An Estimated DSGE Model with a Deflation Steady State

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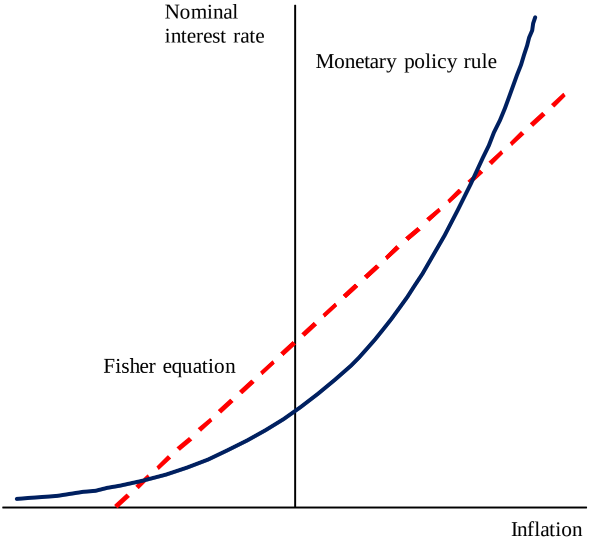
- An increased number of researchers have estimated New Keynesian monetary DSGE models.
  - A central bank follows a Taylor-type monetary policy rule.
    - The nominal interest rate is adjusted when inflation deviates from a given target.
  - The economy fluctuates around the steady state where actual inflation coincides with the targeted inflation.
- Benhabib, Schmitt-Grohé, and Uribe (2001) argue that there exists another steady state when the zero lower bound (ZLB) on the nominal interest rate is taken into account.
  - Called a deflation steady state, where the inflation rate is negative and the nominal interest rate is very close to zero.

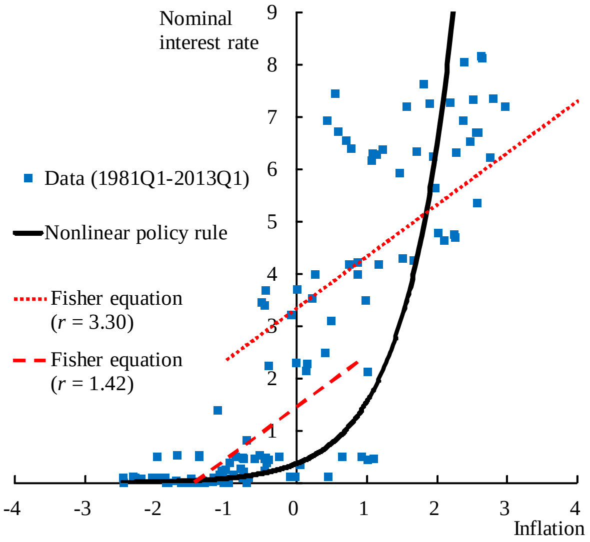
Two Steady States



Objective

- Estimate a DSGE model with a deflation steady state for the Japanese economy.
  - Existing studies have estimated DSGE models with a targeted-inflation steady state.
  - Motivated by Bullard (2010):
    - Points out the possibility that the Japanese economy has been stuck in a deflation equilibrium.

Interest rate and inflation in Japan



- Estimate a medium-scale DSGE model, along the lines of Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003, 2007), and Justiniano, Primiceri, and Tambalotti (2010).
- Approximated around the deflation steady state.
- Sample: 1999Q1 to 2013Q1 in Japan.
  - BOJ conducted the zero interest rate policy, with the exception of August 2000–March 2001 and July 2006–December 2008.
  - Inflation rate was almost always negative.

Strategy (cont.)

- Equilibrium is indeterminate around the deflation steady state.
  - i.e., there are an infinite number of equilibrium trajectories that converge to the deflation steady state.
  - Because of a passive monetary policy which is constrained by the ZLB on the nominal interest rate.
- Following Lubik and Schorfheide (2004), a set of specific equilibria is selected using Bayesian methods.

Main Findings

- Shocks to preferences, investment adjustment costs, and external demand do not necessarily have an inflationary effect.
  - In contrast to a standard model with a targeted-inflation steady state.
- Provides a novel view about the flattening of the short-run Phillips curve in Japan.
  - Argued by Nishizaki and Watanabe (2000) and De Veirman (2009).
    - Based on the estimation of reduced-form Phillips curves.
  - The slope of the Phillips curve itself does not become flat.
  - Rather, the ambiguity of the inflation responses leads to a weak comovement between inflation and output.

Main Findings (cont.)

- An economy in the deflation equilibrium could be unexpectedly volatile because of sunspot shocks.
- Show that the effect of sunspot shocks to Japan's business cycle fluctuations is quite marginal.
- Sunspot shocks contribute to stabilize the economy over the business cycles.
  - Macroeconomic stability during the zero interest period was a result of good luck.

Contributions

- The first benchmark model to empirically investigate the deflationary economy constrained by the zero lower bound.
  - cf. Sugo and Ueda (2007); Kaihatsu and Kurozumi (2010); Fueki, Fukunaga, Ichiue, and Shirota (2010); Hirakata, Sudo, and Ueda (2011); Iwata (2011); Hirose and Kurozumi (2012); Ichiue, Kurozumi, and Sunakawa (2013).
- Although our model does not consider the ZLB explicitly, the effect of ignoring it is mitigated around the deflation steady state.
  - The slopes of the monetary policy rule with respect to inflation and output are very flat.

- Contributes to the literature on the estimation of DSGE models under equilibrium indeterminacy.
  - There have been still few papers that estimate indeterminate models.
  - Exceptions: Hirose (2007, 2008, 2013); Belaygorod and Dueker (2009); Bhattarai, Lee, and Park (2012a, 2012b); Zheng and Guo (2013).
- The first empirical work that applies Lubik and Schorfheide's approach to the estimation of a medium-scale DSGE model.
  - Numerically computes a continuity solution proposed by Lubik and Schorfheide (2004).
    - Impulse responses of endogenous variables to fundamental shocks are continuous at the boundary between the determinacy and indeterminacy regions.

Related Literature

- The most closely related paper is Aruoba, Cuba-Borda, and Schorfheide (2013).
  - Consider Markov switching between the targeted-inflation and deflation steady state in a simple New Keynesian DSGE model.
  - Estimate whether the US and Japan have been in either the targeted-inflation or deflation regime.
- Find that Japan shifted into a deflation regime in 1999 and remained there since then.
  - Validates our assumption that Japan has been stuck in a deflation equilibrium during our sample period (1999–2013).
- Focus on the estimation of the timing of the regime change, given the parameters pre-estimated for the sample from 1981 to 1994.
  - We estimate parameters using data since 1999 and investigates the economic properties around the deflation steady state.

- A medium-scale DSGE model along the lines of Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003, 2007), and Justiniano, Primiceri, and Tambalotti (2010)

1. Households' preferences are specified as in Erceg Guerrieri, and Gust (2006), which ensures the existence of the balanced growth path under the CRRA utility function.
2. Following Greenwood, Hercowitz, and Huffman (1988), a higher utilization rate of capital leads to a higher depreciation rate of capital.
    - Supported by Sugo and Ueda (2007): Replicate a negative correlation between capital utilization and rental cost observed in the Japanese data.
3. The equilibrium conditions are approximated around the deflation steady state.

- Each household $h \in [0,1]$ maximizes the utility function

$$E_t \sum_{j=0}^{\infty} \beta^j e^{z^b_{t+j}} \left\{ \frac{\left(C_{t+j}(h) - \gamma C_{t+j-1}(h)\right)^{1-\sigma}}{1-\sigma} - \frac{Z_{t+j}^{1-\sigma} e^{z^l_{t+j}} l_{t+j}(h)^{1+\chi}}{1+\chi} \right\},$$

and the profit function

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{\Lambda(h)_{t+j}}{\Lambda(h)_t} \left(R^k_{t+j}(h) u_{t+j}(h) K_{t+j-1}(h) - I_{t+j}(h)\right).$$

  - As in Erceg Guerrieri, and Gust (2006), labor disutility includes $Z_t^{1-\sigma}$, which ensures the existence of the balanced growth path.

- Capital accumulation:

$$K_t(h) = \{1 - \delta(u_t(h))\} K_{t-1}(h) + \left\{1 - S\left(\frac{I_t(h)}{I_{t-1}(h)} \frac{e^{z_t^i}}{z}\right)\right\} I_t(h).$$

  - Following Greenwood, Hercowitz, and Huffman (1988), $\delta(\cdot)$ has the properties of $\delta' > 0$ and $\delta'' > 0$.

- Budget constraint:

$$\begin{aligned} & C_t(h) + I_t(h) + \frac{B_t(h)}{P_t} \\ = \quad & W_t(h) l_t(h) + R_t^k(h) u_t(h) K_{t-1}(h) + R_{t-1}^n \frac{B_{t-1}(h)}{P_t} + T_t(h). \end{aligned}$$

- In monopolistically competitive labor markets, nominal wages are set on a staggered basis à la Calvo (1983).
  - In each period, a fraction $1-\xi_w \in (0,1)$ of wages is reoptimized, while the remaining fraction ξ_w is set by indexation to the balanced growth rate z as well as a weighted average of past inflation π_{t-1} and steady-state inflation π.

$$\max_{W_t(h)} E_t \sum_{j=0}^{\infty} (\beta\xi_w)^j \left\{ \begin{array}{c} \Lambda_{t+j} l_{t+j|t}(h) \frac{P_t W_t(h)}{P_{t+j}} \prod_{k=1}^{j} \left(z\pi_{t+k-1}^{\gamma_w} \pi^{1-\gamma_w} \right) \\ -\frac{e^{z_{t+j}^b} Z_{t+j}^{1-\sigma} e^{z_{t+j}^l} l_{t+j|t}(h)^{1+\chi}}{1+\chi} \end{array} \right\},$$

subject to the labor demand function

$$l_{t+j|t}(h) = l_{t+j} \left\{ \frac{P_t W_t(h)}{P_{t+j} W_{t+j}} \prod_{k=1}^{j} \left(z\pi_{t+k-1}^{\gamma_w} \pi^{1-\gamma_w} \right) \right\}^{-\frac{1+\lambda_{t+j}^w}{\lambda_{t+j}^w}}.$$

- The representative final-good firm produces output Y_t under perfect competition by choosing a combination of intermediate inputs $\{Y_t(f)\}$, $f \in [0, 1]$ so as to maximize the profit

$$P_t Y_t - \int_0^1 P_t(f) Y_t(f) df,$$

subject to a CES production technology

$$Y_t = \left(\int_0^1 Y_t(f)^{1/(1+\lambda_t^p)} df \right)^{1+\lambda_t^p}.$$

- Market clearing condition for final good:

$$Y_t = C_t + I_t + g Z_t e^{z_t^g}.$$

- Each intermediate-good firm f produces one kind of differentiated goods $Y_t(f)$ by choosing a cost-minimizing pair of capital and labor services $\{u_t K_{t-1}(f), l_t(f)\}$ subject to the production function

$$Y_t(f) = (Z_t l_t(f))^{1-\alpha} (u_t K_{t-1}(f))^{\alpha} - \phi Z_t.$$

  - Technology level Z_t follows the the nonstationary stochastic process

$$\log Z_t = \log z + \log Z_{t-1} + z_t^z.$$

- Intermediate-good firms set prices of their products on a staggered basis à la Calvo (1983).
  - In each period, a fraction $1-\xi_p \in (0,1)$ of intermediate-good firms reoptimizes prices, while the remaining fraction ξ_p indexes prices to a weighted average of past and steady-state inflation.

$$\max_{P_t(f)} E_t \sum_{j=0}^{\infty} \xi_p^j \left(\beta^j \frac{\Lambda_{t+j}}{\Lambda_t}\right) \left\{ \frac{P_t(f)}{P_{t+j}} \prod_{k=1}^{j} \left(\pi_{t+k-1}^{\gamma_p} \pi^{1-\gamma_p}\right) - mc_{t+j} \right\} Y_{t+j|t}(f),$$

subject to the final-good firm's demand function

$$Y_{t+j|t}(f) = Y_{t+j} \left\{ \frac{P_t(f)}{P_{t+j}} \prod_{k=1}^{j} \left(\pi_{t+k-1}^{\gamma_p} \pi^{1-\gamma_p}\right) \right\}^{-\frac{1+\lambda_{t+j}^p}{\lambda_{t+j}^p}}.$$

- The central bank adjusts the nominal interest rate following a monetary policy rule

$$R_t^n = R^n\left(\pi_t, \frac{Y_t}{Z_t}, R_{t-1}^n, z_t^r\right).$$

  - The functional form of $R^n(\cdot)$ is not specified at this stage.
  - Three assumptions as in Benhabib, Schmitt-Grohé, and Uribe (2001):

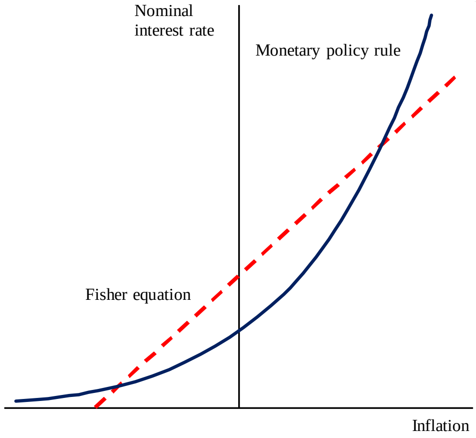
1. $\partial R^n / \partial \pi_t \geq 0$; $\partial R^n / \partial (Y_t / Z_t) \geq 0$; $\partial R^n / \partial R_{t-1}^n \geq 0$.
2. The ZLB constraint on the nominal interest rate: $R^n(\cdot) > 1$ for all $\{\pi_t, Y_t/Z_t, R_{t-1}^n, z_t^r\}$.
3. Around the inflation target, the monetary policy rule satisfies the Taylor principle.

Detrending

- Because the log level of technology has a unit root with drift, the equilibrium conditions are rewritten in terms of stationary variables detrended by Z_t:

$$y_t = Y_t/Z_t, c_t = C_t/Z_t, w_t = W_t/Z_t, \lambda_t = \Lambda_t Z_t^{\sigma}, i_t = I_t/Z_t, k_t = K_t/Z_t$$

- Then, we can compute the steady states for the detrended variables.



- The model is approximated around the deflation steady state.

$$\tilde{R}_t^n = \psi_r \tilde{R}_{t-1}^n + (1-\psi_r)\left(\psi_\pi \tilde{\pi}_t + \psi_y \tilde{y}_t\right) + z_t^r. \tag{1}$$

- Appears to be the same as a standard Taylor-type monetary policy rule.
- However, ψ_π and ψ_y are very small because of the ZLB.

  $\Longrightarrow$Does not satisfy the Taylor principle.

  $\Longrightarrow$Equilibrium indeterminacy
- Remark: Does not take account of the ZLB constraint explicitly.
  - However, the effect of ignoring the ZLB should be marginal near the deflation steady state.
    - The slopes of the monetary policy rule with respect to inflation and output are very flat.

Log-linearized Equilibrium Conditions

$$\left(1-\frac{\beta\gamma}{z^{\sigma}}\right)\tilde{\lambda}_t = -\frac{\sigma z}{z-\gamma}\left\{\tilde{c}_t - \frac{\gamma}{z}\left(\tilde{c}_{t-1} - z_t^z\right)\right\} + z_t^b$$

$$+\frac{\beta\gamma}{z^{\sigma}}\left\{\frac{\sigma z}{z-\gamma}\left(E_t\tilde{c}_{t+1} + E_t z_{t+1}^z - \frac{\gamma}{z}\tilde{c}_t\right) - E_t z_{t+1}^b\right\}, \quad (2)$$

$$\tilde{\lambda}_t = E_t\tilde{\lambda}_{t+1} - \sigma E_t z_{t+1}^z + \tilde{R}_t^n - E_t\tilde{\pi}_{t+1}, \quad (3)$$

$$\tilde{u}_t = \mu\left(\tilde{r}_t^k - \tilde{q}_t\right), \quad (4)$$

$$\frac{1}{\zeta}\left(\tilde{\imath}_t - \tilde{\imath}_{t-1} + z_t^z + z_t^i\right) = \tilde{q}_t + \frac{\beta z^{1-\sigma}}{\zeta}\left(E_t\tilde{\imath}_{t+1} - \tilde{\imath}_t + E_t z_{t+1}^z + E_t z_{t+1}^i\right), \quad (5)$$

$$\tilde{q}_t = E_t\tilde{\lambda}_{t+1} - \tilde{\lambda}_t - \sigma E_t z_{t+1}^z + \frac{\beta}{z^{\sigma}}\left\{R^k E_t\tilde{R}_{t+1}^k + (1-\delta)E_t\tilde{q}_{t+1}\right\}, \quad (6)$$

$$\tilde{k}_t = \frac{1-\delta}{z}\left(\tilde{k}_{t-1} - z_t^z\right) + \frac{R^k}{z}\tilde{u}_t + \left(1-\frac{1-\delta}{z}\right)\tilde{\imath}_t, \quad (7)$$

$$\begin{aligned} &\tilde{w}_t - \tilde{w}_{t-1} + \tilde{\pi}_t - \gamma_w \tilde{\pi}_{t-1} - z_t^z \\ &= \beta z^{1-\sigma} \left(E_t \tilde{w}_{t+1} - \tilde{w}_t + E_t \tilde{\pi}_{t+1} - \gamma_w \tilde{\pi}_t + E_t z_{t+1}^z \right) \\ &+ \frac{(1-\xi_w)(1-\xi_w \beta z^{1-\sigma})\lambda^w}{\xi_w \{\lambda^w + \chi(1+\lambda^w)\}} \left(\chi \tilde{l}_t - \tilde{\lambda}_t - \tilde{w}_t + z_t^b \right) + z_t^w, \end{aligned} \tag{8}$$

$$\tilde{y}_t = \frac{c}{y}\tilde{c}_t + \frac{i}{y}\tilde{\imath}_t + \frac{g}{y} z_t^g, \tag{9}$$

$$\tilde{mc}_t = (1-\alpha)\tilde{w}_t + \alpha \tilde{R}_t^k, \tag{10}$$

$$\tilde{w}_t - \tilde{R}_t^k = \tilde{u}_t + \tilde{k}_{t-1} - \tilde{l}_t - z_t^z, \tag{11}$$

$$\tilde{y}_t = (1+\lambda^p) \left\{ (1-\alpha)\tilde{l}_t + \alpha \left(\tilde{u}_t + \tilde{k}_{t-1} - z_t^z \right) \right\}, \tag{12}$$

$$\tilde{\pi}_t - \gamma_p \tilde{\pi}_{t-1} = \beta z^{1-\sigma} \left(E_t \tilde{\pi}_{t+1} - \gamma_p \tilde{\pi}_t \right) + \frac{(1-\xi_p)(1-\xi_p \beta z^{1-\sigma})}{\xi_p} \tilde{mc}_t + z_t^p. \tag{13}$$

- Seven fundamental shocks:
  1. z_t^z: Technology
  2. z_t^b: Preference
  3. z_t^i: Investment adjustment cost
  4. z_t^g: External demand
  5. z_t^w: Wage markup
  6. z_t^p: Price markup
  7. z_t^r: Monetary policy

- Each of the shocks follows the stationary AR(1) process:

$$z_t^x = \rho_x z_{t-1}^x + \varepsilon_t^x, \; x \in \{z, b, i, g, w, p, r\}. \tag{14}$$

  - $\varepsilon_t^x \sim i.i.d. N(0, \sigma_x^2)$

- Log-linearized system of equations:

$$\Gamma_0(\theta) s_t = \Gamma_1(\theta) s_{t-1} + \Psi_0(\theta) \varepsilon_t + \Pi_0(\theta) \eta_t. \tag{15}$$

- The full set of rational expectations solutions (Lubik and Schorfheide, 2003):

$$s_t = \Phi_1(\theta) s_{t-1} + \Phi_\varepsilon(\theta, \tilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t. \tag{16}$$

  - $\zeta_t \sim i.i.d.N(0, \sigma_\zeta^2)$: Sunspot shock
  - $\tilde{M}$: Arbitrary matrix
    - The model has multiple solutions, and different solutions exhibit different propagation of fundamental shocks.

Solution under Indeterminacy (cont.)

- Need to pin down $\tilde{M}$ to specify the law of motion for the endogenous variables under indeterminacy.
- Components of the arbitrary matrix $\tilde{M}$ are estimated using Bayesian methods, following Lubik and Schorfheide (2004).
  - Construct a prior distribution that is centered on a particular solution $M^*(\theta)$.
    - i.e., replace $\tilde{M}$ with $M^*(\theta) + M$ and set the prior mean for M equal to zero.

Particular Solutions

- Two particular solutions:
  1. Continuity solution: $M^*(\theta)$ is chosen such that $\partial s_t / \partial \varepsilon_t$ is continuous at the boundary between the determinacy and indeterminacy regions.
     - Proposed by Lubik and Schorfheide (2004).
  2. Orthogonality solution: The contributions of fundamental shocks ε_t and sunspot shocks ζ_t to the forecast errors η_t are orthogonal.
     - Obtained by setting $M^*(\theta) = 0$.
     - Often used in the literature because it can be directly obtained with the algorithm described in Sims (2002).
- Conduct Bayesian model comparison to investigate which particular solution is well fitted to the data.

- Bayesian estimation
- Data: log difference of real GDP, real consumption, real investment and real wage; the log of hours worked; the log difference of the GDP deflator; the overnight call rate.
- Sample period: 1999Q1–2013Q1
  - BOJ conducted the zero interest rate policy, with the exception of August 2000–March 2001 and July 2006–December 2008
  - Inflation rate was almost always negative.

Bayesian inference (cont.)

- Measurement equations:

$$\begin{bmatrix} 100\Delta \log Y_t \\ 100\Delta \log C_t \\ 100\Delta \log I_t \\ 100\Delta \log W_t \\ 100 \log l_t \\ 100\Delta \log P_t \\ 100 \log R_t^n \end{bmatrix} = \begin{bmatrix} \bar{z} \\ \bar{z} \\ \bar{z} \\ \bar{z} \\ \bar{l} \\ \bar{\pi} \\ \bar{r} + \bar{\pi} \end{bmatrix} + \begin{bmatrix} \tilde{y}_t - \tilde{y}_{t-1} + z_t^z \\ \tilde{c}_t - \tilde{c}_{t-1} + z_t^z \\ \tilde{\imath}_t - \tilde{\imath}_{t-1} + z_t^z \\ \tilde{w}_t - \tilde{w}_{t-1} + z_t^z \\ \tilde{l}_t \\ \tilde{\pi}_t \\ \tilde{R}_t^n \end{bmatrix}.$$

- Fixed parameters: $\delta = 0.06/4$; $\alpha = 0.37$; $\lambda^w = 0.2$; $g/y = 0.248$
- Priors: Justiniano, Primiceri, and Tambalotti (2010), Smets and Wouters (2007), and Sugo and Ueda (2008).
  - $\bar{z}, \bar{l}, \bar{\pi}, \bar{r}$: Centered at the sample mean.

Parameter	Distribution	Mean	S.D.
σ: Relative risk aversion	Gamma	1.500	0.375
γ: Habit persistence	Beta	0.500	0.100
χ: Inv. elasticity of labor supply	Gamma	2.000	0.750
$1/\zeta$: Elasticity of the investment adj. cost	Gamma	4.000	1.000
μ: Inv. elasticity of the utilization rate adj. cost	Gamma	1.000	0.500
γ_w: Wage indexation	Beta	0.500	0.150
ξ_w: Wage stickiness	Beta	0.660	0.100
γ_p: Price indexation	Beta	0.500	0.150
ξ_p: Price stickiness	Beta	0.660	0.100
λ_p: Steady-state price markup	Gamma	0.150	0.050
ψ_r: Interest rate smoothing	Beta	0.900	0.100
ψ_π: Policy response to inflation	Gamma	0.200	0.100
ψ_y: Policy response to output	Gamma	0.200	0.100
$\bar{z}$: Steady-state output growth rate	Normal	0.145	0.025
$\bar{l}$: Steady-state hours worked	Normal	0.000	0.050
$\bar{\pi}$: Steady-state inflation rate	Normal	-0.332	0.050
$\bar{r}$: Steady-state real interest rate	Normal	0.361	0.050
$\rho_z, \rho_b, \rho_i, \rho_g, \rho_w, \rho_p, \rho_r$: Persistence of shocks	Beta	0.500	0.150
$\sigma_z, \sigma_b, \sigma_i, \sigma_g, \sigma_w, \sigma_p, \sigma_r, \sigma_\varsigma$: S.D. of shocks	Inv. gamma	0.500	∞
$M_z, M_b, M_i, M_g, M_w, M_p, M_r$: Arbitrary parameters	Normal	0.000	0.500

Model Selection

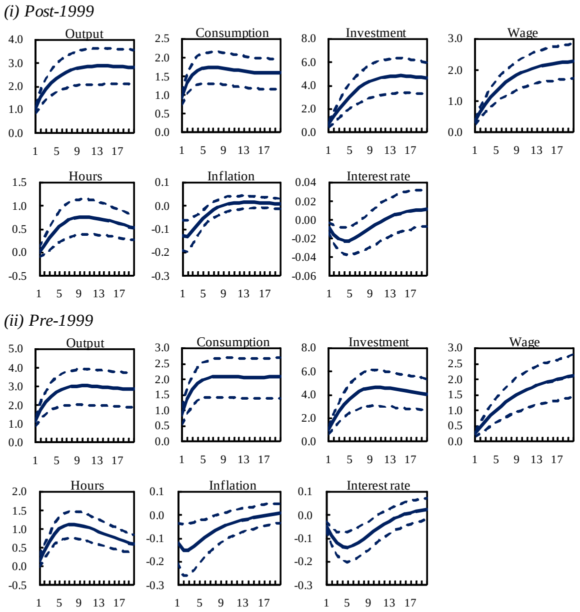
- The model is estimated based on two particular solutions:
  1. $\mathcal{M}_c$: Based on the continuity solution
  2. $\mathcal{M}_o$: Based on the orthogonality solution
- Investigate which solution is empirically more plausible by computing marginal data densities:
  1. $\log p\left(\mathcal{Y}^T|\mathcal{M}_c\right) = -371.8$
  2. $\log p\left(\mathcal{Y}^T|\mathcal{M}_o\right) = -373.4$
- Bayes factor: $\frac{p\left(\mathcal{Y}^T|\mathcal{M}_c\right)}{p(\mathcal{Y}^T|\mathcal{M}_o)} = 4.648$
  - According to Jeffreys (1961), interpreted as "substantial" evidence in favor of the continuity solution.

	Post-1999 (Continuity)		Pre-1999	
Parameter	Mean	90% interval	Mean	90% interval
σ	0.736	[0.528, 0.940]	1.833	[1.232, 2.410]
γ	0.351	[0.244, 0.461]	0.620	[0.494, 0.752]
χ	1.889	[0.790, 2.923]	3.006	[1.743, 4.249]
$1/\zeta$	4.873	[3.106, 6.527]	4.587	[2.925, 6.235]
μ	2.430	[1.227, 3.544]	1.599	[0.997, 2.223]
γ_w	0.286	[0.136, 0.430]	0.327	[0.168, 0.487]
ξ_w	0.732	[0.636, 0.829]	0.857	[0.808, 0.902]
γ_p	0.234	[0.070, 0.387]	0.377	[0.120, 0.648]
ξ_p	0.846	[0.783, 0.910]	0.881	[0.817, 0.946]
λ_p	0.204	[0.098, 0.304]	0.165	[0.089, 0.245]
ψ_r	0.824	[0.726, 0.921]	0.897	[0.861, 0.932]
ψ_π	0.089	[0.020, 0.155]	1.298	[1.075, 1.514]
ψ_y	0.066	[0.014, 0.117]	0.444	[0.236, 0.649]
$\bar{z}$	0.136	[0.097, 0.175]	0.462	[0.424, 0.500]
$\bar{l}$	-0.004	[-0.084, 0.079]	1.168	[1.085, 1.249]
$\bar{\pi}$	-0.312	[-0.386, -0.237]	0.195	[0.118, 0.270]
$\bar{r}$	0.423	[0.352, 0.493]	0.837	[0.758, 0.913]

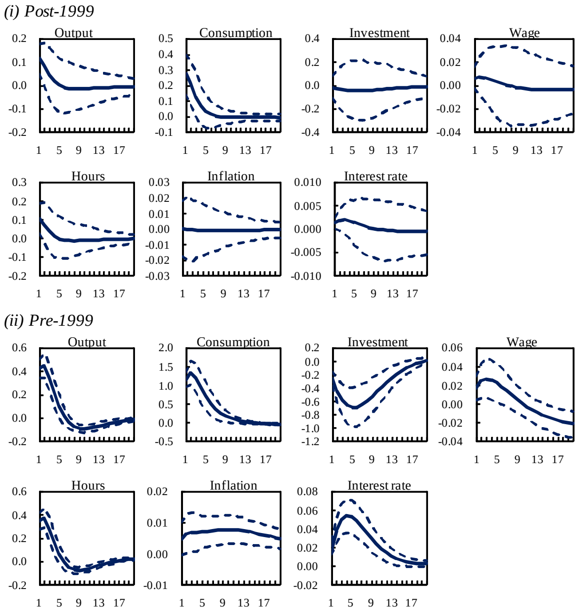
Parameter Estimates (cont.)

Parameter	Post-1999 (Continuity)		Pre-1999	
	Mean	90% interval	Mean	90% interval
ρ_z	0.359	[0.229, 0.483]	0.321	[0.182, 0.459]
ρ_b	0.448	[0.217, 0.683]	0.576	[0.378, 0.773]
ρ_i	0.368	[0.205, 0.525]	0.507	[0.398, 0.613]
ρ_g	0.856	[0.794, 0.921]	0.937	[0.908, 0.969]
ρ_w	0.228	[0.083, 0.359]	0.169	[0.058, 0.276]
ρ_p	0.294	[0.111, 0.468]	0.470	[0.243, 0.702]
ρ_r	0.393	[0.195, 0.585]	0.326	[0.178, 0.479]
σ_z	1.662	[1.346, 1.948]	1.805	[1.459, 2.164]
σ_b	0.339	[0.157, 0.528]	5.977	[3.673, 8.251]
σ_i	4.155	[3.411, 4.894]	5.853	[4.448, 7.234]
σ_g	3.509	[2.936, 4.073]	3.095	[2.588, 3.597]
σ_w	0.333	[0.267, 0.400]	0.397	[0.329, 0.467]
σ_p	0.434	[0.325, 0.543]	0.345	[0.220, 0.472]
σ_r	0.057	[0.047, 0.065]	0.120	[0.100, 0.141]
σ_ζ	0.403	[0.165, 0.628]	-	-
M_z	-0.665	[-1.085, -0.200]	-	-
M_b	0.012	[-0.685, 0.720]	-	-
M_i	0.011	[-0.122, 0.141]	-	-
M_g	-0.077	[-0.156, 0.006]	-	-
M_w	-0.546	[-1.025, -0.078]	-	-
M_p	-0.599	[-1.042, -0.118]	-	-
M_r	0.032	[-0.775, 0.866]	-	-

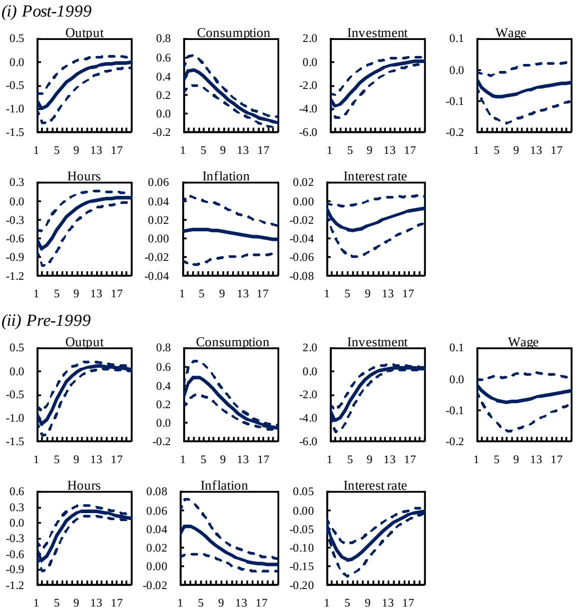
Impulse Responses to Technology Shock



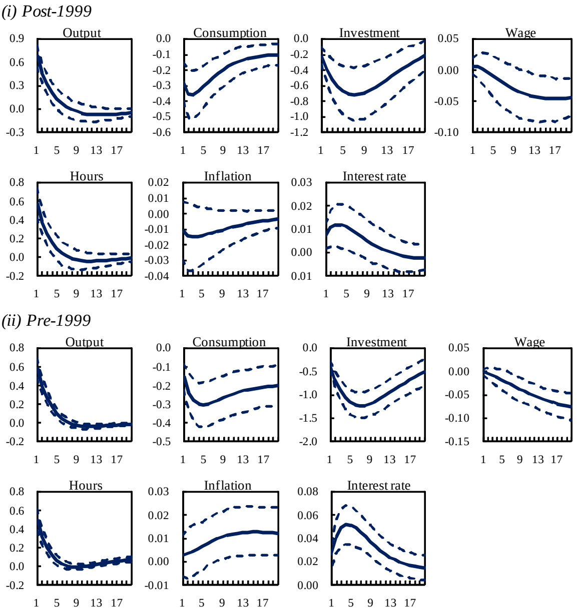
Impulse Responses to Preference Shock



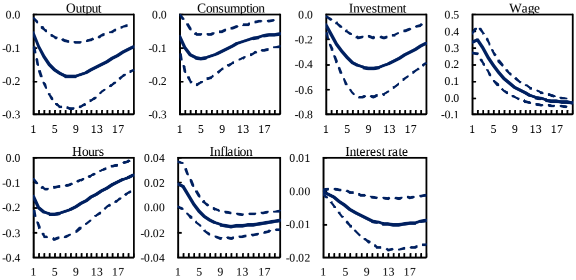
Impulse Responses to Investment Adjustment Cost Shock



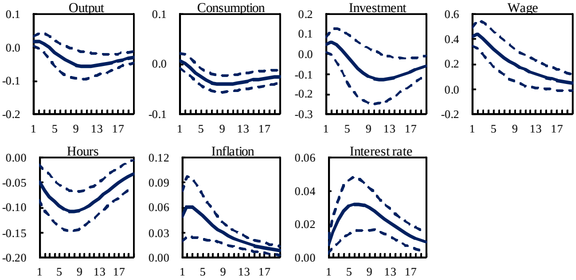
Impulse Responses to External Demand Shock



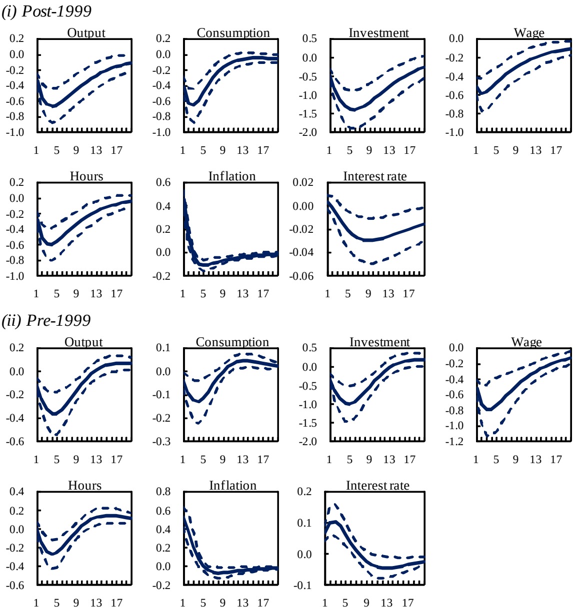
(i) Post-1999



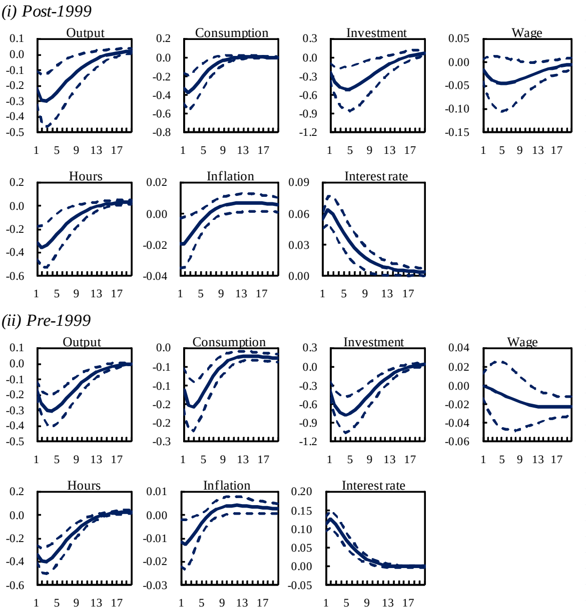
(ii) Pre-1999

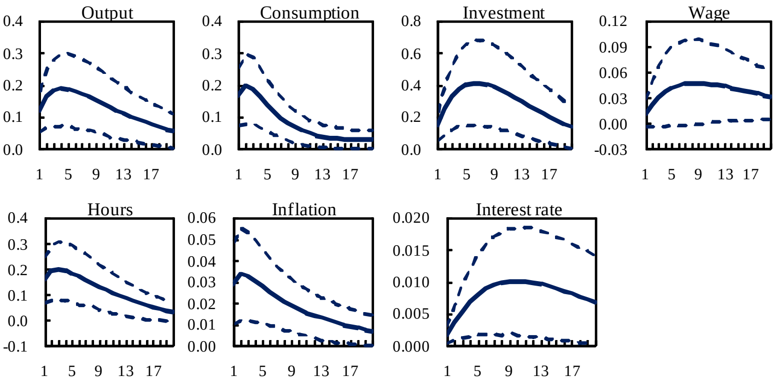


Impulse Responses to Price Markup Shock



Impulse Responses to Monetary Policy Shock





- Sunspot shock has positive effects on expectational variables.
- Such nonfundamental beliefs are self-fulfilling under indeterminacy.

Remarkable Changes in Impulse Responses

- Remarkable changes are found in the responses to the shocks about preferences, investment adj. costs, and external demand.
  - Pre-1999: these shocks have an inflationary effect.
  - Post-1999: the effect on inflation is ambiguous.
- Why inflation can both decrease and increase in response to these shocks?
  - Technically, it comes from the estimated arbitrary matrix M and its parameter uncertainty.

Intuition:

1. Initially, these shocks would have a positive effect on π_t.
2. The central bank would raise R_t^n following a monetary policy rule.
    - Around the deflation steady state, the policy rule does not satisfy the Taylor principle due to the ZLB.$\Rightarrow$An increase in R_t^n is limited.
3. R_t would decrease, which would stimulate demand for goods.
4. π_t would increase.

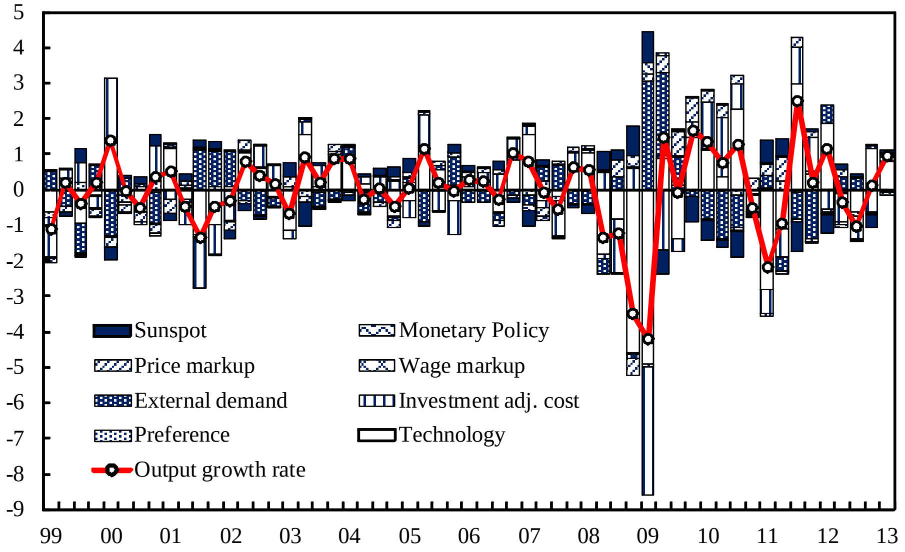
- A loop 2–4 can make the inflation trajectory explosive, which cannot be an equilibrium.
    - Therefore, inflation must decrease in this case.
- If the initial inflationary effect is moderate, the loop does not necessarily give rise to an explosive path.
    - In such a case, an increase in inflation can be an equilibrium.

Flattening of Japan's Phillips curve

- The finding about the changes in inflation responses provides a novel view about the flattening of Japan's short-run Phillips curve.
  - Nishizaki and Watanabe (2000): Japan's Phillips curve became flatter as the inflation rate approached zero.
  - De Veirman (2009): Provides evidence of a gradual flattening of the Phillips curve since the late 1990s.
- Our analysis provides a structural interpretation for their arguments.
  - The slope itself did not become flat.
    - $(1-\xi_p)(1-\xi_p\beta z^{1-\sigma})/\xi_p = 0.028$ for the post-1999 sample and 0.017 for the pre-1999 sample
  - Rather, the ambiguity of the inflation responses leads to a weak comovement between inflation and output.
    - Can be identified as a flattening of the Phillips curve in the estimation of reduced-form equations.

Variance Decompositions

	$\Delta \log Y_t$	$\Delta \log C_t$	$\Delta \log I_t$	$\Delta \log W_t$	$\log l_t$	$\Delta \log P_t$	$\log R_t^n$
Post-1999							
Technology	47.9	56.1	16.5	50.9	58.7	18.5	16.0
Preference	0.6	5.7	0.1	0.0	0.3	0.3	0.4
Investment adj. cost	25.5	8.7	76.8	0.3	14.5	2.0	16.3
External demand	17.0	4.9	1.0	0.1	3.8	0.9	2.1
Wage markup	0.3	0.4	0.3	14.6	3.5	1.3	3.6
Price markup	6.0	14.6	4.1	33.8	13.4	73.9	20.0
Monetary policy	2.1	7.6	0.9	0.2	3.8	0.6	38.4
Sunspot	0.6	1.9	0.4	0.1	2.1	2.6	3.2
Pre-1999							
Technology	50.7	38.6	16.9	35.8	79.2	23.8	40.5
Preference	6.3	54.5	1.0	0.1	1.8	0.3	3.7
Investment adj. cost	29.2	4.6	76.1	0.2	8.3	1.6	24.4
External demand	10.8	1.3	2.1	0.1	4.1	1.4	6.5
Wage markup	0.0	0.0	0.1	22.6	0.6	3.3	2.1
Price markup	1.7	0.4	2.4	41.3	2.4	69.5	11.3
Monetary policy	1.3	0.6	1.5	0.0	3.6	0.1	11.5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Sunspot
Monetary Policy
Price markup
Wage markup
External demand
Investment adj. cost
Preference
Technology
Output growth rate

	$\Delta \log Y_t$	$\Delta \log C_t$	$\Delta \log I_t$	$\Delta \log W_t$	$\log l_t$	$\Delta \log P_t$	$\log R_t^n$
Post-1999							
Technology	47.9	56.1	16.5	50.9	58.7	18.5	16.0
Preference	0.6	5.7	0.1	0.0	0.3	0.3	0.4
Investment adj. cost	25.5	8.7	76.8	0.3	14.5	2.0	16.3
External demand	17.0	4.9	1.0	0.1	3.8	0.9	2.1
Wage markup	0.3	0.4	0.3	14.6	3.5	1.3	3.6
Price markup	6.0	14.6	4.1	33.8	13.4	73.9	20.0
Monetary policy	2.1	7.6	0.9	0.2	3.8	0.6	38.4
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Pre-1999							
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Wage markup	0.0	0.0	0.1	22.6	0.6	3.3	2.1
Price markup	1.7	0.4	2.4	41.3	2.4	69.5	11.3
Monetary policy	1.3	0.6	1.5	0.0	3.6	0.1	11.5

Concluding Remarks

- Estimated a medium-scale DSGE model with a deflation steady state for the Japanese economy.
  - A specific equilibrium path is selected by extending the Bayesian methods developed by Lubik and Schorfheide (2004).
- According to the estimated model, the shocks to preferences, investment adj. costs, and external demand do not necessarily have an inflationary effect.
  - Provides a structural interpretation about the flattening of the short-run Phillips curve in Japan.
- Japan's business cycles are mainly driven by the shocks about technology, investment adj. costs, and external demand.
  - The effect of sunspot shocks turns out to be very small.
  - Rather, the sunspot shocks helped to stabilize the economy during the period.

- Our analysis assumes that the Japanese economy has been stuck in the deflation equilibrium since 1999.
- However, Japan will possibly return to the targeted-inflation steady state at some time in the future.
- To consider such a steady-state change, regime switching between the two steady states, as in Aruoba, Cuba-Borda, and Schorfheide (2013), must be incorporated.
  - Bianchi (2013) estimates a DSGE model switching between determinacy and indeterminacy regimes using a particular solution proposed by Farmer, Waggoner, and Zha (2011).
  - Farmer, Waggoner, and Zha (2009) provide a sunspot solution for indeterminate equilibria in Markov switching RE models.