\sum_{k=0}^\infty \beta^k \mathrm{E}_t \left(C^{-1}_{t+k} SP_{H,t+k} \right)}{C_t^{-1} R_{t-1} \frac{B^n_{H,t-1}}{P_t}}, \tag{22}$$

where $SP_{H,t} \equiv \frac{P_{H,t}}{P_t} \left(\tau_{H,t} Y_{H,t} - G_{H,t} \right)$ denotes the (nominal) fiscal surplus in terms of CPI, which we simply refer to hereafter as the real fiscal surplus. Eq.(22) is quite similar to the central equation of the Fiscal Theory of the Price Level (FTPL). On the RHS in Eq.(22), the numerator is the net present value of the sum of fiscal surplus in terms of the marginal utility of consumption, namely solvency, and the denominator is the past nominal government debt with interest payments divided by current CPI, namely the government's liability. The LHS is unity. Thus, Eq.(22) implies that solvency definitely corresponds to the government's liability. If solvency is less than the government's liability, CPI increases, that is, inflation occurs so that the government's liability is mitigated. Because Eq.(22) can be rewritten as:

$$1 = \frac{\sum_{k=0}^\infty \beta^k \mathrm{E}_t \left(C^{-1}_{t+k} SP_{H,t+k} \right)}{C_t^{-1} R_{t-1} B_{H,t-1} \Pi_t^{-1}},$$

this mechanism can be easily understood by paying attention to the denominator on the RHS.

Eq.(22) can be rewritten as follows:

$$R_{t-1}\Pi_t^{-1}B_{H,t-1} = SP_{H,t} + \beta \mathrm{E}_t\left(\frac{C_t}{C_{t+1}}\Pi_{t+1}^{-1}\right)R_t B_{H,t}.$$

Log-linearizing this equality yields:

$$c_t = \mathrm{E}_t\left(c_{t+1}\right) - \hat{r}_t + \mathrm{E}_t\left(\pi_{t+1}\right) - b_{H,t} + \frac{1}{\beta}b_{H,t-1} + \frac{1}{\beta}\hat{r}_{t-1} - \frac{1}{\beta}\pi_t - \frac{1-\beta}{\beta}sp_{H,t}. \tag{23}$$

Unlike the familiar log-linearized Euler equation, the terms for government debt and the fiscal surplus appear in Eq.(23), because this equality as well as the government budget constraint is log-linearized. The sign for the fiscal surplus term is negative and this implies that taxation decreases consumption and government expenditure stimulates consumption.

Next, we discuss the government budget constraint in country F where there is default risk. The appropriate transversality condition for country F's government debt is given by:

$$\lim_{j\to\infty}\beta^{t+j+1}\mathrm{E}_t\left[R_{t+j}\Gamma\left(\widetilde{df}_{F,t+j}\right)\left(1-\delta_{t+j+1}\right)\frac{P_{t+j}B_{F,t+j}}{P_{t+j+1}}\right] = 0.$$

Similar to Eq.(22), iterating the second equality in Eq.(21) forward and imposing the appropriate transversality condition for country F's government debt, we have:

$$1-\delta_t = \frac{\sum_{k=0}^{\infty}\beta^k\mathrm{E}_t\left(C_{t+k}^{-1}SP_{F,t+k}\right)}{C_t^{-1}R_{t-1}\Gamma\left(\widetilde{df}_{F,t-1}\right)\frac{B_{F,t-1}^n}{P_t}}. \tag{24}$$

Eq.(24) implies that an increase in CPI, namely inflation, does not necessarily occur if solvency is less than the government's debt. Not only inflation, but also default, mitigates the government's liability. Suppose that CPI is constant and there is no inflation. In this situation, if the solvency is about equal to the government's liability, the RHS is less than unity. On the other hand, the LHS is definitely less than unity through an increase in the default rate. In other words, if the government falls insolvent while CPI is strictly stable, default is inevitable. Uribe[11] pointed out that there is a trade-off between inflation stabilization and suppressing default by introducing default, namely sovereign risk, into the central equation of the FTPL. Similar to Uribe[11], at first glance Eq.(24) also implies that there is such a trade-off. Furthermore, he calibrates his model and compares it with the Taylor rule, which stabilizes inflation and the interest rate peg. Under the interest rate peg, the interest rate on risky assets corresponds to the risk-free asset interest rate pegged to the steady-state rate. This calibration further shows that default ceases just one period after the shock decreased the fiscal surplus, although default continues under the Taylor rule. This result implies that a Taylor rule to stabilize inflation includes the unwelcome possibility of magnifying sovereign risk, and this calls for an interest rate peg to counter default. Although Uribe[11] ignores the welfare perspective of these actions, his policy implications are persuasive. Paying attention to just Eq.(24), which is similar to Uribe's[11] model, we seem to obtain policy implications quite similar to those in Uribe[11].

Eq.(24) can be rewritten as a second-order differential equation as follows:

$$R_{t-1}\Gamma\left(\widetilde{df}_{t-1}\right)\Pi_t^{-1}\left(1-\delta_t\right)B_{F,t-1} = SP_{F,t} + \beta\mathrm{E}_t\left[\left(\frac{C_t}{C_{t+1}}\Pi_{t+1}^{-1}\right)R_t\Gamma\left(\widetilde{df}_t\right)\left(1-\delta_{t+1}\right)B_{F,t}\right].$$

Log-linearizing this equality yields:

$$\begin{aligned} c_t &= \mathrm{E}_t\left(c_{t+1}\right) - \hat{r}_t + \mathrm{E}_t\left(\pi_{t+1}\right) + \mathrm{E}_t\left(\delta_{t+1}\right) - \frac{\tilde{\omega}_\phi}{1-\beta} b_{F,t} + \frac{\hat{\omega}_\phi}{(1-\beta)\beta} b_{F,t-1} + \frac{\phi}{(1-\beta)\beta} b_{F,t-2} \\ &+ \frac{\tilde{\omega}_\phi}{(1-\beta)\beta}\hat{r}_{t-1} + \frac{\phi}{(1-\beta)\beta}\hat{r}_{t-2} - \frac{\tilde{\omega}_\phi}{(1-\beta)\beta}\pi_t - \frac{\phi}{(1-\beta)\beta}\pi_{t-1} - \frac{1}{\beta}\delta_t \\ &- \frac{1-\beta}{\beta} sp_{F,t}, \end{aligned} \tag{25}$$

with $\tilde{\omega}_\phi \equiv 1-\beta(1+\phi)$ and $\hat{\omega}_\phi \equiv 1-\beta[1+\phi(1+\beta)]$. Unlike Eq.(23), the expected default rate appears in the fourth term on the RHS in Eq.(25) because of the presence of sovereign risk in country F. However, when we assume $\delta_t = 0$ for all t and $\phi = 0$, that is, there is neither sovereign risk nor an interest rate spread, Eq.(25) corresponds to Eq.(18) with the exception that $b_{F,t}$ and $sp_{F,t}$ replaces $b_{H,t}$ and $sp_{H,t}$. The sign for the fiscal surplus term is negative and this implies that taxation decreases consumption and government expenditure stimulates consumption.

By log-linearizing the definition of the fiscal surplus, we have:

$$\begin{aligned} sp_{H,t} &= -(1-\alpha)s_t + \frac{\beta\tau}{(1-\beta)\varsigma_B} y_{H,t} + \frac{\beta\tau}{(1-\beta)\varsigma_B}\hat{\tau}_{H,t} - \frac{\beta\varsigma_G}{(1-\beta)\varsigma_B} g_{H,t} \\ sp_{F,t} &= \alpha s_t + \frac{\beta\tau}{(1-\beta)\varsigma_B} y_{F,t} + \frac{\beta\tau}{(1-\beta)\varsigma_B}\hat{\tau}_{F,t} - \frac{\beta\varsigma_G}{(1-\beta)\varsigma_B} g_{F,t}. \end{aligned} \tag{26}$$

Eq.(26) includes not only terms for output, the tax gap and government expenditure but also the TOT. Because the fiscal surplus $SP_{H,t}$ and $SP_{F,t}$ is the nominal fiscal surplus deflated by the CPI, the TOT appears in Eq.(26).

2.4 Equilibrium

2.4.1 Market-Clearing Conditions

Up to now, our model's features do not include market-clearing conditions, which in our analysis are quite similar to that in Gali and Monacelli[9] and Ferrero[8].

The market-clearing conditions in a currency union are given by:

$$\begin{aligned} Y_t(h) &= C_t(h) + C_t^*(h) + G_t(h), \\ Y_t(f) &= C_t(f) + C_t^*(f) + G_t(f), \end{aligned}$$

where $C_{H,t}^*(h)$ and $C_{F,t}^*(f)$ denote foreign demand for country H's goods and country F's goods, respectively. By combining Eqs.(6), (7), (13) and these market-clearing conditions, we have:

$$\begin{aligned} Y_{H,t} &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + G_{H,t}, \\ Y_{F,t} &= \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t + G_{F,t}. \end{aligned} \tag{27}$$

By log-linearizing Eq.(27), we obtain:

$$\begin{aligned} y_{H,t} &= (1-\varsigma_G)c_t + (1-\alpha)\eta(1-\varsigma_G)s_t + \varsigma_G g_{H,t}, \\ y_{F,t} &= (1-\varsigma_G)c_t - \alpha\eta(1-\varsigma_G)s_t + \varsigma_G g_{F,t}. \end{aligned} \tag{28}$$

Combining each equality in Eq.(28) yields:

$$y_t = (1-\varsigma_G)c_t + \alpha\varsigma_G g_{H,t} + (1-\alpha)\varsigma_G g_{F,t}, \tag{29}$$

where $y_t \equiv \alpha y_{H,t} + (1-\alpha) y_{F,t}$ denotes the (logarithmic) weighted-average output, namely, output in the currency union. Eq.(29) is the (logarithmic) currency union's market-clearing condition. Similar to Eq.(29), we have:

$$s_t = \frac{1}{\eta(1-\varsigma_G)} y_{H,t} - \frac{1}{\eta(1-\varsigma_G)} y_{F,t} - \frac{\varsigma_G}{\eta(1-\varsigma_G)} g_{H,t} + \frac{\varsigma_G}{\eta(1-\varsigma_G)} g_{F,t}, \tag{30}$$

such that TOT corresponds to the output differential between countries H and F, not including the government expenditure differential.

2.5 Welfare Costs

We derive the period welfare cost function from the welfare criterion. Because of the distorted steady state, we derive the welfare criterion following Benigno and Woodford[3]. We obtain the welfare cost function by subtracting a second-order approximated FONC for firms from a second-order approximated utility function to eliminate the linear terms that lead to the incorrect evaluation of welfare. [7]

The higher values of this welfare criterion correspond to the lower values of the welfare cost function, which is given by:

$$\mathcal{W}^W = -\mathcal{L}^W + \Upsilon_0 + \text{t.i.p.} + o\left(\|\xi\|^3\right),$$

where $\mathcal{W}^W \equiv \alpha \sum_{t=0}^{\infty} \beta^t (U_{H,t} - U) + (1-\alpha) \sum_{t=0}^{\infty} \beta^t (U_{F,t} - U)$ denotes the welfare criterion, which is the weighted average of the deviation of utility from its steady state, Υ_0 denotes a transitory component consisting of the average initial price dispersion, t.i.p. are terms independent of policy and $o\left(\|\xi\|^3\right)$ are terms of order three or higher and:

$$\mathcal{L}^W \equiv \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t L_t^W$$

denotes the weighted-average welfare costs. Furthermore:

$$L_t^W \equiv \frac{\alpha}{2} \Lambda_\pi \pi_{H,t}^2 + \frac{1-\alpha}{2} \Lambda_\pi \pi_{F,t}^2 + \frac{1}{2} \Lambda_x x_t^2 + \frac{\alpha(1-\alpha)}{2} \Lambda_z z_t^2 \tag{31}$$

denotes the weighted average welfare costs, $x_t \equiv y_t - y_t^e$ is the welfare-relevant output gap, $y_t^e \equiv \frac{\alpha\omega_{15}}{\omega_{14}} a_{H,t} + \frac{(1-\alpha)\omega_{15}}{\omega_{14}} a_{F,t} + \frac{\alpha\omega_{17}}{\omega_{14}} g_{H,t} + \frac{(1-\alpha)\omega_{17}^*}{\omega_{14}} g_{F,t}$ is the (logarithmic) efficient level of weighted-average output, $z_t \equiv s_t - s_t^e$ is the percentage deviation of TOT from its efficient level, $s_t^e \equiv \frac{\omega_{19}}{\omega_{18}} a_{H,t} - \frac{\omega_{19}}{\omega_{18}} a_{F,t} - \frac{\omega_{20}}{(1-\varsigma_G)\omega_{18}} g_{H,t} + \frac{\omega_{20}^*}{(1-\varsigma_G)\omega_{18}} g_{F,t}$ is the (logarithmic) efficient level of TOT, $\Lambda_\pi \equiv \frac{\varepsilon\{1-\Phi[1-\kappa(1-\varsigma_G)]\}}{\kappa(1-\varsigma_G)}$ is the weight of GDP inflation in welfare costs and $\Lambda_x \equiv \frac{\omega_{14}}{(1-\varsigma_G)^2}$ is the weight on the welfare-relevant output gap and $\Lambda_z \equiv (1-\varsigma_G)\eta^2(1-\Phi)(1+\varphi) + \Phi\kappa\tilde{\Omega}_1$ denotes the weight on the percentage deviation of TOT from its efficient level with $\omega_{14} \equiv (1-\Phi)(1+\varphi)(1-\varsigma_G) + \Phi\kappa\tilde{\Omega}_6$, $\omega_{15} \equiv (1-\varsigma_G)(1+\phi)[1-\Phi(1-\kappa\omega_{12})]$, $\omega_{17} \equiv \Phi\kappa\varsigma_G\left[(1-\alpha)\tilde{\Omega}_6 - \tilde{\Omega}_5\right]$, $\omega_{17}^* \equiv \Phi\kappa\varsigma_G\left(\alpha\tilde{\Omega}_6 - \tilde{\Omega}_5\right)$, $\omega_{18} \equiv (1-\varsigma_G)\eta^2(1-\Phi)(1+\varphi) + \Phi\kappa\tilde{\Omega}_1$, $\omega_{19} \equiv (1+\varphi)\{\eta(1-\Phi) + \Phi\kappa[\eta(1-\varsigma_G)+\varphi]\}$, $\omega_{20} \equiv$

[7] Sutherland[?] and Benigno and Woodford[3] derive the second-order approximated utility function without the presence of a linear term under the distorted steady state. Thus, we follow Benigno and WoodfordBenignoWoodford05 to derive the period loss function because of the distorted steady state in our model. Note that Woodford[12] discusses how the presence of linear terms generally leads to the incorrect evaluation of welfare. A simple example of this result is proposed by Kim and Kim[?].

$\eta\left(1-\varsigma_G\right)\left(1-\Phi\right)\left(1+\varphi\right)+\Phi\kappa\varsigma_G\left(\tilde{\Omega}_3+\alpha\tilde{\Omega}_4\right)$, $\omega_{20}^*\equiv\eta\left(1-\varsigma_G\right)\left(1-\Phi\right)\left(1+\varphi\right)+\Phi\kappa\varsigma_G\left[\tilde{\Omega}_3^*+\left(1-\alpha\right)\tilde{\Omega}_4^*\right]$, $\omega_{12}\equiv\left(1-\varsigma_G\right)\left(1+\varphi\right)-\varsigma_G$, $\Phi\equiv 1-\frac{1-\tau}{\frac{\varepsilon}{\varepsilon-1}}$ denotes the steady-state wedge between the marginal rate of substitution between consumption and leisure and the marginal product of labor and $\tilde{\Omega}_1$, $\tilde{\Omega}_3$, $\tilde{\Omega}_3^*$, $\tilde{\Omega}_4$, $\tilde{\Omega}_4^*$, $\tilde{\Omega}_5$ and $\tilde{\Omega}_6$ are combinations of the structural parameters defined in the Appendix. Hereafter, we simply refer to the percentage deviation of TOT from its efficient level z_t as the TOT gap.

2.6 Welfare-Relevant Output Gap and Dynamics

2.6.1 NKISs

In our model, the welfare-relevant output gap is the deviation of output not from the natural level of output but rather from the efficient level of output. Combining the definition of the welfare-relevant output gap, the efficient level of output, the definition of the TOT gap, and the efficient level of TOT and Eqs.(23), (26), (30) and (31) yields:

$$\begin{aligned} x_t \quad = \quad & \frac{\varsigma_B}{\omega_1}\mathrm{E}_t\left(x_{t+1}\right)-\frac{\left(1-\varsigma_G\right)}{\omega_1}\hat{r}_t+\frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}\mathrm{E}_t\left(\pi_{t+1}\right)-\frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}b_{H,t}+\frac{\left(1-\varsigma_G\right)\varsigma_B}{\beta\omega_1}\hat{r}_{t-1} \\ & +\frac{\left(1-\varsigma_G\right)\varsigma_B}{\beta\omega_1}b_{H,t-1}+\frac{\left(1-\varsigma_G\right)\varsigma_B}{\beta\omega_1}\pi_t-\frac{\left(1-\alpha\right)\left(1-\varsigma_G\right)}{\beta\omega_1}z_t-\frac{\left(1-\varsigma_G\right)\tau}{\omega_1}\hat{\tau}_{H,t}+\psi_{H,t}, \quad (32)\end{aligned}$$

where $\psi_{H,t}\equiv-\frac{\Omega_1}{\Omega_\psi}a_{H,t}-\frac{(1-\alpha)\Omega_2}{\Omega_\psi}a_{F,t}+\frac{\Omega_3}{\Omega_\psi}g_{H,t}-\frac{(1-\alpha)\Omega_4}{\Omega_\psi}g_{F,t}$ denotes the demand shock in country H with Ω_ψ, Ω_1, Ω_2, Ω_3 and Ω_4 being the combinations of the structural parameters defined in the Appendix.

Combining the definition of the welfare-relevant output gap, the efficient level of output, the definition of the TOT gap, and the efficient level of TOT and Eqs.(25), (26), (30) and (31) yields:

$$\begin{aligned} x_t \quad = \quad & \frac{\varsigma_B}{\omega_1}\mathrm{E}_t\left(x_{t+1}\right)-\frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}\hat{r}_t+\frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}\mathrm{E}_t\left(\pi_{t+1}\right)+\frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}\mathrm{E}_t\left(\delta_{t+1}\right)-\frac{\left(1-\varsigma_G\right)\varsigma_B\tilde{\omega}_\phi}{\omega_1\left(1-\beta\right)}b_{F,t} \\ & +\frac{\left(1-\varsigma_G\right)\varsigma_B\hat{\omega}_\phi}{\omega_1\left(1-\beta\right)\beta}b_{F,t-1}+\frac{\left(1-\varsigma_G\right)\varsigma_B\phi}{\omega_1\left(1-\beta\right)}b_{F,t-2}+\frac{\left(1-\varsigma_G\right)\varsigma_B\tilde{\omega}_\phi}{\omega_1\left(1-\beta\right)\beta}\hat{r}_{t-1}+\frac{\left(1-\varsigma_G\right)\varsigma_B\phi}{\omega_1\left(1-\beta\right)\beta}\hat{r}_{t-2} \\ & -\frac{\left(1-\varsigma_G\right)\varsigma_B\tilde{\omega}_\phi}{\omega_1\left(1-\beta\right)\beta}\pi_t-\frac{\left(1-\varsigma_G\right)\varsigma_B\phi}{\omega_1\left(1-\beta\right)\beta}\pi_{t-1}-\frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1\beta}\delta_t+\frac{\alpha\left(1-\varsigma_G\right)\omega_2}{\beta\omega_1}z_t-\frac{\left(1-\varsigma_G\right)\tau}{\omega_1}\hat{\tau}_{F,t} \\ & +\psi_{F,t}, \quad (33)\end{aligned}$$

where $\psi_{F,t}\equiv-\frac{\Omega_1^*}{\Omega_\psi}a_{F,t}-\frac{\alpha\Omega_2}{\Omega_\psi}a_{H,t}+\frac{\Omega_3^*}{\Omega_\psi}g_{F,t}-\frac{\alpha\Omega_4^*}{\Omega_\psi}g_{H,t}$ denotes the demand shock in country F with Ω_1^*, Ω_3^* and Ω_4^* being combinations of the structural parameters defined in the Appendix. Eq.(33) is the NKIS in country F. Although Eq.(33) is quite complicated because of the sovereign risk, Eq.(33) is simplified to:

$$\begin{aligned} x_t \quad = \quad & \frac{\varsigma_B}{\omega_1}\mathrm{E}_t\left(x_{t+1}\right)-\frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}\hat{r}_t+\frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}\mathrm{E}_t\left(\pi_{t+1}\right)-\frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1}b_{F,t}+\frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1\beta}\hat{r}_{t-1} \\ & +\frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1\beta}b_{F,t-1}-\frac{\left(1-\varsigma_G\right)\varsigma_B}{\omega_1\beta}\pi_t+\frac{\alpha\left(1-\varsigma_G\right)\omega_2}{\beta\omega_1}z_t-\frac{\left(1-\varsigma_G\right)\tau}{\omega_1}\hat{\tau}_{F,t}+\psi_{F,t},\end{aligned}$$

if we assume $\phi=0$ and $\delta_t=0$ for all t, that is, there is no sovereign risk. This equality is similar to Eq.(32).

2.6.2 NKPCs

Similar to the NKISs, the NKPCs are derived by combining the definition of the welfare-relevant output gap, the efficient level of output, the definition of TOT deviation from its efficient level, and the efficient level of TOT and Eqs.(17), (19), (29) and (30). The NKPC in country H is given by:

$$\pi_{H,t} = \beta \mathrm{E}_t \left(\pi_{H,t+1}\right) + \kappa \left(1+\varphi\right) x_t + \kappa \left(1-\alpha\right) \left(1+\varphi\right) z_t + \frac{\kappa\tau}{1-\tau}\hat{\tau}_{H,t} + \epsilon_{H,t}, \tag{34}$$

where $\epsilon_{H,t} \equiv \frac{\kappa\Omega_7}{\Omega_\epsilon} a_{H,t} + \frac{\kappa(1-\alpha)\Omega_8}{\Omega_\epsilon} a_{F,t} + \frac{\kappa\Omega_5}{(1-\varsigma_G)\Omega_\epsilon} g_{H,t} + \frac{\kappa(1-\alpha)\Omega_9^*}{(1-\varsigma_G)\Omega_\epsilon} g_{F,t}$ denotes the cost-push shock in country H with Ω_ϵ, Ω_5, Ω_7 and Ω_9^* being the combinations of the structural parameters defined in the Appendix. Because of the distorted steady state, the cost-push shock appears in the NKPC and inflation–output gap trade-off is no longer dissolved by monetary and fiscal policy.

Similar to Eq.(34), The NKPC in country F is given by:

$$\pi_{F,t} \quad = \quad \beta \mathrm{E}_t \left(\pi_{F,t+1}\right) + \kappa \left(1+\varphi\right) x_t - \kappa\alpha \left(1+\varphi\right) z_t + \frac{\kappa\tau}{1-\tau}\hat{\tau}_{F,t} + \epsilon_{F,t}, \tag{35}$$

where $\epsilon_{F,t} \equiv \frac{\kappa\Omega_7^*}{\Omega_\epsilon} a_{F,t} + \frac{\kappa\alpha\Omega_8}{\Omega_\epsilon} a_{H,t} + \frac{\kappa\Omega_5^*}{(1-\varsigma_G)\Omega_\epsilon} g_{F,t} + \frac{\kappa\alpha\Omega_9}{(1-\varsigma_G)\Omega_\epsilon} g_{H,t}$ denotes the cost-push shock in country F with Ω_5^*, Ω_7^* and Ω_9 being combinations of the structural parameters defined in the Appendix.

2.6.3 The TOT Gap

Plugging the definitions of the welfare-relevant output gap and the TOT gap into Eq.(5), we have:

$$\Delta z_t = \pi_{F,t} - \pi_{H,t} + \frac{\omega_{21}}{\left(1-\tau\right)\omega_{18}}\Delta\hat{\tau}_{H,t} - \frac{\omega_{21}}{\left(1-\tau\right)\omega_{18}}\Delta\hat{\tau}_{F,t} + \Delta\xi_t, \tag{36}$$

with $\xi_t \equiv -\frac{\omega_1 9}{\omega_{18}} a_{H,t} + \frac{\omega_1 9}{\omega_{18}} a_{F,t} + \frac{\omega_{20}}{(1-\varsigma_G)\omega_{18}} - \frac{\omega_{20}^*}{(1-\varsigma_G)\omega_{18}} g_{F,t}$ being the TOT shock, which implies that the GDP inflation differential and changes in the tax rate affect changes in the TOT deviation from its efficient level.

3 The LQ Problem

To clarify the role of the tax gap as a policy instrument, we analyze the taxation regime and the non-taxation regime. Under the taxation regime, country F's government changes the tax rate in country F to minimize welfare costs while the non-taxation regime, country F's government, does not change the tax rate. Thus, under the taxation regime, the available policy instruments are the nominal interest rate $\hat{r}_t$, the tax gap in country H $\hat{\tau}_{H,t}$ and the tax gap in country F $\hat{\tau}_{F,t}$ while under the non-taxation regime the policy instruments are the nominal interest rate $\hat{r}_t$ and the tax gap in country H $\hat{\tau}_{H,t}$. Note that the non-taxation regime depicts the actual situation of taxation failure in Greece, as discussed.

The non-taxation regime lacks one of the available policy instruments. Thus, we forecast that welfare costs in the non-taxation regime will be larger than in the taxation regime. The welfare costs are minimized by stabilizing the fluctuation in GDP inflation, the welfare-relevant output gap and the TOT gap. Especially, the weight on the welfare-relevant output gap Λ_x is the largest weight in the period loss function Eq.(31). In fact, under our plausible parameterization, discussed in Section 4.1, to calibrate the model, the weight on GDP inflation Λ_π, the weight on the welfare-relevant output gap Λ_x and the weight on the TOT gap Λ_z are 151.08, 4.28 and 45.22, respectively. Clearly, optimal policy is consistent with inflation-stabilization policy. Because the taxation regime has more policy instruments available to minimize welfare costs, which is consistent with an inflation-stabilization policy, the taxation regime is more aggressive in stabilizing inflation than the non-taxation regime.

The policy authority minimizes weighted-average welfare costs subject to Eqs.(4), (12), (32)–(36) and the log-linearized average and relative government budget constraints.[8] Under the taxation regime, the policy authorities choose the sequence $\{x_t, z_t, \pi_{H,t}, \pi_{F,t}, \pi_t, \hat{r}_t, \hat{\tau}_{H,t}, \hat{\tau}_{F,t}, b_{H,t}, b_{F,t}, \delta_t\}_{t=0}^{\infty}$ while in the non-taxation regime the policy authorities select the sequence $\{x_t, z_t, \pi_{H,t}, \pi_{F,t}, \pi_t, \hat{r}_t, \hat{\tau}_{H,t}, b_{H,t}, b_{F,t}, \delta_t\}_{t=}^{\infty}$

3.1 Taxation Regime

Now, we discuss the FONCs under taxation. The FONC for the welfare-relevant output gap is given by:

$$\begin{aligned} x_t \quad = \quad & -\frac{1}{\Lambda_x}\mu_{1,t} - \frac{1}{\Lambda_x}\mu_{2,t} + \frac{\kappa(1+\varphi)}{\Lambda_x}\mu_{3,t} + \frac{\kappa(1+\varphi)}{\Lambda_x}\mu_{4,t} - \frac{\tau\tilde{\phi}_1}{\Lambda_x\varsigma_B\bar{\omega}_\phi}\mu_{5,t} - \frac{\tau\tilde{\phi}_0}{\Lambda_x\varsigma_B\bar{\omega}_\phi}\mu_{6,t} \\ & + \frac{\varsigma_B}{\beta\Lambda_x\omega_1}\mu_{1,t-1} + \frac{\varsigma_B}{\beta\Lambda_x\omega_1}\mu_{2,t-1}, \end{aligned} \tag{37}$$

with $\bar{\omega} \equiv 1-(\beta-\phi)$ where $\mu_{1,t}$, $\mu_{2,t}$, $\mu_{3,t}$, $\mu_{4,t}$, $\mu_{5,t}$ and $\mu_{6,t}$ are the Lagrange multipliers for Eqs.(32)–(35) and the log-linearized average and relative government budget constraints, respectively. Because we assume that the policy authorities commit their policy following Woodford's [12] timeless perspective, a lagged Lagrange multiplier appears.

The FONC for the TOT gap is given by:

$$\begin{aligned} \alpha(1-\alpha)z_t \quad = \quad & -\frac{(1-\alpha)(1-\varsigma_G)\omega_2}{\beta\Lambda_z\omega_1}\mu_{1,t} + \frac{\alpha(1-\varsigma_G)\omega_2}{\beta\Lambda_z\omega_1}\mu_{2,t} + \frac{(1-\alpha)\kappa(1+\varphi)}{\Lambda_z}\mu_{3,t} - \frac{\alpha\kappa(1+\varphi)}{\Lambda_z}\mu_{4,t} \\ & -\frac{\alpha(1-\alpha)\tilde{\phi}_0\omega_2}{\beta\Lambda_z\varsigma_B\bar{\omega}_\phi}\mu_{5,t} - \frac{\tilde{\phi}_2\omega_2}{\beta\Lambda_z\varsigma_B\bar{\omega}_\phi}\mu_{6,t} - \frac{1}{\Lambda_z}\mu_{7,t} + \frac{\beta}{\Lambda_z}\mathrm{E}_t(\mu_{7,t+1}), \end{aligned} \tag{38}$$

where $\mu_{7,t}$ is Lagrange multiplayer on Eq.(36).

The FONCs for GDP inflation are given by:

$$\alpha\pi_{H,t} \quad = \quad -\frac{1}{\Lambda_\pi}(\mu_{3,t}-\mu_{3,t-1}) - \frac{1}{\Lambda_\pi}\mu_{7,t} + \frac{\alpha}{\Lambda_\pi}\mu_{8,t} \tag{39}$$

$$(1-\alpha)\pi_{F,t} \quad = \quad -\frac{1}{\Lambda_\pi}(\mu_{4,t}-\mu_{4,t-1}) + \frac{1}{\Lambda_\pi}\mu_{7,t} + \frac{1-\alpha}{\Lambda_\pi}\mu_{8,t}, \tag{40}$$

where $\mu_{8,t}$ is the Lagrange multiplier for Eq.(4). Lagrange multipliers $\mu_{3,t}$ and $\mu_{4,t}$ appear in Eqs.(37)–(40), that is, there is a relationship between the stabilization of the welfare-relevant output gap, the TOT gap and GDP inflation.

The FONC for the nominal interest rate is given by:

$$\begin{aligned} \mu_{1,t}+\mu_{2,t} \quad = \quad & \frac{(1-\beta)\omega_1}{(1-\varsigma_G)\varsigma_B}\mu_{9,t} + \mathrm{E}_t(\mu_{1,t+1}) + \frac{\tilde{\omega}_\phi}{1-\beta}\mathrm{E}_t(\mu_{2,t+1}) + \frac{\omega_1}{(1-\varsigma_G)\varsigma_G}\mathrm{E}_t(\mu_{5,t+1}) \\ & -\frac{\beta(\gamma-2\phi)\omega_1}{(1-\varsigma_G)\varsigma_G}\mathrm{E}_t(\mu_{9,t+1}), \end{aligned} \tag{41}$$

where $\mu_{9,t}$ is the Lagrange multiplier for Eq.(12).

[8] These government budget constraints are derived by log-linearizing Eq.(21).

The FONCs for the tax gap are given by:

$$\begin{aligned}\mu_{1,t} &= \frac{\kappa\omega_1}{(1-\varsigma_G)(1-\tau)}\mu_{3,t} - \frac{\alpha\omega_1}{(1-\varsigma_G)\varsigma_B}\mu_{5,t} - \frac{\omega_1}{(1-\varsigma_G)\varsigma_B}\mu_{6,t} + \frac{\omega_1\omega_{21}}{(1-\varsigma_G)\tau(1-\tau)\omega_{18}}\mu_{7,t} \\ &\quad - \frac{\beta\omega_1\omega_{21}}{(1-\varsigma_G)\tau(1-\tau)\omega_{18}}\mu_{7,t+1}, \qquad (42)\\ \mu_{2,t} &= \frac{\kappa\omega_1}{(1-\varsigma_G)(1-\tau)}\mu_{4,t} - \frac{(1-\alpha)(1-\beta)\omega_1}{(1-\varsigma_G)\varsigma_B\bar{\omega}_\phi}\mu_{5,t} + \frac{(1-\alpha)(1-\beta)\omega_1}{(1-\varsigma_G)\varsigma_B\bar{\omega}_\phi}\mu_{6,t} \\ &\quad - \frac{\omega_1\omega_{21}}{(1-\varsigma_G)\tau(1-\tau)\omega_{18}}\mu_{7,t} + \frac{\beta\omega_1\omega_{21}}{(1-\varsigma_G)\tau(1-\tau)\omega_{18}}\mu_{7,t+1}, \qquad (43)\end{aligned}$$

which shows the linkages between the NKISs, the NKPCs and the TOT gap via the tax gap. That is, changes in the tax gap simultaneously affect the welfare-relevant output gap, GDP inflation, and the TOT deviation.

The FONC for CPI inflation is given by:

$$\begin{aligned}\mu_{8,t} &= -\frac{(1-\varsigma_G)\varsigma_B}{\beta\omega_1}\mu_{1,t} - \frac{(1-\varsigma_G)\varsigma_B\tilde{\omega}_\phi}{\beta(1-\beta)\omega_1}\mu_{2,t} - \frac{1}{\beta}\mu_{5,t} - (\gamma-2\phi)\mu_{9,t} + \frac{(1-\varsigma_G)\varsigma_B}{\beta\omega_1}\mu_{1,t-1} \\ &\quad + \frac{(1-\varsigma_G)\varsigma_B}{\beta\omega_1}\mu_{2,t-1} - \frac{(1-\varsigma_G)\varsigma_B\phi}{(1-\beta)\omega_1}\mathrm{E}_t(\mu_{2,t+1}), \qquad (44)\end{aligned}$$

which shows that there is a relationship between the CPI inflation rate and the default rate because of the fourth term on the RHS in Eq.(44) and the sign of that term is negative. That is, there is a trade-off between stabilizing inflation and preventing default.

Finally, the FONC for the default rate is given by:

$$\mu_{9,t} = \frac{\phi(1-\varsigma_G)\varsigma_B}{(1-\beta)\omega_1}\mu_{2,t} + \frac{\phi(1-\alpha)\beta}{\bar{\omega}_\phi}\mu_{5,t} + \frac{\phi\beta}{\beta\bar{\omega}_\phi}\mu_{6,t} + \frac{\phi(1-\varsigma_G)\varsigma_B}{(1-\beta)\omega_1}\mu_{2,t-1} + \frac{\phi}{1-\beta}\mu_{9,t-1}, \quad (45)$$

which shows that the default rate is affected by the welfare-relevant output gap in country F and government debt in country F. As shown in Eq.(21), changes in government debt with interest payments correspond to the fiscal deficit. Thus, Eq.(45) implies that the default rate has a relationship with the fiscal surplus in that an increase in the fiscal surplus suppresses the increase in government debt. Because the signs of the second and third terms on the RHS in Eq.(45) are positive, an increase in the tax gap, which suppresses or eliminates government debt, decreases the default rate. When we assume $\phi = 0$, that is, there are no interest rate spreads, Eq.(45) boils down to $\mu_{9,t} = 0$ which implies that the defailt rate dynamics Eq.(12) does no longer work as a constraint in a whole system.

Eqs.(42) and (43) show that the tax gap is a strong policy tool to stabilize inflation because of the positive sign in the first terms of Eqs.(42) and (43). In addition, Lagrange multipliers $\mu_{5,t}$ and $\mu_{6,t}$ appear in (42) and (43). This implies that the tax gap affects the fiscal surplus. The second terms in Eqs.(42) and (43) are negative, that is, an increase in the tax gap increases the (average) fiscal surplus. In addition, the third term on the RHS in Eq.(43) is positive. This implies that an increase in the tax gap in country F decreases government debt in country F compared with country H, because $\mu_{6,t}$, which appears in the third term on the RHS in Eq.(43), is the Lagrange multiplier on the log-linearized relative block government budget constraint. Thus, the tax gap is not only a policy tool to minimize welfare costs via stabilizing inflation, but also controls the default rate via the effect on the fiscal surplus. As we have already pointed out, there is a trade-off between stabilizing inflation and preventing default because of Eq.(44). However, because an increase in the tax gap stabilizes inflation and subdues the fiscal deficit, that trade-off appears obscurely in our model. Rather, stabilizing inflation is consistent with preventing default via controlling the tax gap, namely conducting optimal fiscal policy.

3.2 Non-taxation Regime

Under non-taxation, the tax gap in country F $\tau_{F,t}$ is definitely zero. Country F's government collects the tax but does not change the tax rate nor conduct optimal fiscal policy. Only the central bank and country H's government conduct optimal policy. The FONC for the tax gap in country F, Eq.(43), is no longer available under non-taxation. As a result, stabilizing inflation may be difficult because of a lack of policy tools and Eq.(44) implies that the trade-off between stabilizing inflation and preventing default weakens under non-taxation. At first impression, it appears that default may not be as harsh as the taxation, although inflation is not so stable. However, we obtain the stabilization of inflation through not only monetary policy, but also fiscal policy. An increase in the tax gap then not only stabilizes inflation but also suppresses default via improvement in the fiscal balance. Thus, we cannot necessarily say that optimal policy increases the default rate, even under non-taxation, although the effect on suppressing default is smaller.

4 Numerical Analysis

4.1 Parameterization

We run a series of dynamic simulations and adopt the following benchmark parameterization. Following the analysis of optimal monetary and fiscal policy in a currency union in Okano[10], we set the values for the subjective discount factor β, the elasticity of substitution across goods ε, the inverse of the labor supply elasticity φ, the steady-state share of government bonds to output σ_B and the steady-state share of government expenditure to output σ_G, the steady-state tax rate τ, the persistence of productivity shocks and the persistence of government spending shocks to 0.99, 11, 3, 2.4, 0.276, 0.705 and 0.8, respectively,[9] Although Okano[10] assumes the share of population in country H equals 0.5, we set α to 0.68, which corresponds to the GDP share of the PIIGS among the 17 countries in the Euro area in 2013Q4. In addition, we set the elasticity of substitution between goods produced in countries H and F θ to 4.5, following Ferrero[8].

Because of the introduction of default risk, there are several novel parameters to be specified, including the spread of the nominal interest rate and the elasticity of the interest rate spread to a one percent change in the fiscal deficit or government debt γ. Following Benigno[1], we set ϕ equal to 0.1.[10] Although, we cannot find any empirical results concerning the elasticity of the interest rate spread to a one percent change in the fiscal deficit or government debt γ, we set it to 2.0, which implies that the interest rate spread increases by 200 basis points when the fiscal deficit increases by 1% in the steady state.

[9] $\sigma_B = 2.4$ is consistent with quarterly time periods in the model and implies that the annual steady-state debt–output ratio is 0.6.

[10] Benigno[1] regards ϕ as interest rate spreads in the steady state.

4.2 Macroeconomic Dynamics

4.2.1 Macroeconomic Volatility and Correlation

To examine the impulse response functions (IRFs), we consider a one percent change in productivity in countries H and F and a one percent change in government expenditure in countries H and F. Before showing the IRFs themselves, we discuss macroeconomic volatility (Tab. 1). The volatility of the default rate under the taxation model is 1.85, which is lower than it under non-taxation 3.14. In addition, the volatility of GDP and CPI inflation under taxation are lower than under non-taxation. That is, the volatilities of CPI inflation in the taxation and the non-taxation are 0.21 and 0.84, repectively and those of GDP inflation in country H in the taxation and the non-taxation are 0.14 and 1.15, respectively. The volatility of GDP inflation in country F is 0.50 in both the taxation and the non-taxation. These results means that there is not necessarily a trade-off between stabilizing inflation and suppressing the default rate. Underlying this result, the channel between the fiscal surplus and the default rate has an important role in suppressing the latter. Lastly, the volatility of the fiscal surplus in country F under taxation is higher than under non-taxation.

We calculate the correlation between selected variables, the inflation, the default rate and the fiscal surplus and this identifies which variables contribute to suppressing the default rate (Tab. 2). Note that IRFs and correlations are calculated given no-persistence in the shocks to sharpen the intuition about the mechanism for the propagation of shocks following Ferrero[8]. The correlation between CPI inflation in both countries and the default rate is -0.84 in the taxation. This intimates that the higher is inflation, the lower is the default rate and vice versa. However, the correlation between the fiscal surplus in country F and the default rate is -0.72 in the taxation. This shows that the fiscal surplus channel cannot be ignore from the view point of surpressing default rate and if the changes in fiscal surplus is larger than the CPI inflation, the fiscal surplus channel is more important than the inflation channel from that view point. An increase in the tax gap then decreases GDP inflation on the NKPCs, as shown in Eq.(35) while this increase decreases the default rate via an increase in the fiscal surplus in country F, as shown in Eqs.(25) and (26). Thus, Uribe's [11] policy implication is not applicable in our model and the latter effect, namely the fiscal surplus channel, overcomes the former effect, namely the CPI inflation channel in our parameterization. At the least, in our setting stabilizing inflation is then consistent with suppressing the default rate.

4.2.2 IRFs under Taxation

We can understand our result using the IRFs. We first discuss IRFs to shocks in country H. Here, an increase in government expenditure in country H increases the fiscal surplus in country F increases fiscal surplus in country H and decreases government debt in country H (Panels 9 and 11 in Fig.3). To surpress inflationary pressure in country H, the tax gap in country H is hiked to absorb this pressure (Panle 7 in Fig.3). Because an increase in the tax gap covers government expenditure, the fiscal surplus rises and the government debt goes down. Similar to the tax gap in country H, the tax gap in country F is hiked to surpress inflationary pressure generated in country H (Panel 8 in Fig.3). However, an increase in the nominal interes rate absorbs that pressure and the tax gap in country H declines in the next period (Panel 7 and 8 in Fig.3). Thus, in country F, the government debt increases and the fiscal surplus decreases (Panels 10 and 12 in Fig.3).

Deteriorating in the fiscal balance in country F induces an increase in default rate (Panel 13 in Fig.3).

While an increase in government expenditure in country H, increases the default rate, an increase in productivity in country H does not necessarily increase the default rate. Rather, the default rate becomes negative (Panel 13 in Fig.3). While a negative default rate appears unrealistic, advanced redemption corresponds to a negative default rate in our model. An increase in the productivity of country H increases the fiscal surplus in both countries via a currency union-wide economic boom (Panels 11 and 12 in Fig.3). To stabilize GDP inflation, both countries then hike the tax gap on the NKPCs, Eqs.(34) and (35) (Panels 3, 4, 11 and 12 in Fig.3). Because CPI inflation climbs slightly via the increase in GDP inflation in both countries, the default rate becomes negative combined with an increase in the fiscal surplus in country F.

We now discuss the IRFs to shocks in country F. An increase in government expenditure in country F provides pressure to increase GDP inflation in country F and the tax gap in country F is hiked to stabilize this increase (Panel 8 in Fig. 4). This response is consistent with the result in Ferrero[8] and Okano[10], who show that fiscal policy plays an important role. An increase in the tax gap then increases the fiscal surplus in country F (Panel 12 in Fig. 4). The default rate slightly rises but almost zero (Panel 13 in Fig. 4). This slight increase in the default rate is an effect on slightly negative fiscal balance in country F (Panel 12 in Fig.fig:4).

An increase in productivity in country F generates pressure to increase GDP inflation in both countries and this is absorbed by an increase in the tax gap in both countries (Panels 7 and 8 in Fig.4). However, because of the distorted steady state, GDP inflation is not completely stabilized and CPI inflation increases somewhat (Panels 2, 3 and 5 in Fig.4). An increase in productivity and the tax gap then increases the fiscal surplus. These increases make the default rate negative along with the increase in the CPI inflation (Panel 13 in Fig.4).

4.2.3 IRFs under Non-taxation

The IRFs under non-taxation are quite different from those under the taxation, although the underlying mechanism is quite similar. The IRFs under non-taxation take much time to settle. Except for the case of an increase in productivity in country F, the default rate under non-taxation does not settle faster than the default rate under taxation (Panel 13 in Figs.5 and 6). This stems from the lack of a specific policy tool, that is, the tax gap in country F is zero over time (Panel 8 in Fig.5 and 6). Because the nominal interest rate and the tax gap in country H take over the role of the tax gap in country F, the nominal interest rate takes more time to settle and the tax gap in country H becomes more volatile (Panels 5 and 7 in Figs.5 and 6). Given the tax gap in country F is zero, the fiscal surplus in country F then takes more time to settle and this prolongs the settling of the default rate after the shocks (Panel 13 in Figs.5 and 6).

Needless to say, the lack of a specific policy tool increases the welfare cost. Following Gali and Monacelli[9] and Okano[10], we set $\beta \to 1$ for the weighted-average welfare costs and obtain:

$$\mathcal{L}^W = \frac{\alpha}{2}\Lambda_\pi \text{var}\left(\pi_{H,t}\right) + \frac{1-\alpha}{2}\Lambda_\pi \text{var}\left(\pi_{F,t}\right) + \frac{1}{2}\Lambda_x \text{var}\left(x_t\right) + \frac{\alpha\left(1-\alpha\right)}{2}\Lambda_z \text{var}\left(z_t\right).$$

As discussed, under our plausible parameterization, the weight on GDP inflation Λ_π, the weight on the welfare-relevant output gap Λ_x and the weight on the TOT gap Λ_z are 151.08, 4.28 and 45.22, respectively. Thus, the welfare costs under taxation and non-taxation are 10.79 and 81.90, respectively. The welfare costs under non-taxation are then about 7.6 times much as under taxation.

Combined with the low default rate, there is then no reason to avoid conducting optimal monetary and fiscal policy, which is equivalent to conducting inflation stabilization policy via optimal taxation.

5 Conclusion

While sovereign risk continues to smolder throughout the Eurozone, the ECB is hesitating to stabilize inflation. However, as we show in our analysis, optimal monetary and fiscal policy itself, namely stabilizing inflation, does not worsen the default rate. Because the central bank's role is limited in a currency union, in an asymmetric two-country model the role of fiscal policy as a tool for minimizing welfare cost is greater than that in a simple currency union setting. In term of policy recommendations, governments should address taxation failure and policy authorities should not hesitate to stabilize inflation.

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Table 1: Macroeconomic Volatility

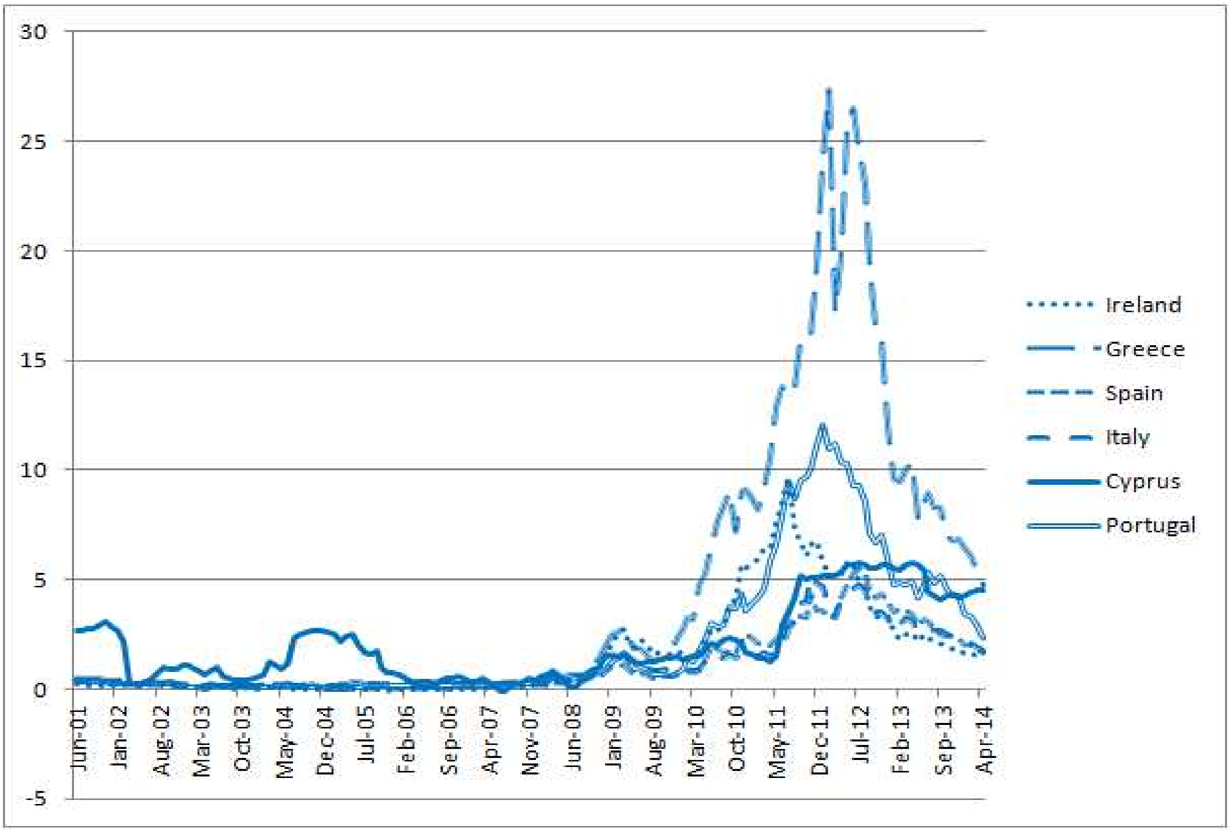
Variable	Taxation	Non-taxation
x_t	1.05	1.08
z_t	0.53	1.05
π_t	0.21	0.84
$\pi_{H,t}$	0.14	1.15
$\pi_{F,t}$	0.50	0.50
$\hat{r}_t$	0.49	0.71
$\hat{\tau}_{H,t}$	9.53	49.59
$\hat{\tau}_{F,t}$	26.87	NA
$b_{H,t}$	28.41	45.10
$b_{F,t}$	0.34	0.35
δ_t	1.85	3.14
$sp_{H,t}$	123.73	604.81
$sp_{F,t}$	323.31	37.77

Note: Standard Deviation in Percentages.

Table 2: Correlation of Selected Variables under Taxation

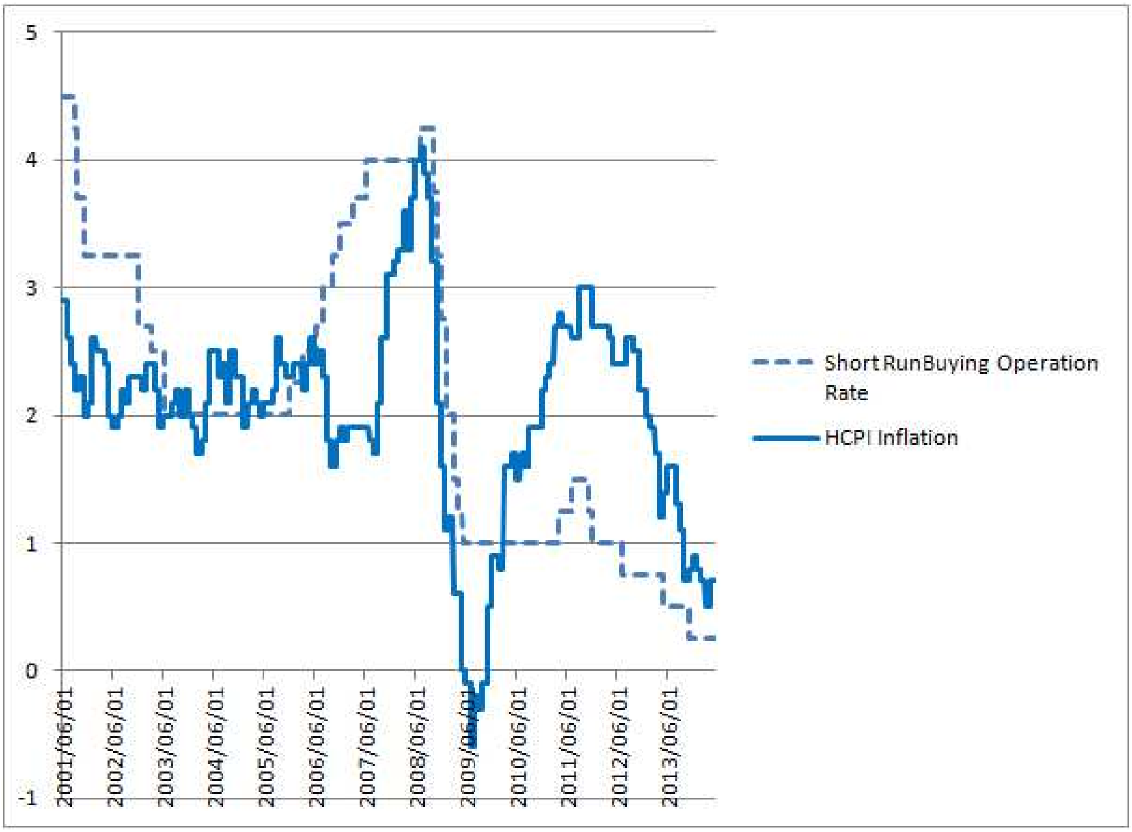
Variable	Regime	π_t	$\pi_{H,t}$	$\pi_{F,t}$	δ_t	$sp_{H,t}$	$sp_{F,t}$
x_t	Taxation	0.14	0.27	0.06	0.02	-0.72	-0.04
	Non-taxation	0.00	-0.02	0.25	-0.06	-0.15	-0.43
z_t	Taxation	0.53	0.73	0.34	-0.07	-0.08	0.71
	Non-taxation	-0.43	-0.41	-0.36	0.44	-0.57	-0.74
π_t	Taxation	1.00	0.79	0.95	-0.84	0.22	0.93
	Non-taxation	1.00	0.99	0.42	-0.99	0.96	0.55
$\pi_{H,t}$	Taxation	0.79	1.00	0.54	-0.41	0.20	0.83
	Non-taxation	1.00	1.00	0.34	-0.99	0.96	0.55
$\pi_{F,t}$	Taxation	0.95	0.54	1.00	-0.82	0.20	0.82
	Non-taxation	0.42	0.34	1.00	-0.34	0.43	0.34
$\hat{r}_t$	Taxation	-0.79	-0.49	-0.82	0.91	-0.17	-0.76
	Non-taxation	-0.92	-0.91	-0.34	0.85	-0.96	-0.61
$\hat{\tau}_{H,t}$	Taxation	0.22	0.23	0.17	-0.03	1.00	0.33
	Non-taxation	0.96	0.95	0.43	-0.91	1.00	0.70
$\hat{\tau}_{F,t}$	Taxation	0.93	0.82	0.82	-0.77	0.33	1.00
	Non-taxation	NA	NA	NA	NA	NA	NA
$b_{H,t}$	Taxation	0.08	0.13	0.04	-0.11	0.11	0.01
	Non-taxation	-0.02	-0.03	0.02	0.00	0.04	0.51
$b_{F,t}$	Taxation	-0.69	-0.88	-0.48	0.22	-0.39	-0.75
	Non-taxation	-0.96	-0.94	-0.51	0.90	-0.98	-0.66
δ_t	Taxation	-0.84	-0.41	-0.93	1.00	-0.07	-0.72
	Non-taxation	-0.99	-0.99	-0.34	1.00	-0.91	-0.49
$sp_{H,t}$	Taxation	0.22	0.20	0.20	-0.07	1.00	0.33
	Non-taxation	0.96	0.96	0.43	-0.91	1.00	0.69
$sp_{F,t}$	Taxation	0.93	0.83	0.82	-0.72	0.33	1.00
	Non-taxation	0.56	0.55	0.34	-0.49	0.69	1.00

Figure 1: Deviation in 10-Year Government Bond Yields from Germany



Source: ECB

Figure 2: Buying Operation Rate and HCPI Inflation



Source: Eurostat and Bloomberg

Figure 3: IRFs to Shocks in Country H under Taxation

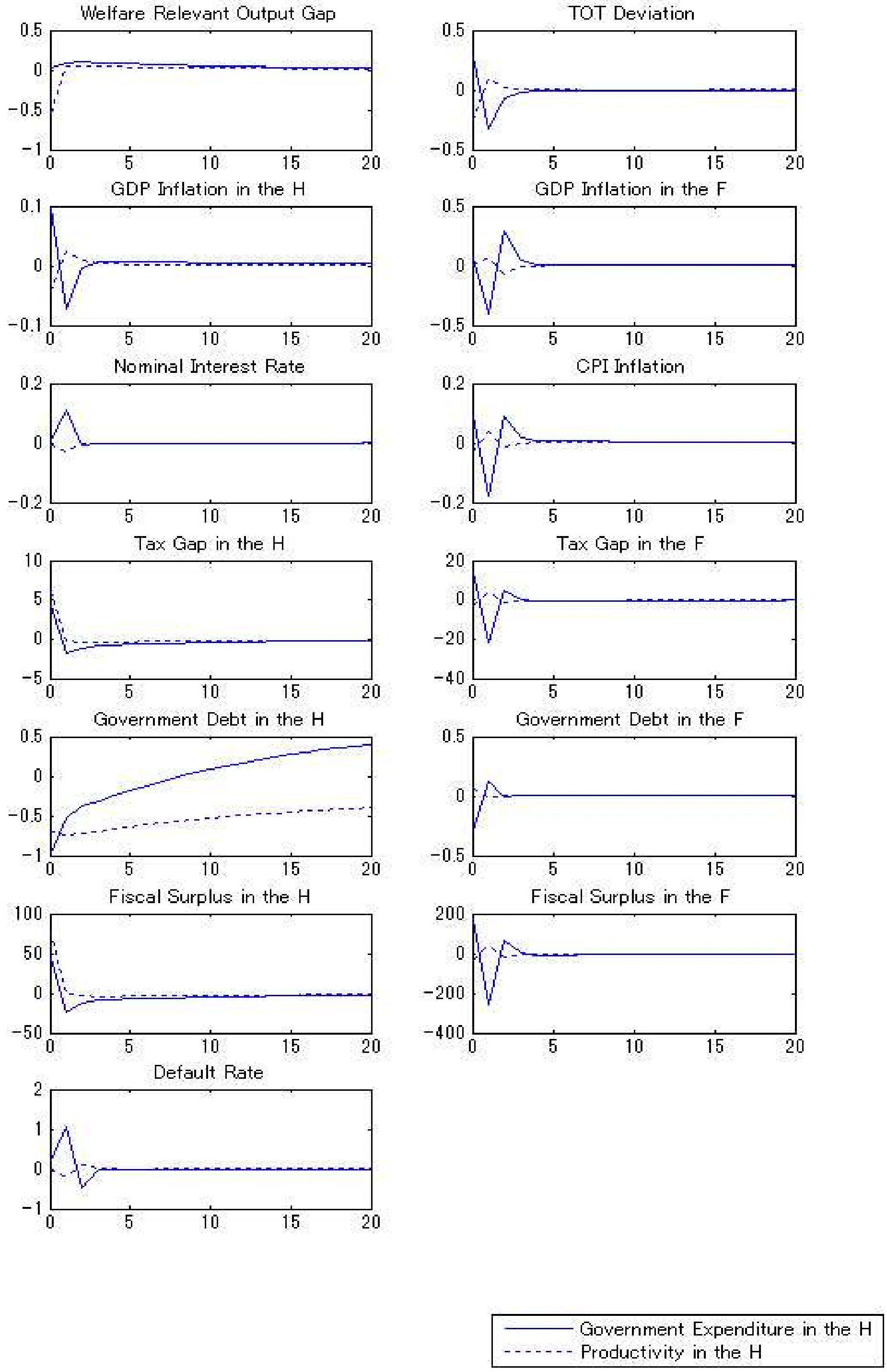


Figure 4: IRFs to Shocks in Country F under Taxation

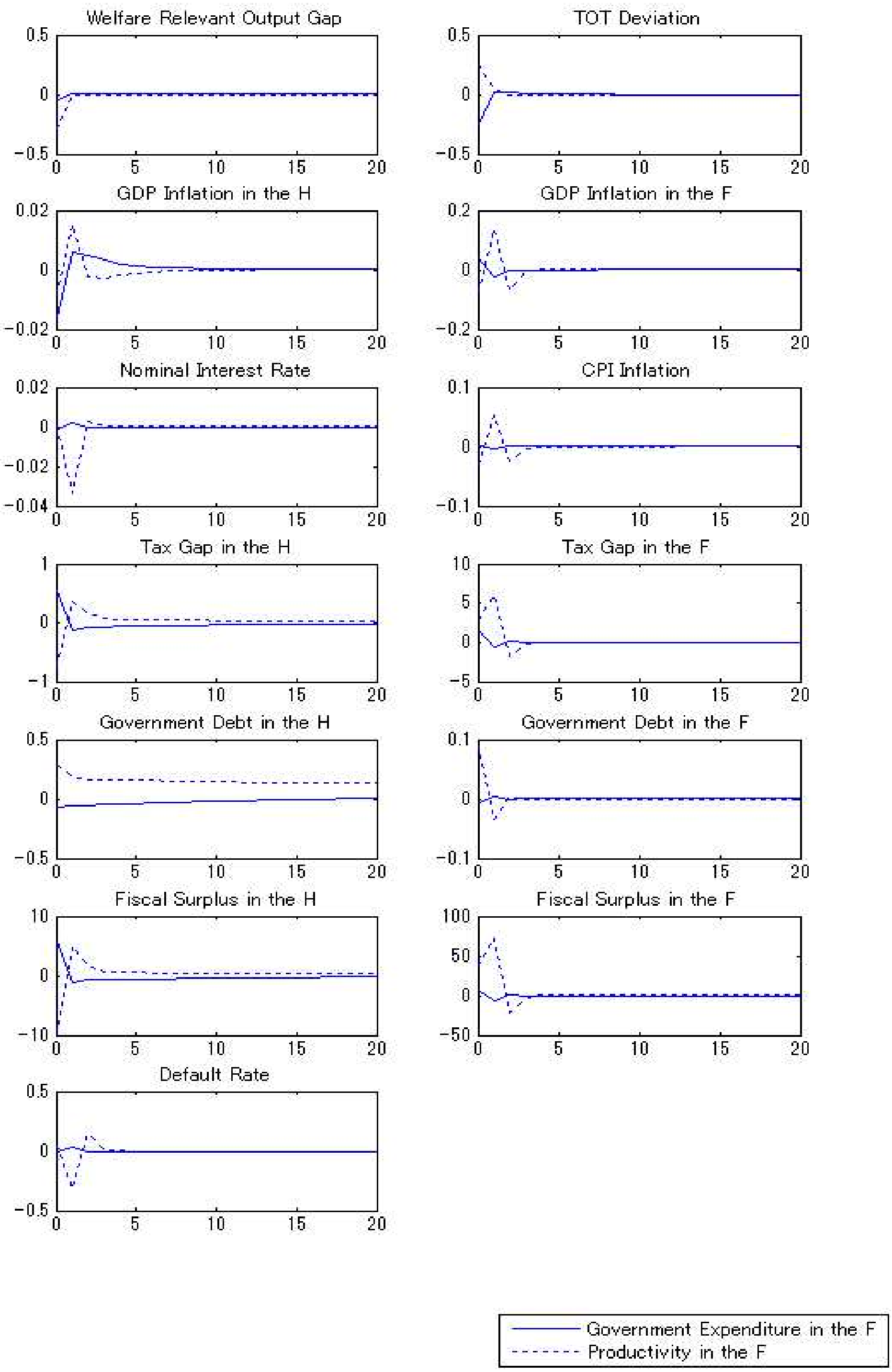


Figure 5: IRFs to Shocks in Country H under Non-taxation

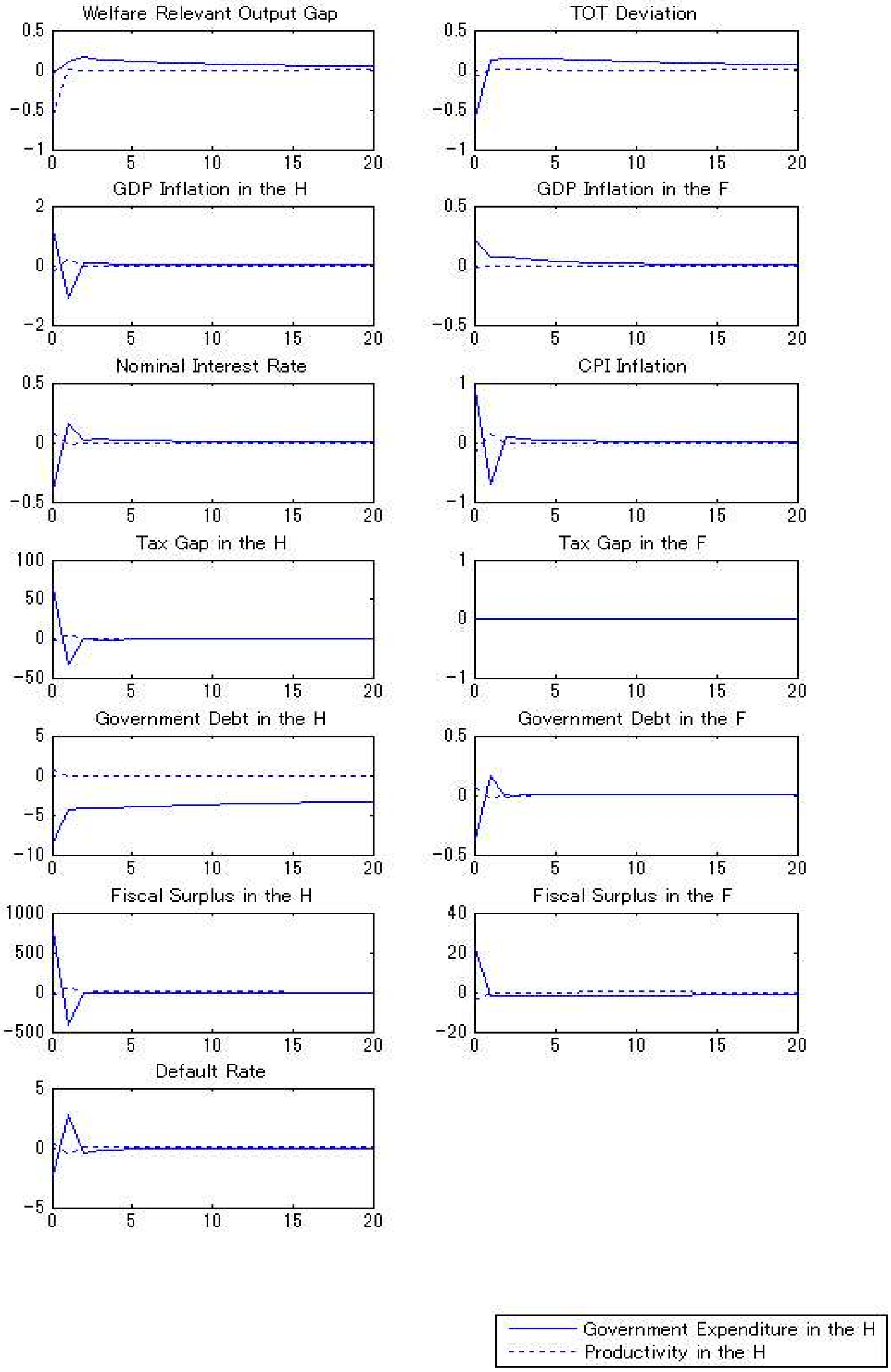


Figure 6: IRFs to Shocks in Country F under Non-taxation